

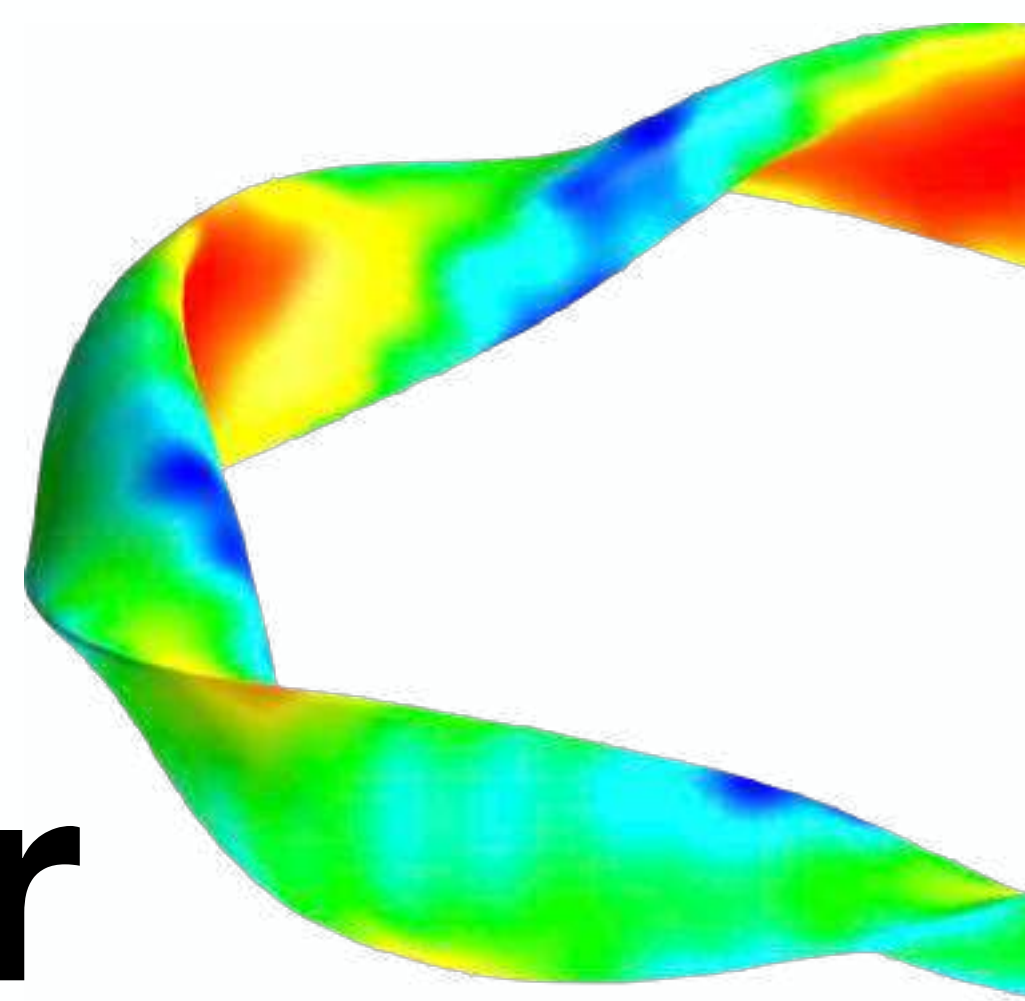
# Geometric Considerations for Zonal Flow Activity in Stellarators

Toward Improved Transport Modeling

Carlos D. Mora Moreno<sup>1</sup>, J.H.E Proll<sup>1</sup>, G.G. Plunk<sup>2</sup>, P. Xanthopoulos<sup>2</sup>,  
Yu. Turkin<sup>2</sup>, J. Geiger<sup>2</sup>

<sup>1</sup>Eindhoven University of Technology, Eindhoven, The Netherlands

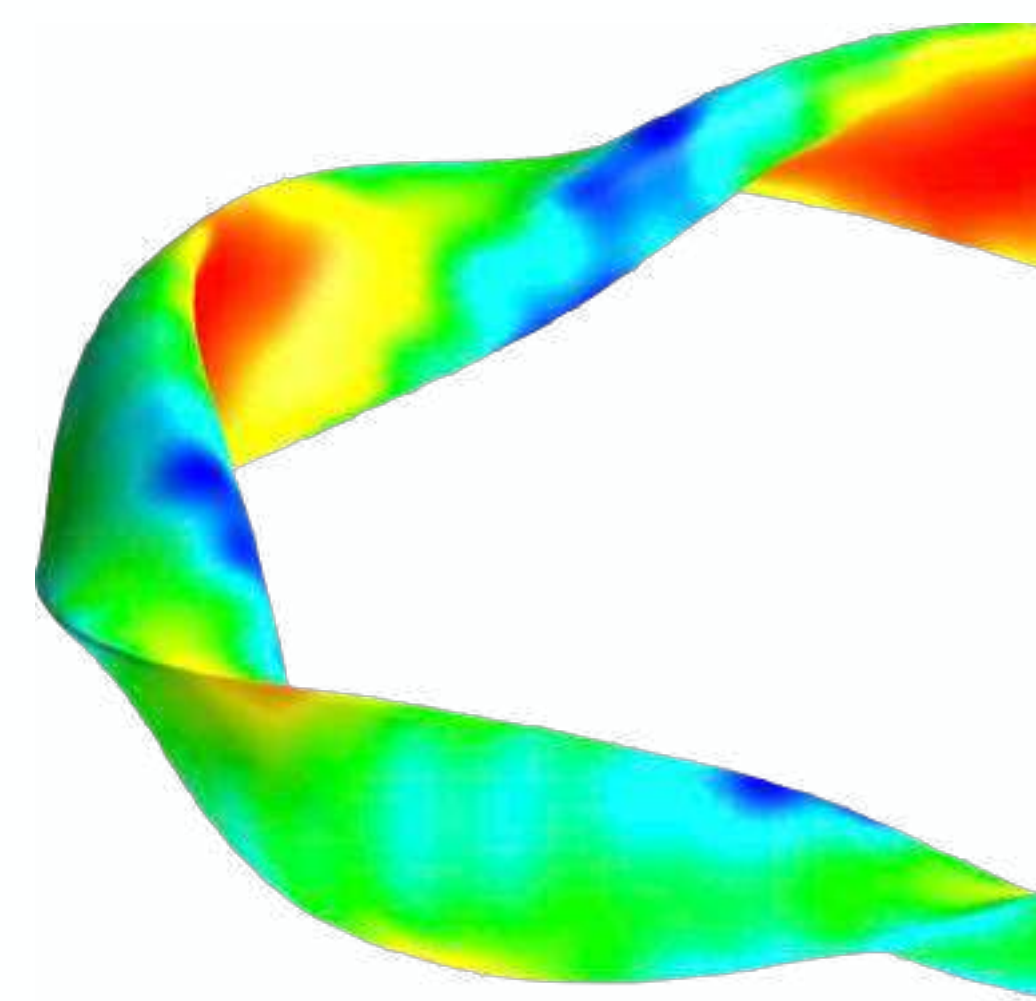
<sup>2</sup>Max-Planck Institute for Plasma Physics, Greifswald, Germany



# Understanding turbulence dynamics

- Turbulence dominates transport.<sup>1</sup>
- Saturation mechanisms?
- Stellarator geometry → decisive.
- Simulations: GENE code.<sup>2,3</sup>
  - ITG, adiabatic electron response.
  - Local: Flux tube domain.
- Craft a predictive model:

$$\chi_i = f(\textit{Geometry}, \textit{Linear physics})$$

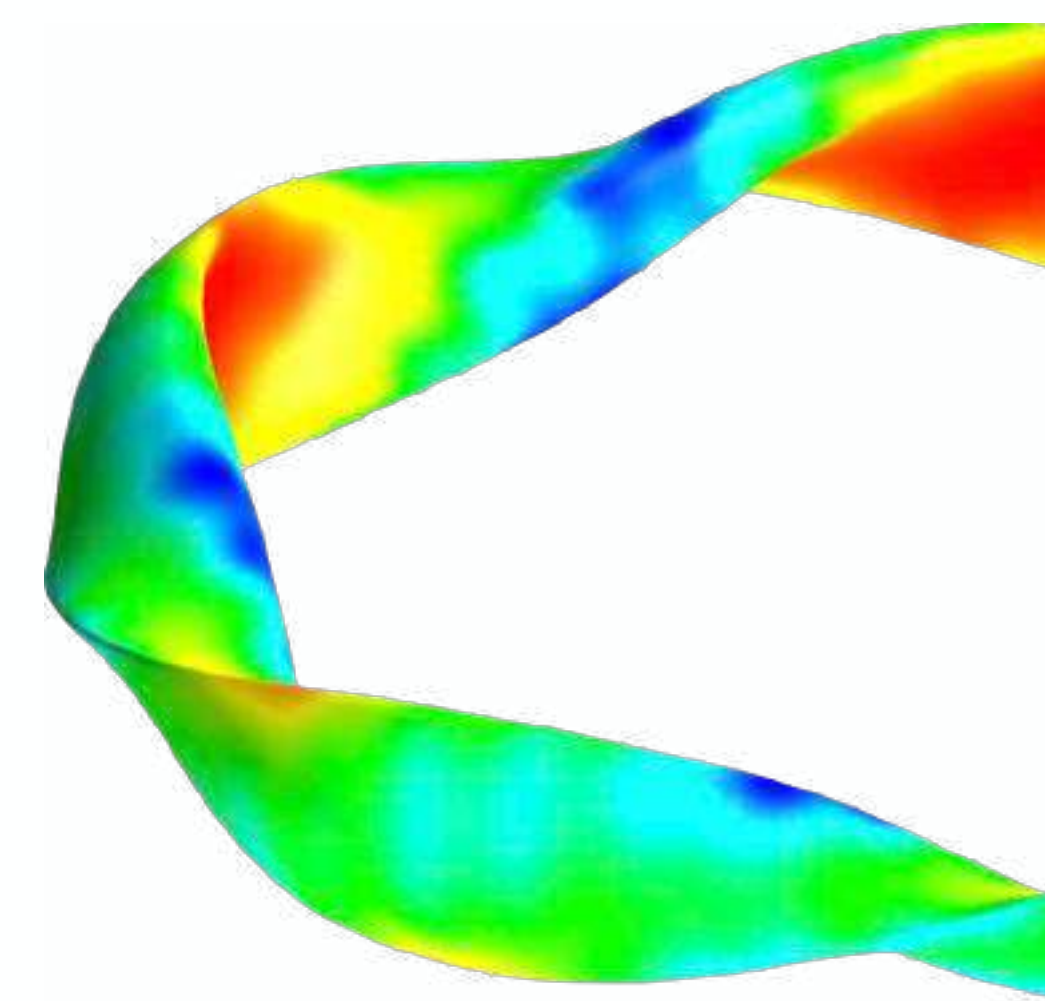


GENE

<sup>1</sup> J.A. Alcusón et al, Proceedings of 45th EPS Conference on Plasma Physics, (Vol. 42A, pp. 841-844). [P2.1088] European Physical Society.

<sup>2</sup> F. Jenko et al, Physics of Plasmas 7, 1904 (2000).

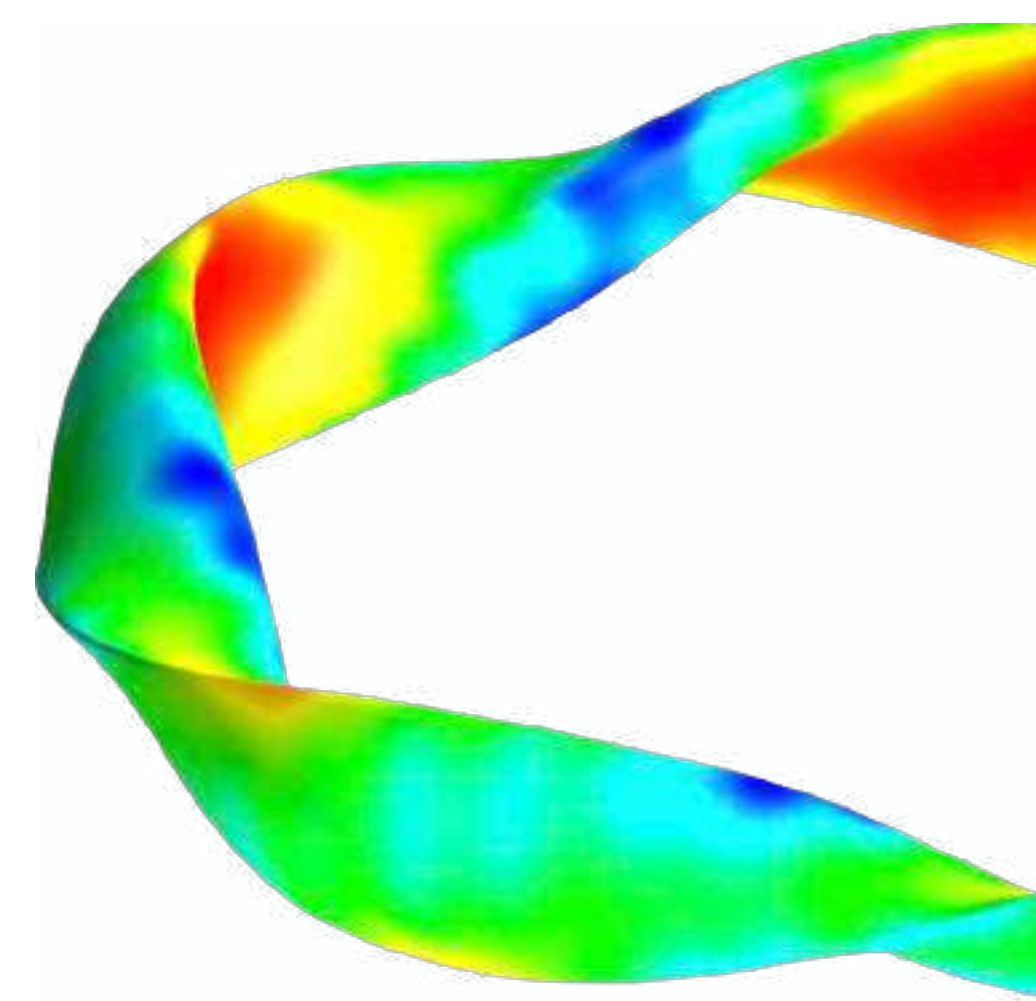
<sup>3</sup> See: <http://www.genecode.org> for details



**What are the geometric features  
that influence saturation?**

# Outline

Geometry in Wendelstein 7-X  
Gyrokinetic turbulence modeling  
Reduced transport modeling  
Summary



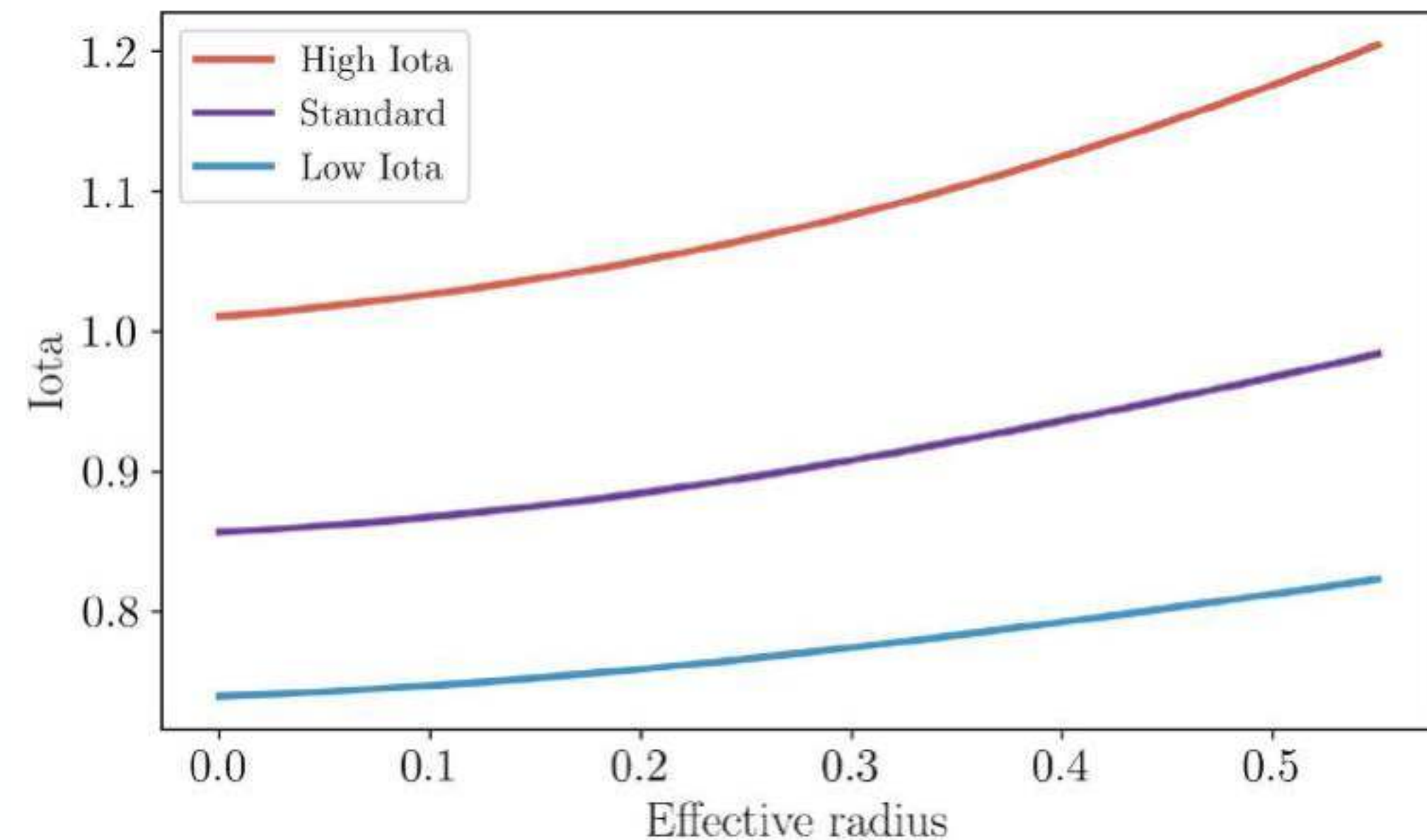
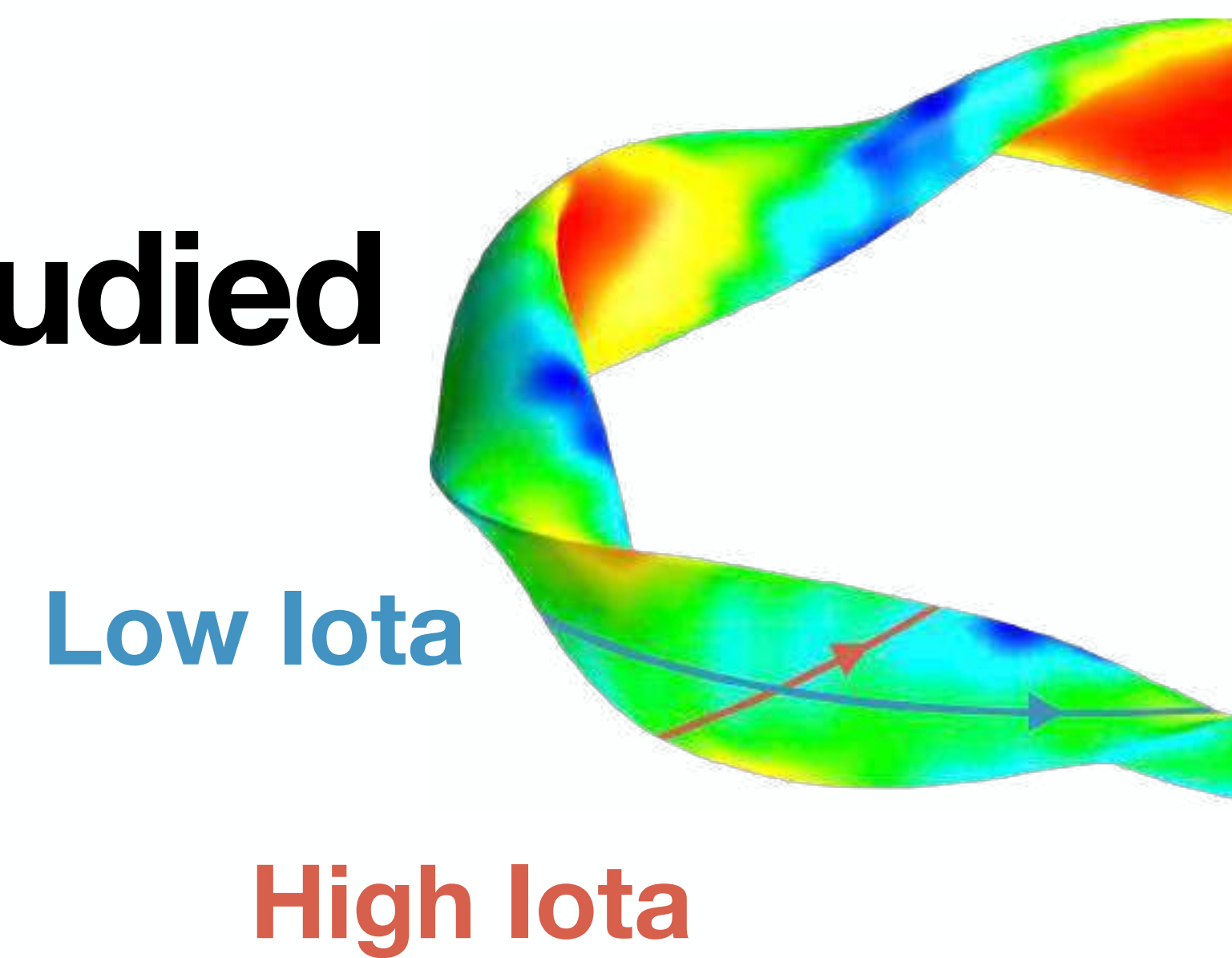
# Configurations of Wendelstein 7-X studied

- Experimental flexibility allows:

- High Iota
- Standard configuration
- Low Iota

where:  $\frac{\iota}{2\pi} = \frac{\text{poloidal turns}}{\text{toroidal turns}} = \frac{1}{q}$

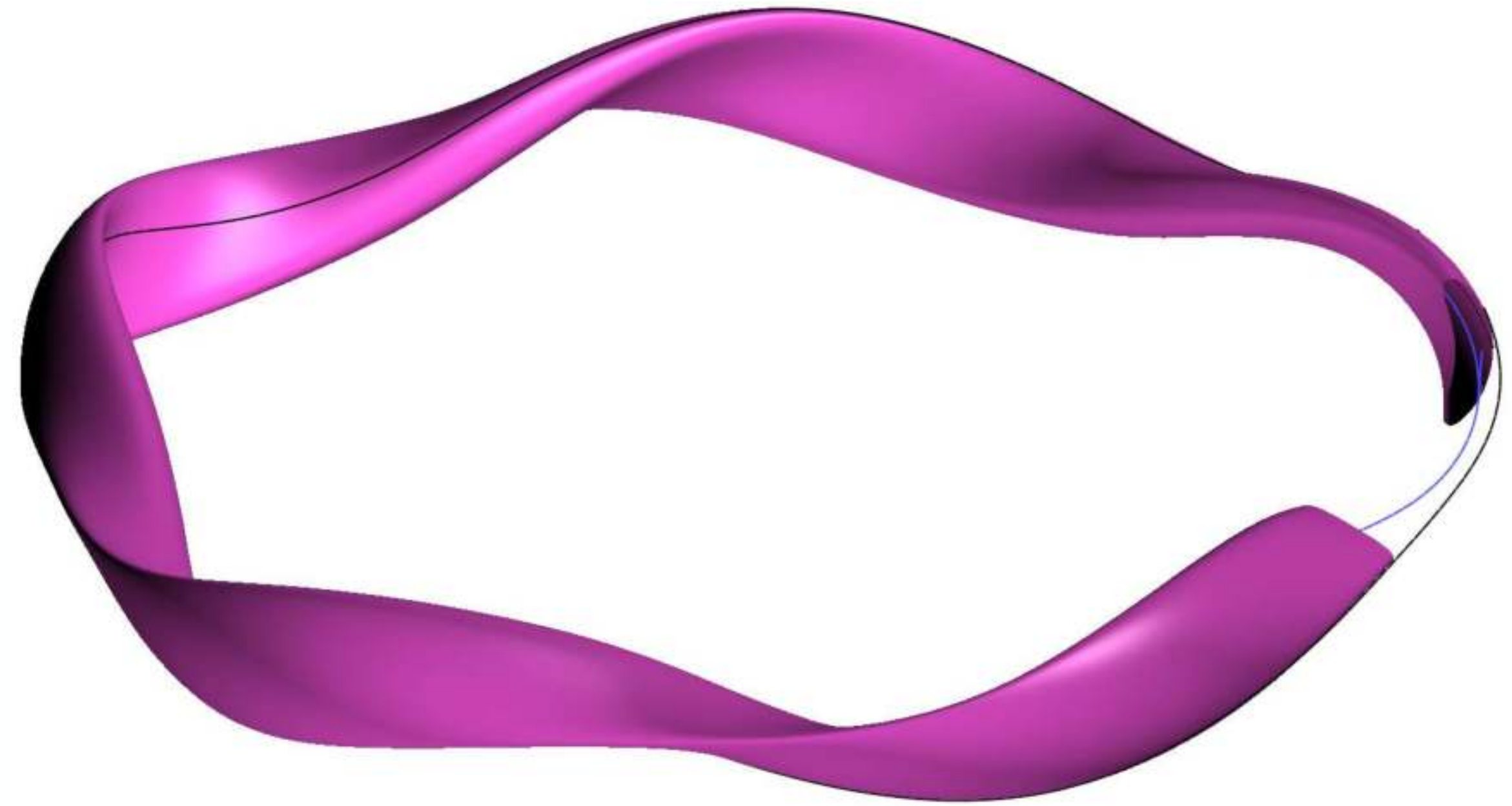
- Impact sample frequency of:
  - Good / bad curvature
  - Periods of geodesic curvature



# Curvature determines turbulence evolution

Curvature definition:

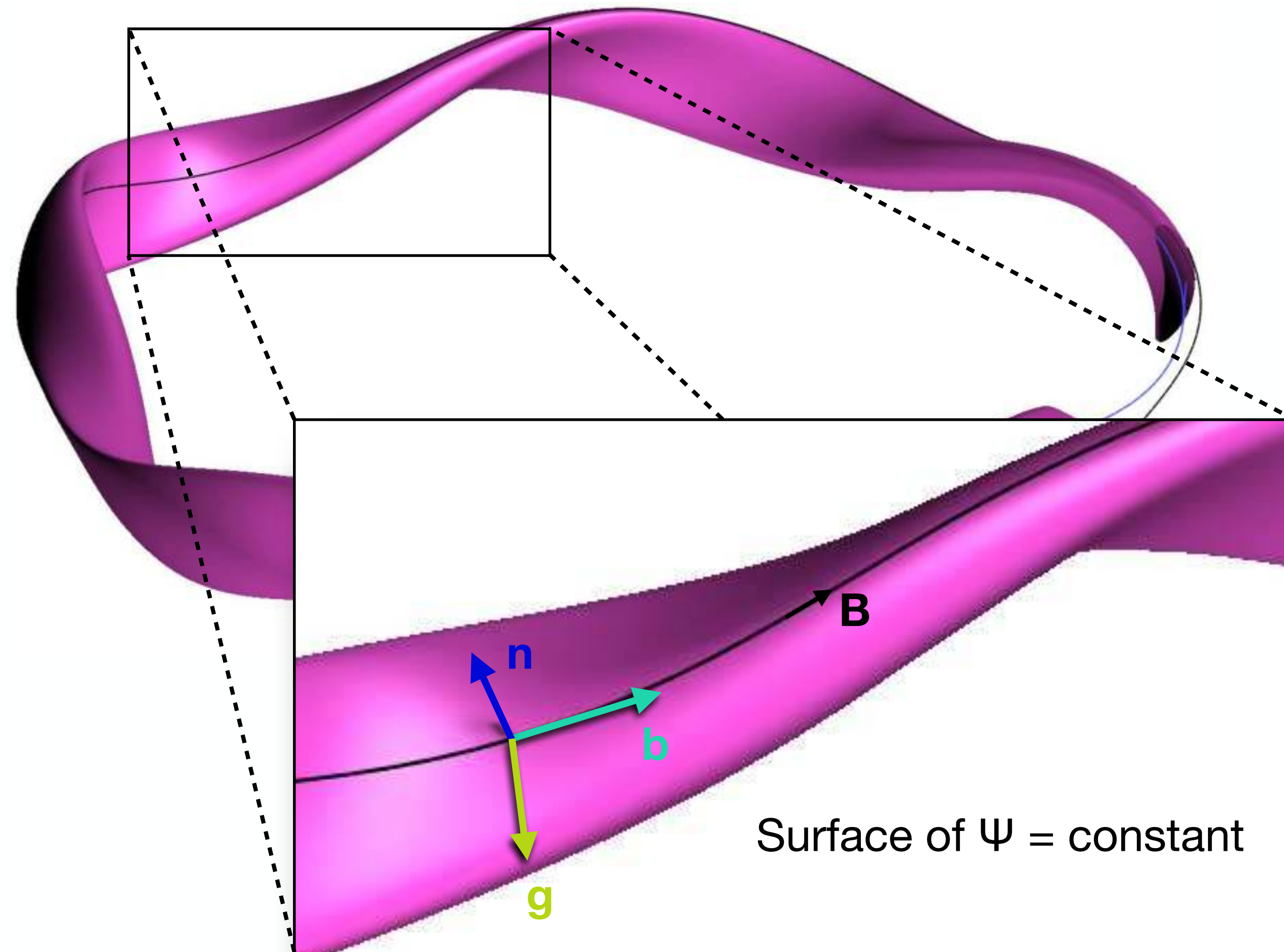
$$\kappa \equiv \mathbf{b} \cdot \nabla \mathbf{b} = \kappa_N + \kappa_G$$



# Curvature determines turbulence evolution

Curvature definition:

$$\kappa \equiv \mathbf{b} \cdot \nabla \mathbf{b} = \kappa_N + \kappa_G$$



# Curvature determines turbulence evolution

Curvature definition:

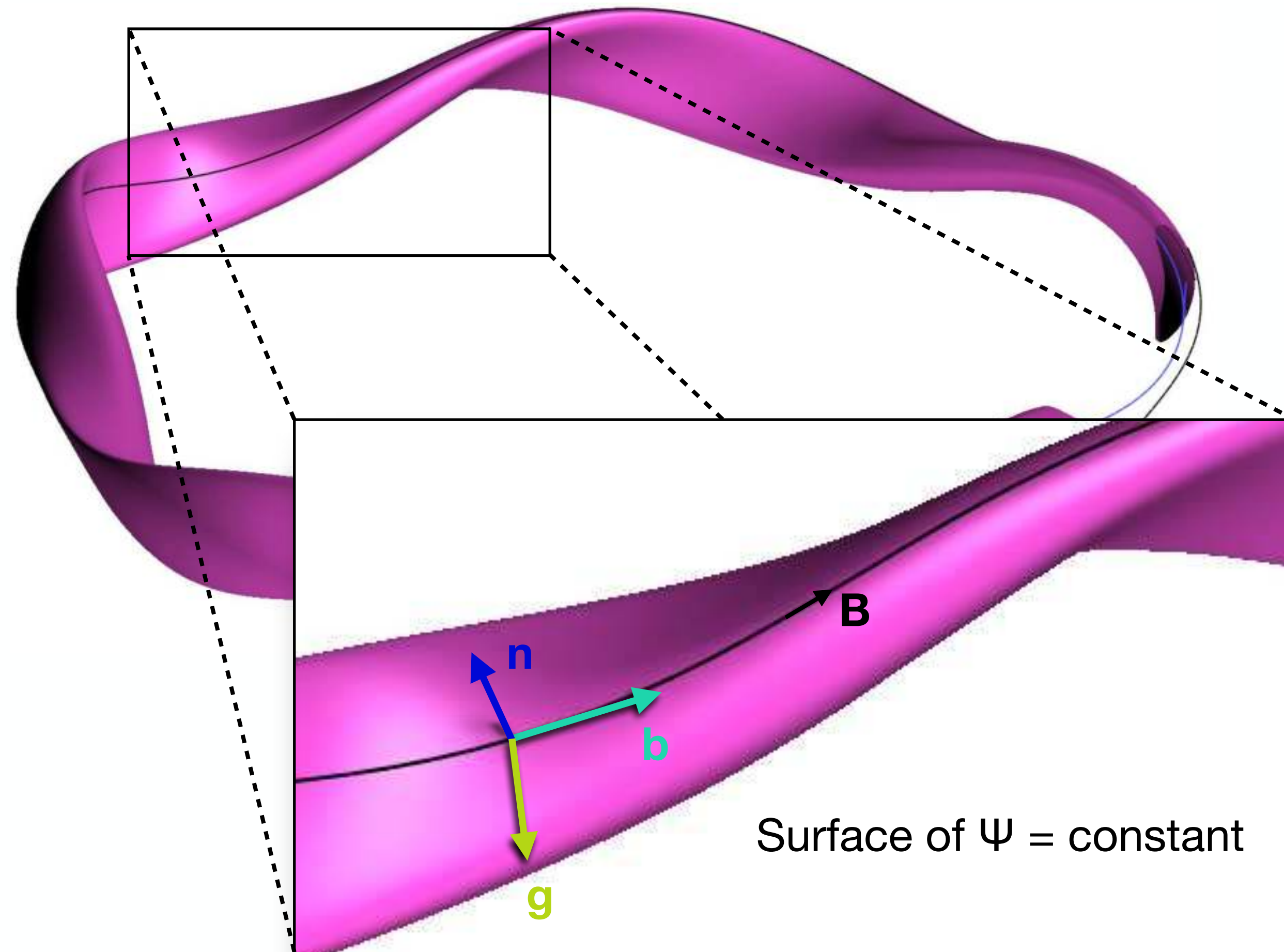
$$\kappa \equiv \mathbf{b} \cdot \nabla \mathbf{b} = \kappa_N + \kappa_G$$

where:

$$\kappa_N = \kappa \cdot \frac{\mathbf{n}}{|\mathbf{n}|}, \quad \kappa_G = \kappa \cdot \frac{\mathbf{g}}{|\mathbf{g}|}$$

and:

$$\mathbf{n} \equiv \nabla \Psi, \quad \mathbf{g} \equiv (\mathbf{B} \times \nabla \Psi)$$





# Curvature features in Wendelstein 7-X

Curvature definition:

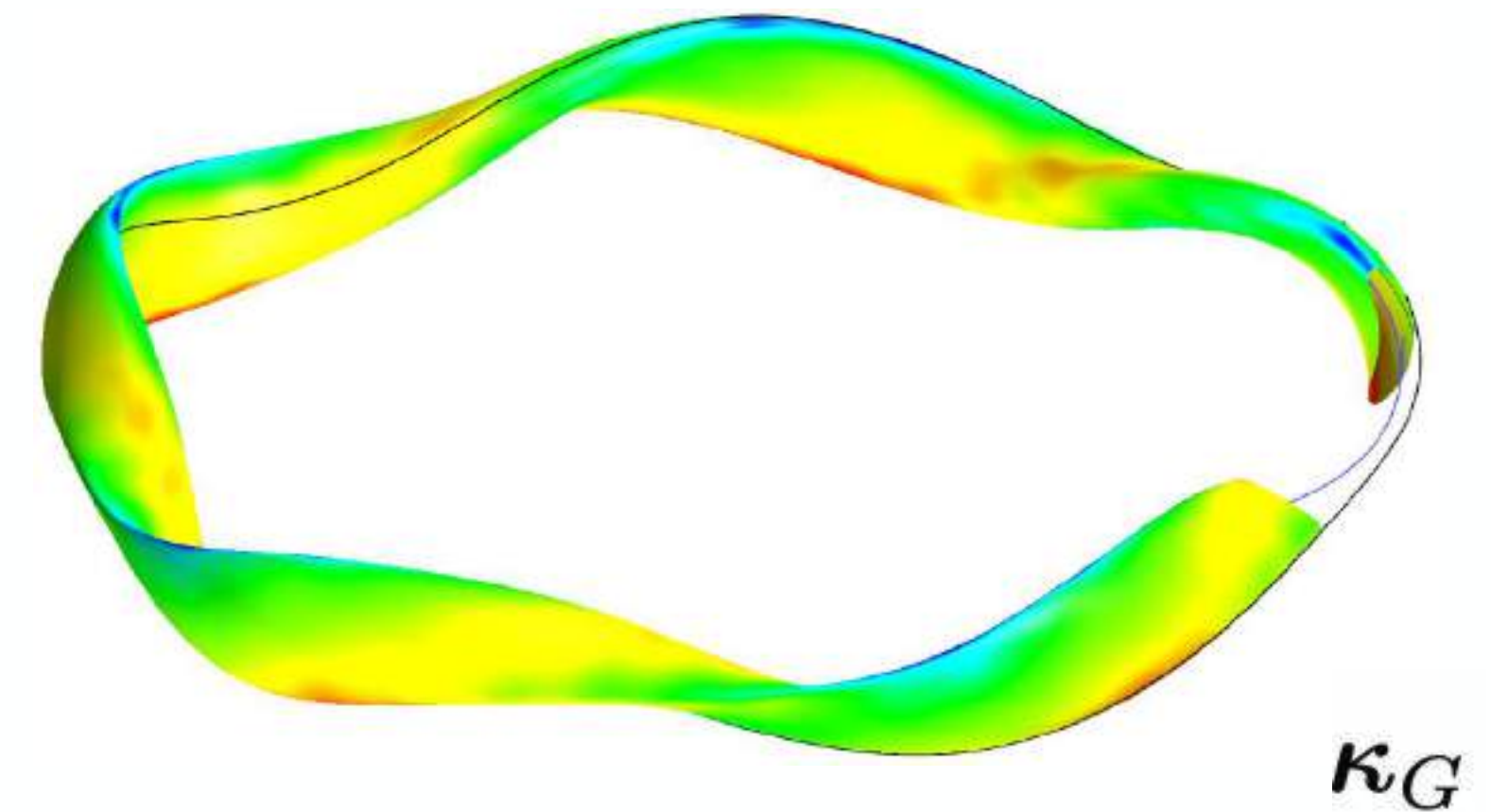
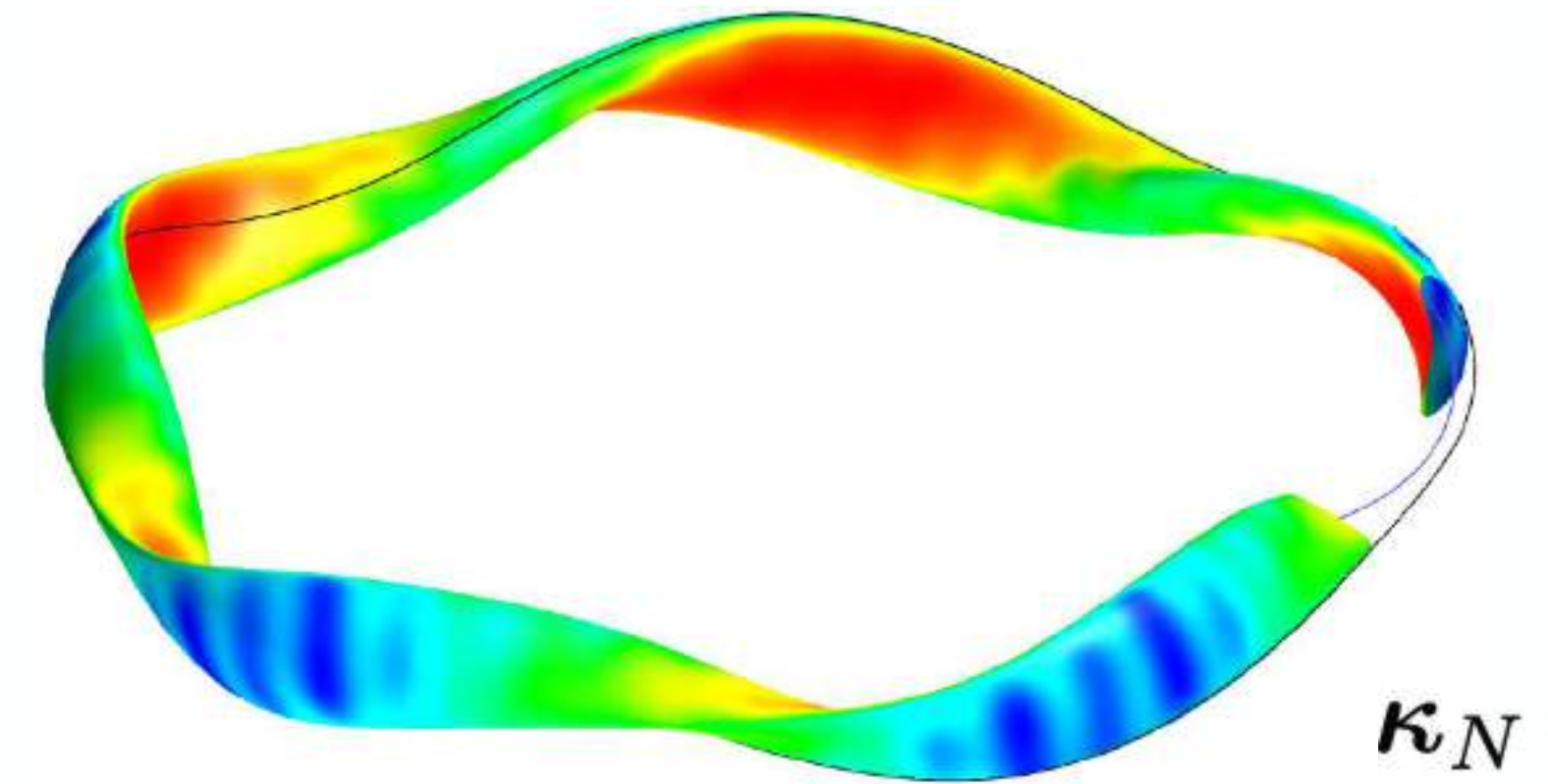
$$\kappa \equiv \mathbf{b} \cdot \nabla \mathbf{b} = \kappa_N + \kappa_G$$

where:

$$\kappa_N = \kappa \cdot \frac{\mathbf{n}}{|\mathbf{n}|}, \quad \kappa_G = \kappa \cdot \frac{\mathbf{g}}{|\mathbf{g}|}$$

and:

$$\mathbf{n} \equiv \nabla \Psi, \quad \mathbf{g} \equiv (\mathbf{B} \times \nabla \Psi)$$



# Curvature features in Wendelstein 7-X

Curvature definition:

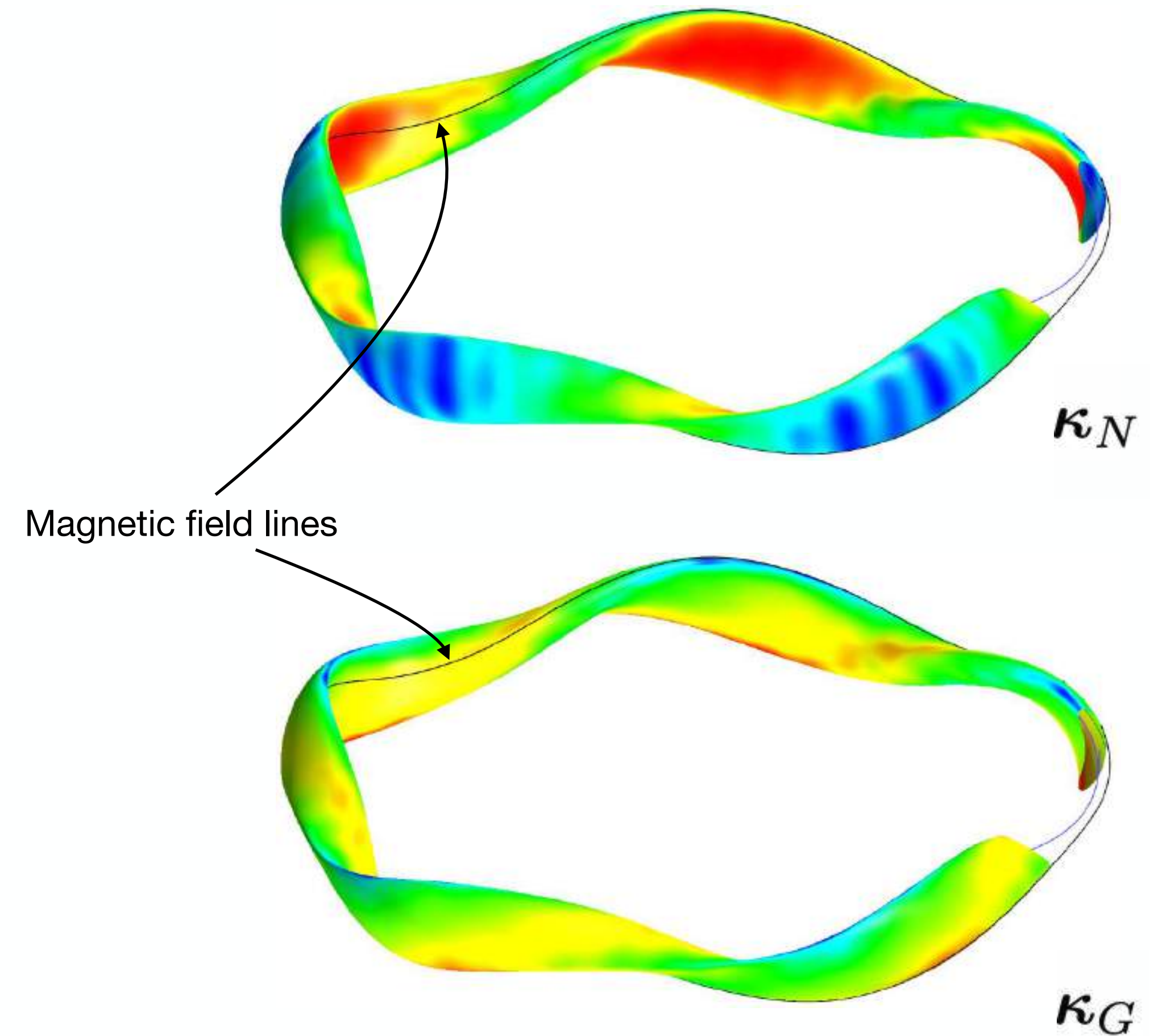
$$\kappa \equiv \mathbf{b} \cdot \nabla \mathbf{b} = \kappa_N + \kappa_G$$

where:

$$\kappa_N = \kappa \cdot \frac{\mathbf{n}}{|\mathbf{n}|}, \quad \kappa_G = \kappa \cdot \frac{\mathbf{g}}{|\mathbf{g}|}$$

and:

$$\mathbf{n} \equiv \nabla \Psi, \quad \mathbf{g} \equiv (\mathbf{B} \times \nabla \Psi)$$



# Curvature features in Wendelstein 7-X

Curvature definition:

$$\boldsymbol{\kappa} \equiv \mathbf{b} \cdot \nabla \mathbf{b} = \boldsymbol{\kappa}_N + \boldsymbol{\kappa}_G$$

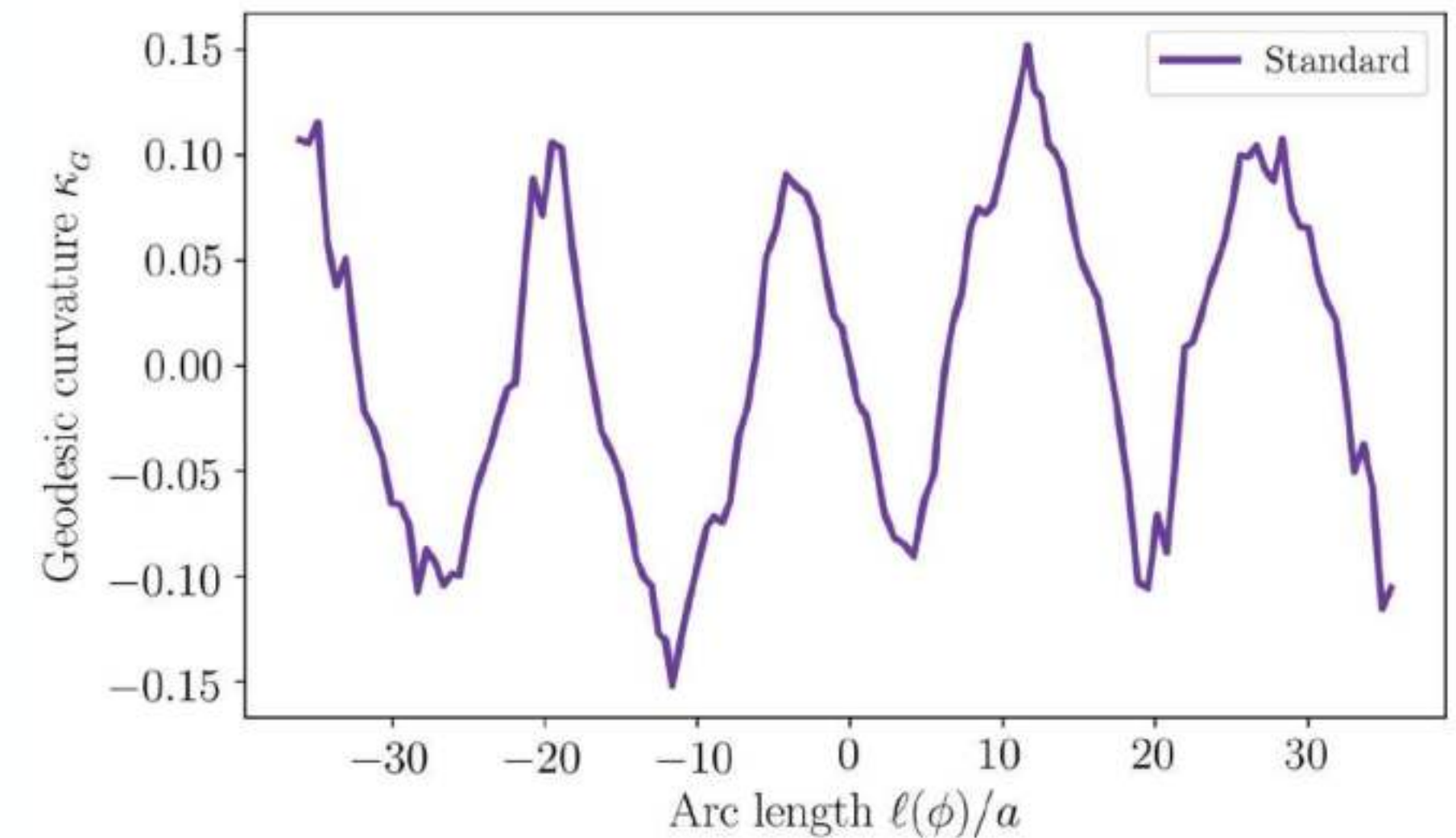
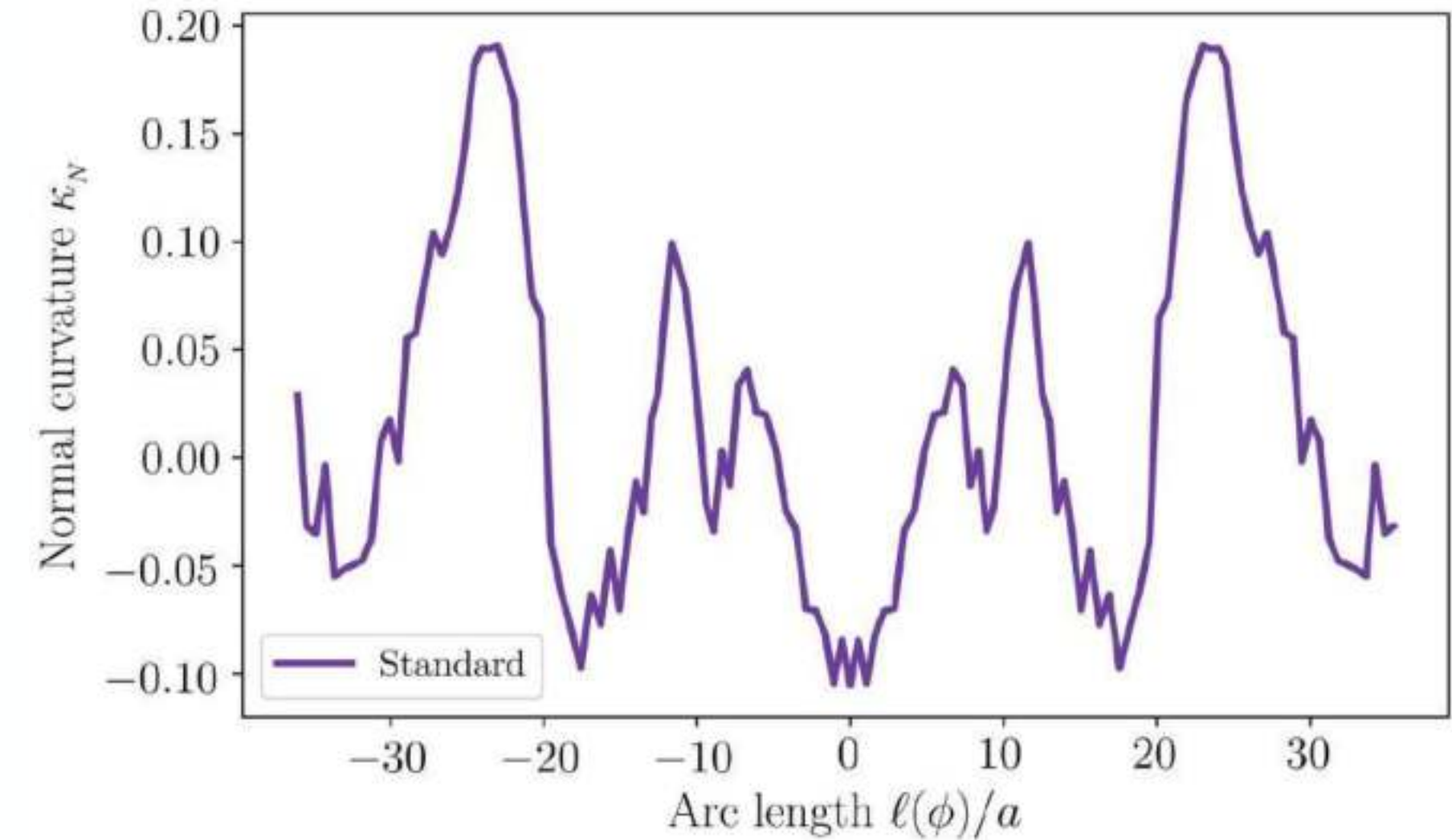
where:

$$\boldsymbol{\kappa}_N = \boldsymbol{\kappa} \cdot \frac{\mathbf{n}}{|\mathbf{n}|}, \quad \boldsymbol{\kappa}_G = \boldsymbol{\kappa} \cdot \frac{\mathbf{g}}{|\mathbf{g}|}$$

and:

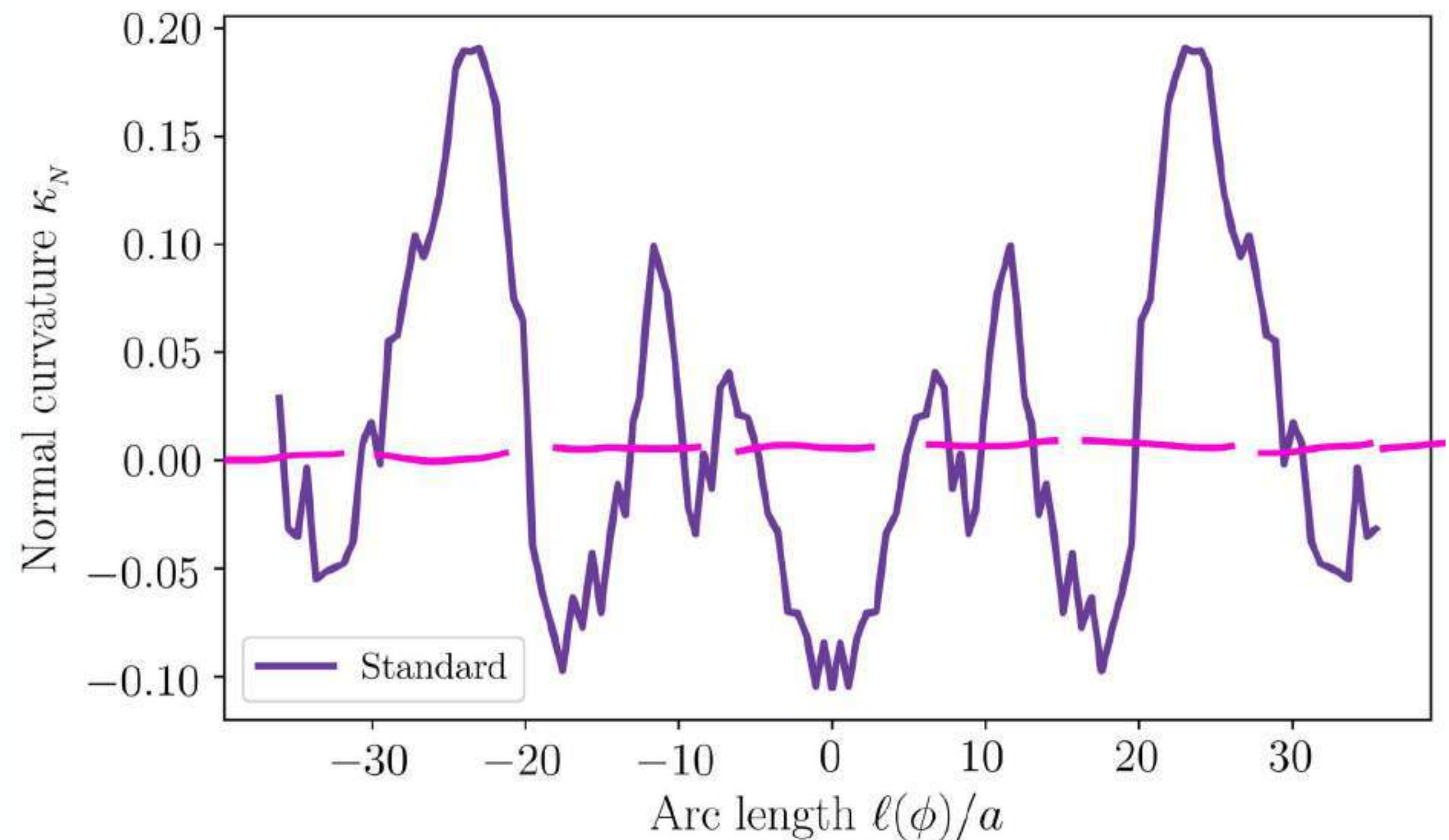
$$\mathbf{n} \equiv \nabla \Psi, \quad \mathbf{g} \equiv (\mathbf{B} \times \nabla \Psi)$$

Different iota: ballooning angle  $\rightarrow$  arc length



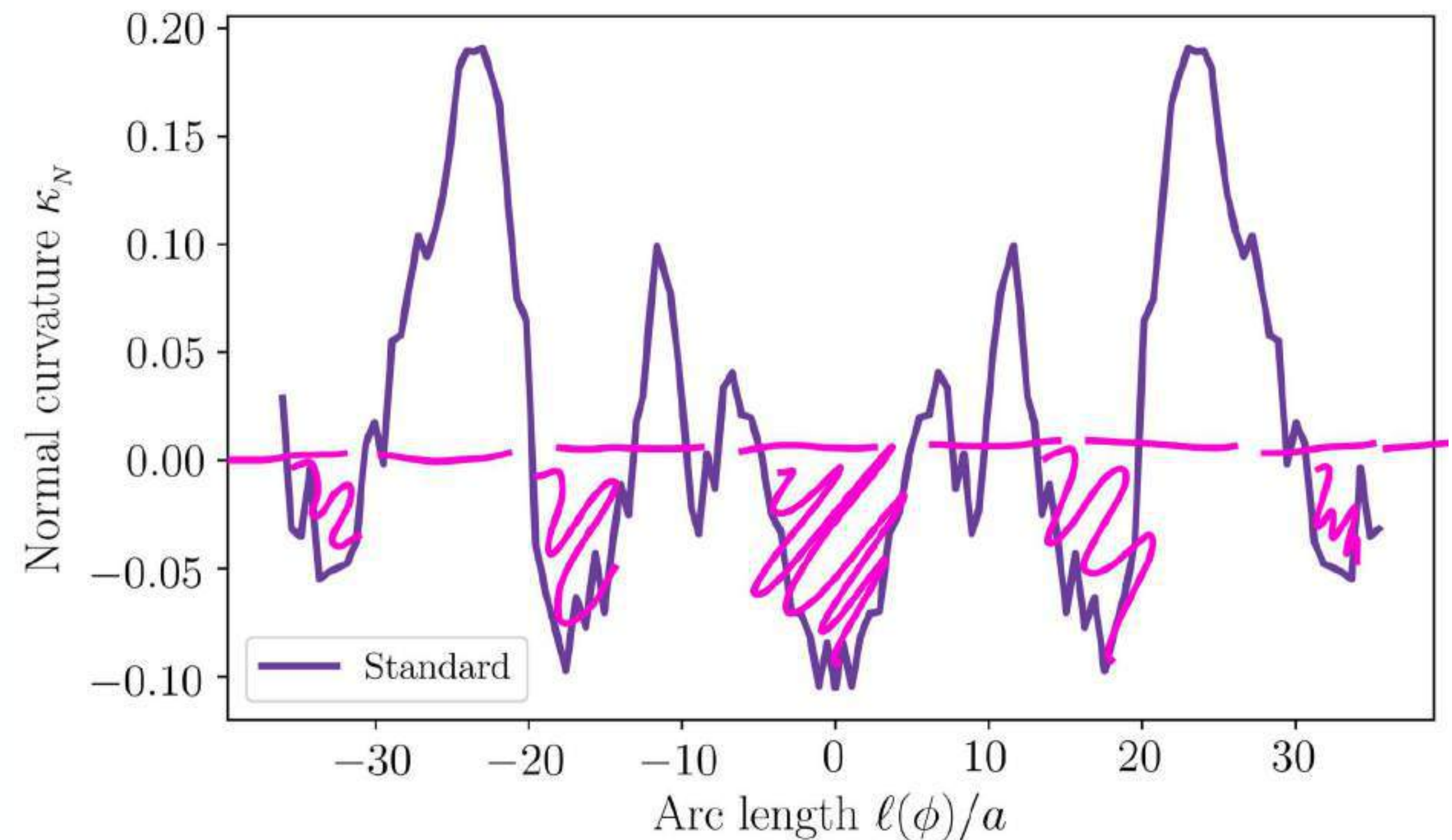
# Primary modes are localized in “bad” curvature regions

- Negative regions: instability growth.
- High particle drifts



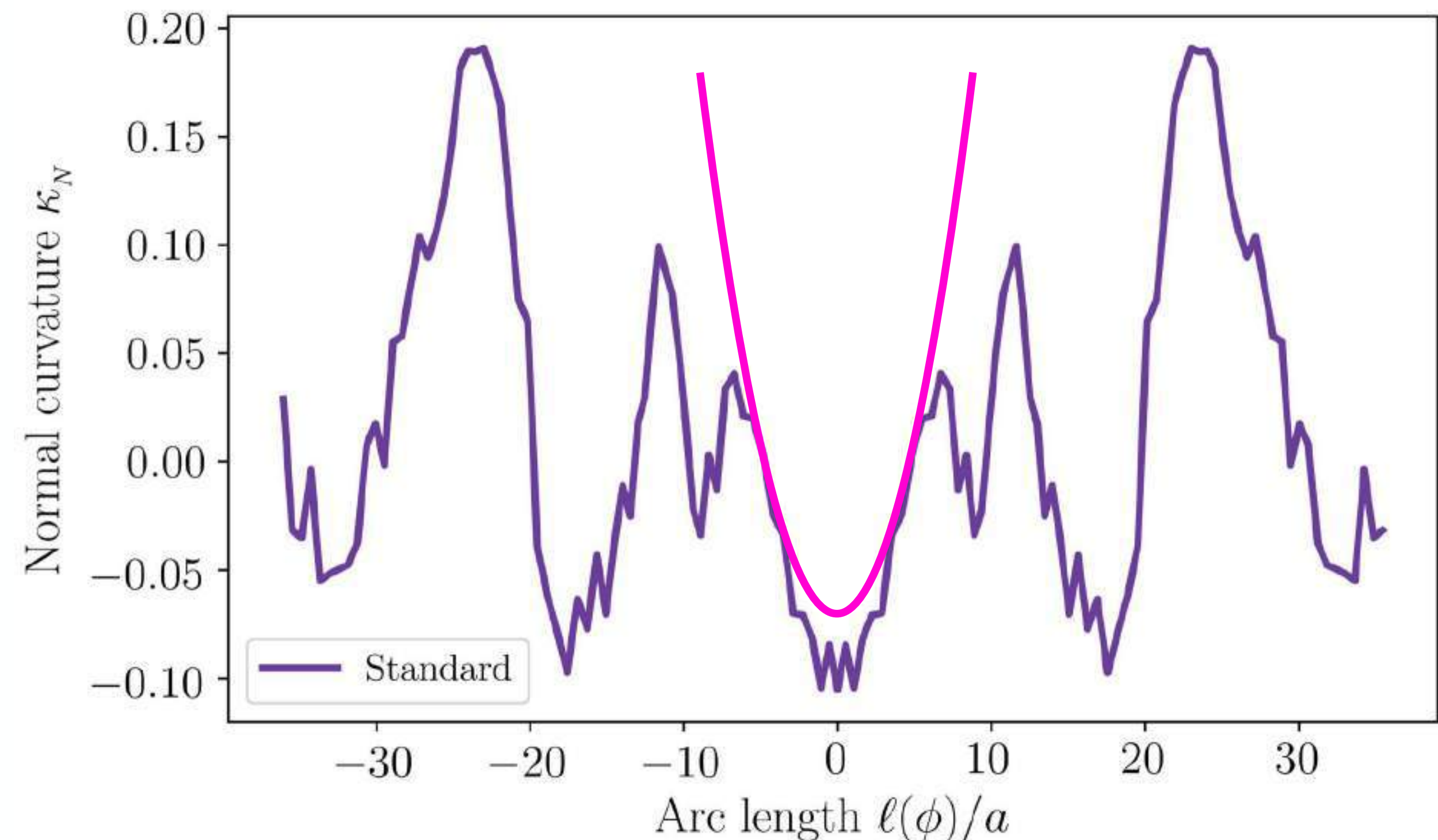
# Primary modes are localized in “bad” curvature regions

- Negative regions: instability growth.
- High particle drifts
- Exact eigenfunctions peak at the center.



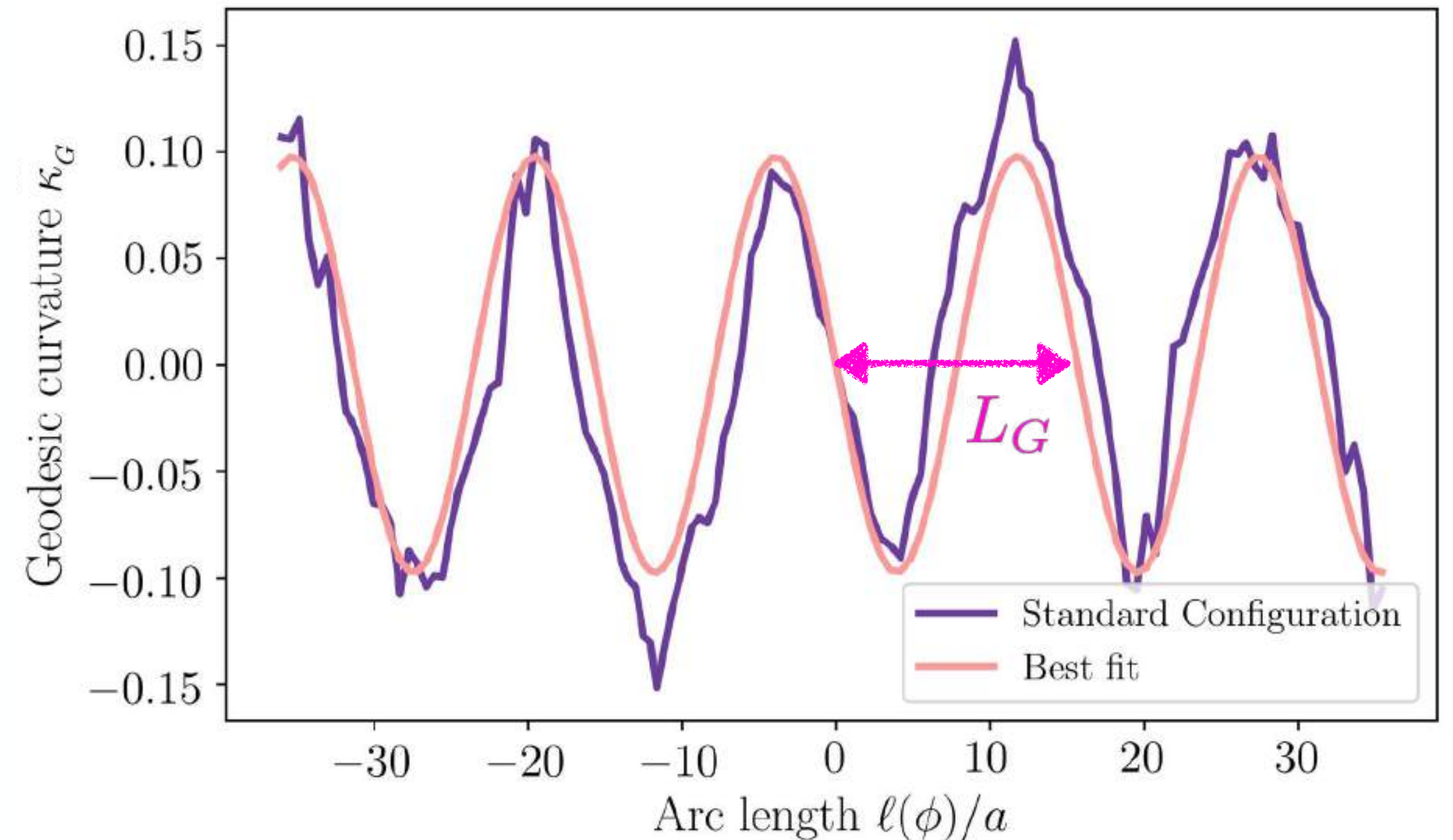
# Primary modes are localized in “bad” curvature regions

- Negative regions: instability growth.
- High particle drifts
- Exact eigenfunctions peak at the center.
- Gaussian envelope: *Drift well.*

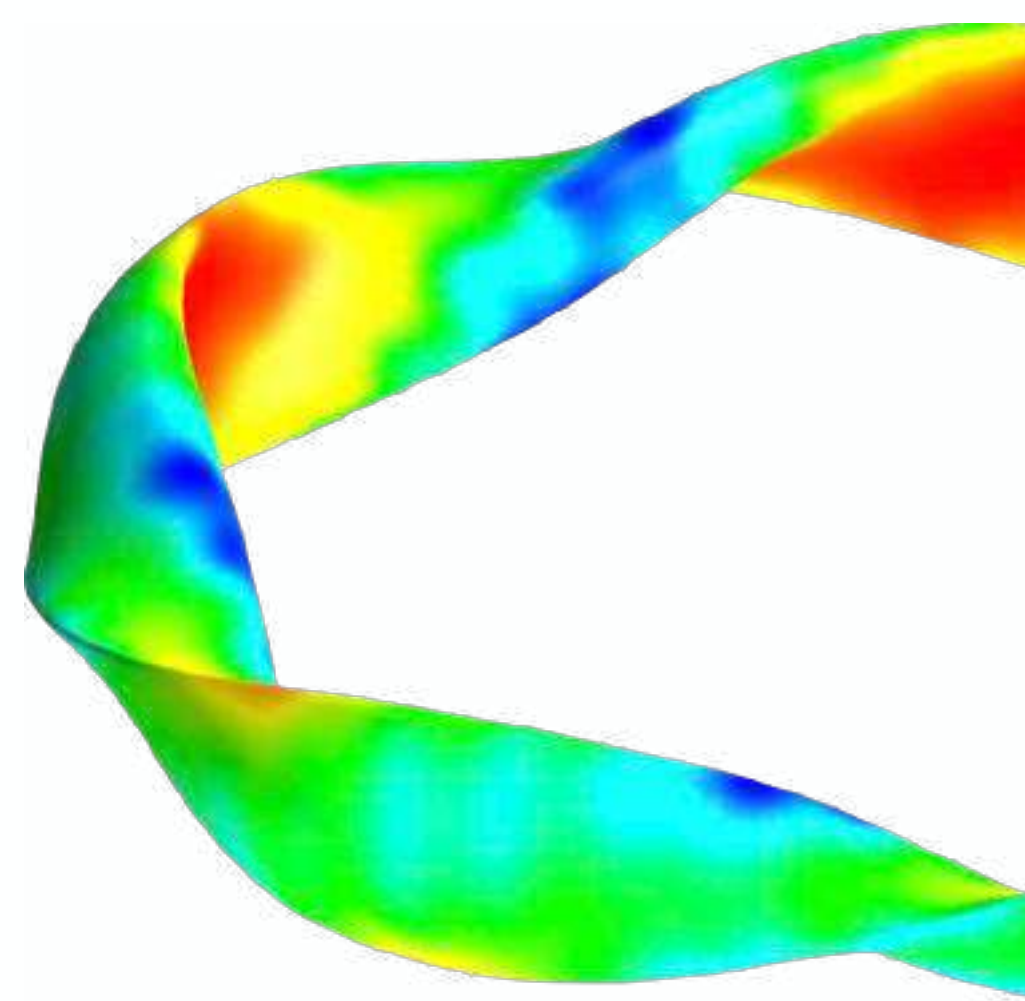


# Mode coupling through curvature is characterized by periodicity

- $\kappa_G$  couples  $k_y = 0$  mode to acoustic wave.
- Geodesic scale:  $k_{||}$  of the mode.
- Characteristic geodesic scale  $L_G$ .



$$\kappa_{G,fit} = |\kappa_G| \sin(2\pi \ell/L_G)$$



Geometry in Wendelstein 7-X

Gyrokinetic turbulence modeling

Reduced transport modeling

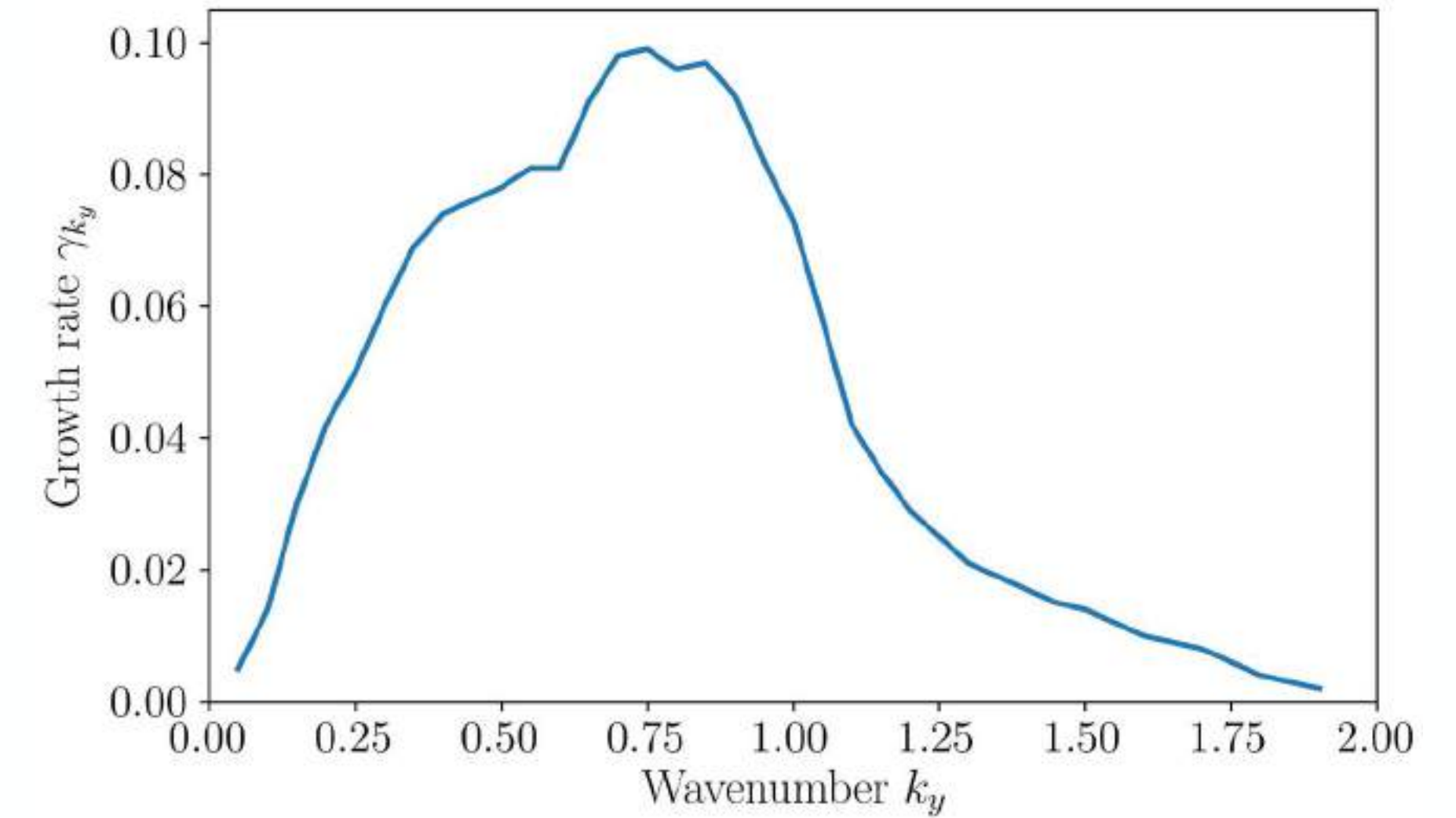
Summary



# Linear estimates of transport ignore saturation

- ITG-driven turbulence.
- Simple quasilinear model:

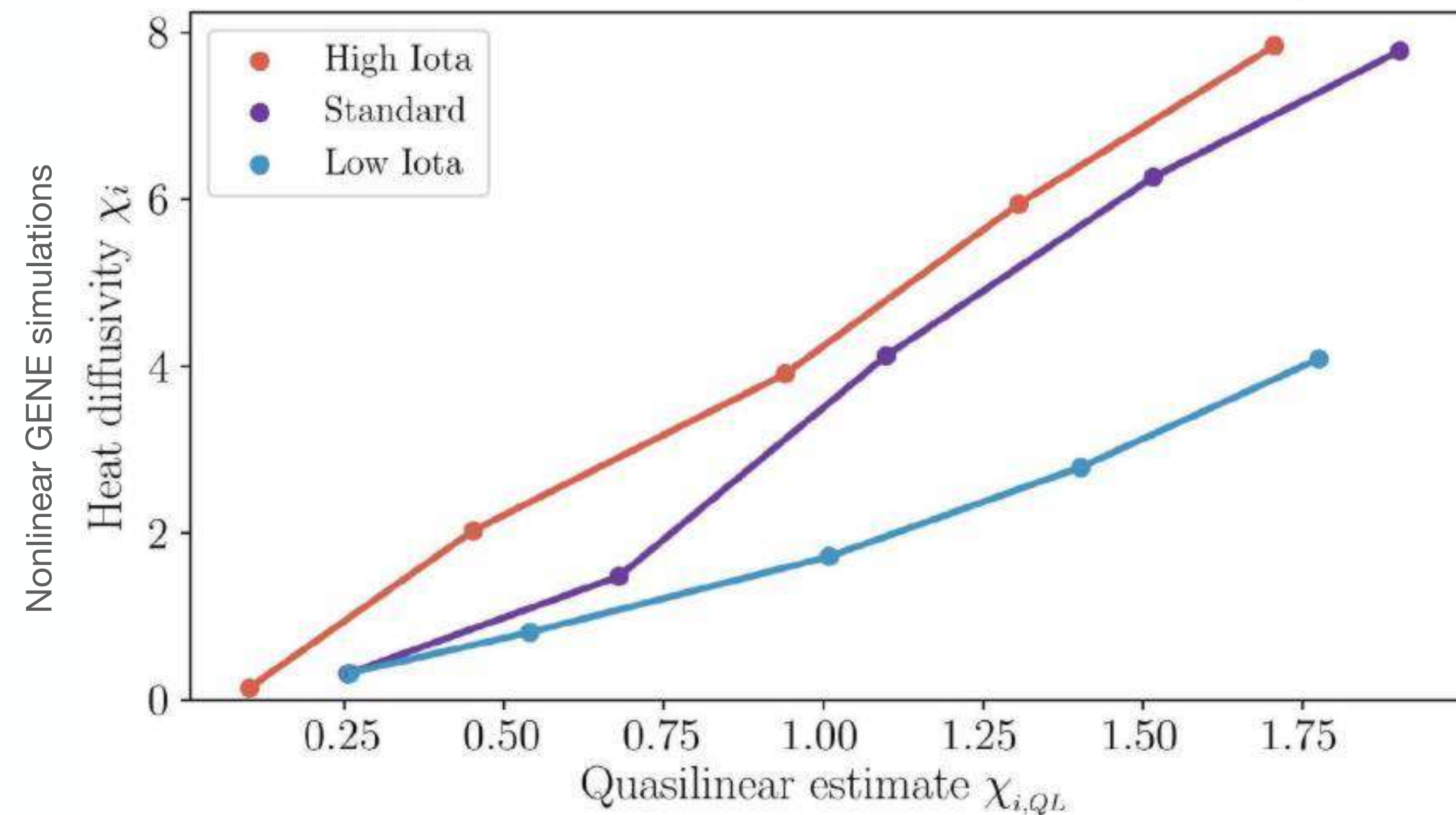
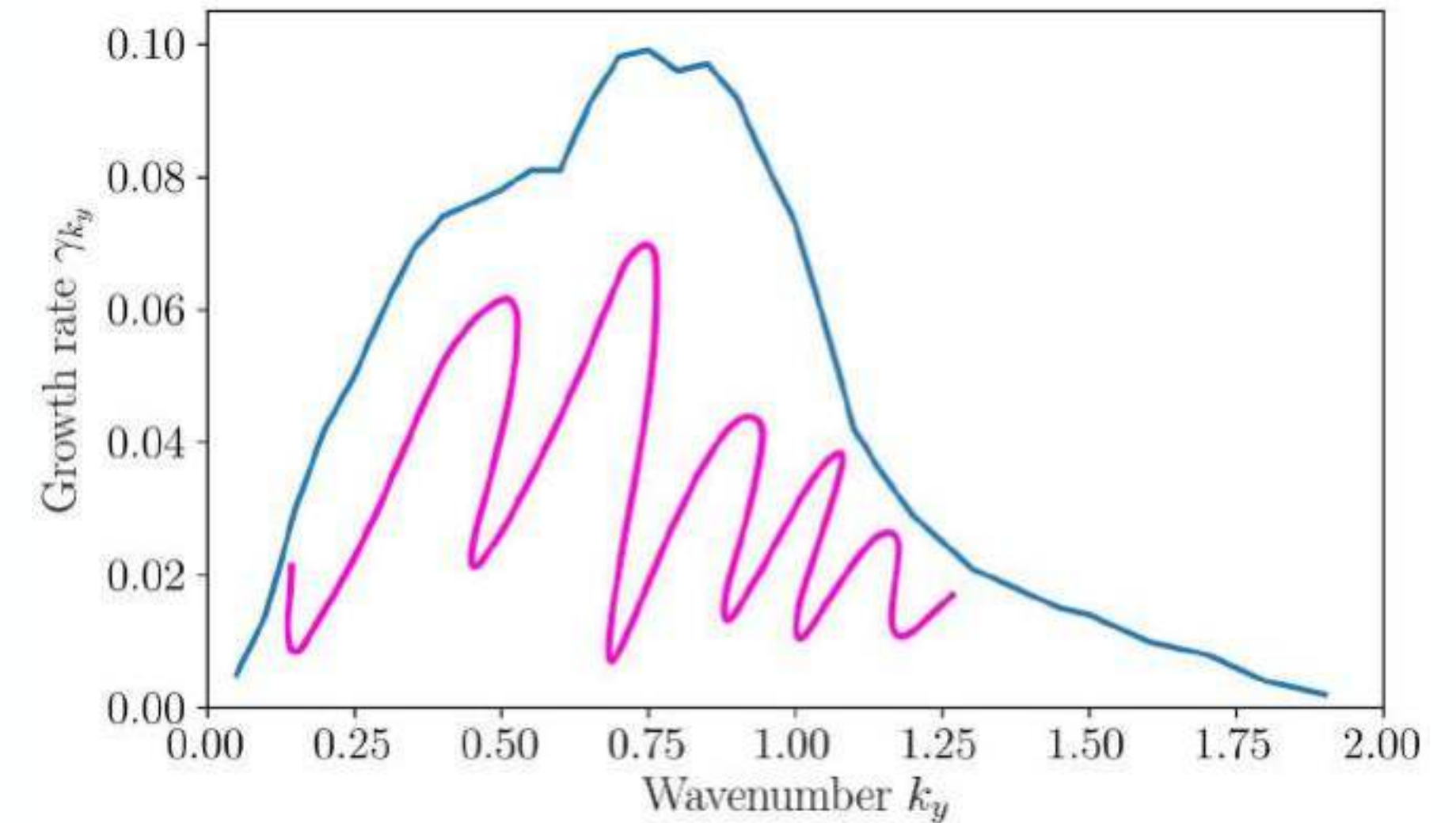
$$\chi_{QL} = C_1 \sum_{k_y} \frac{\gamma_{k_y}}{k_y^2} \times \Delta k_y$$



# Linear estimates of transport ignore saturation

- ITG-driven turbulence.
- Simple quasilinear model:

$$\chi_{QL} = C_1 \sum_{k_y} \frac{\gamma_{k_y}}{k_y^2} \times \Delta k_y$$

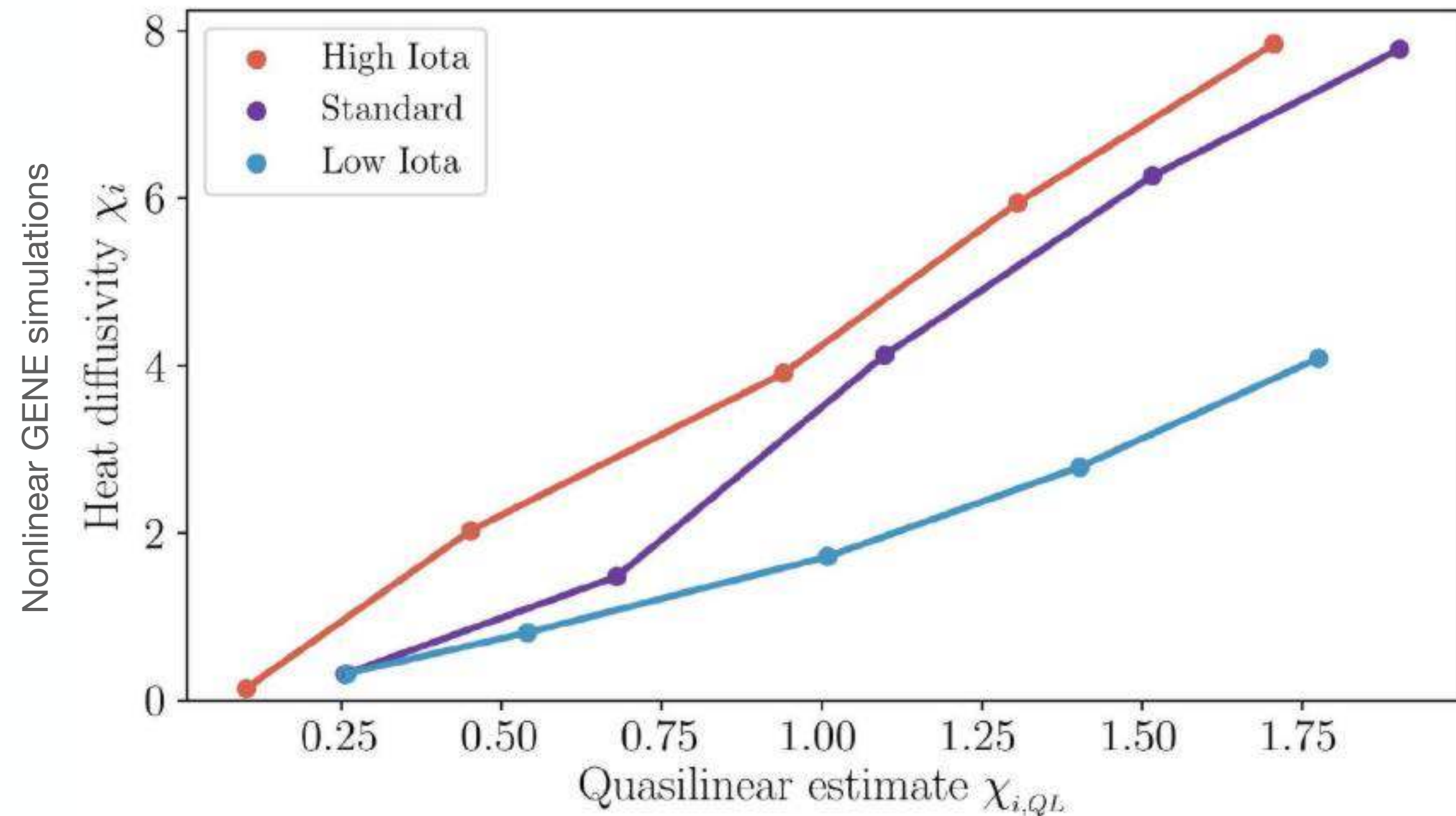
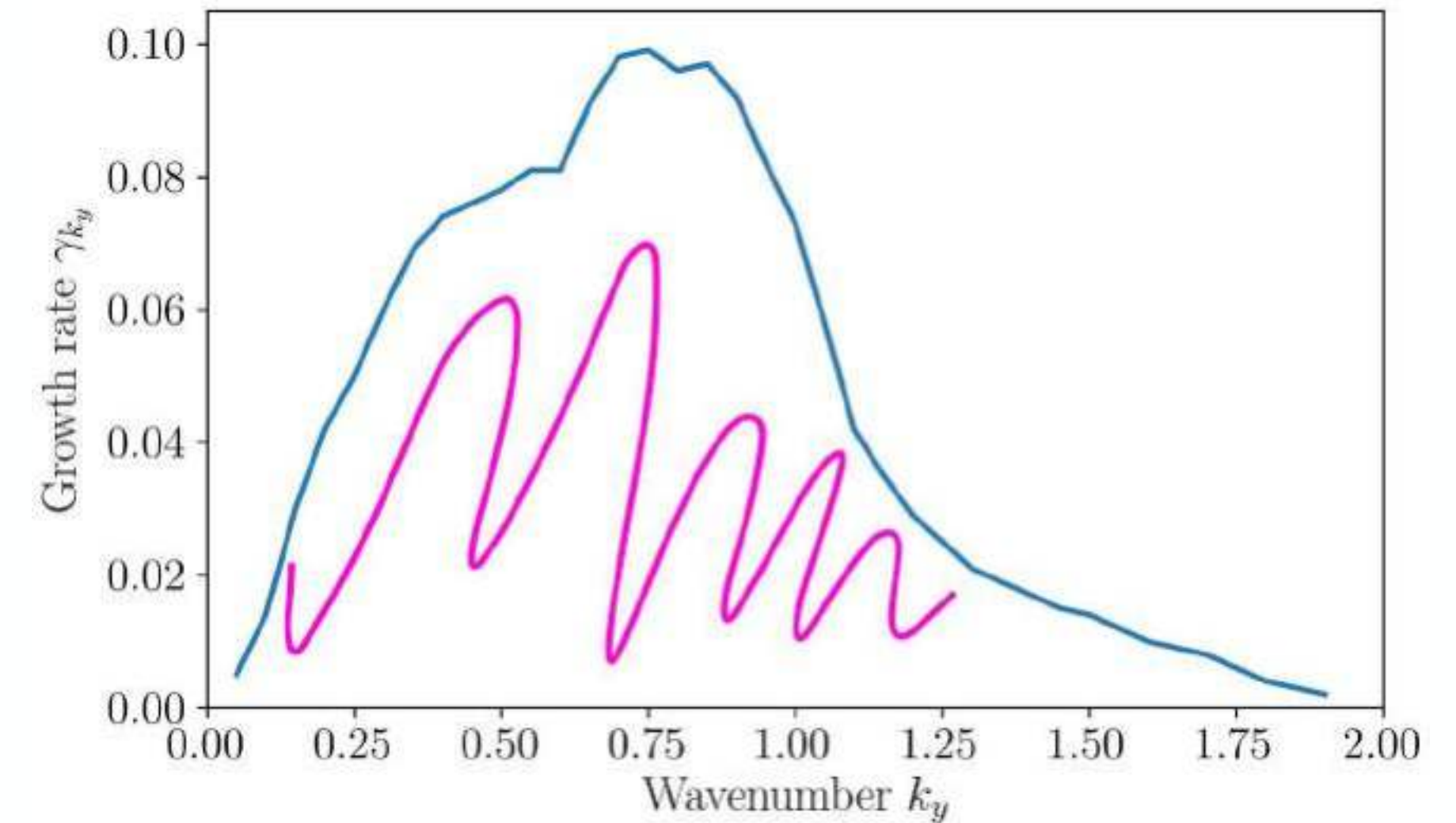


# Linear estimates of transport ignore saturation

- ITG-driven turbulence.
- Simple quasilinear model:

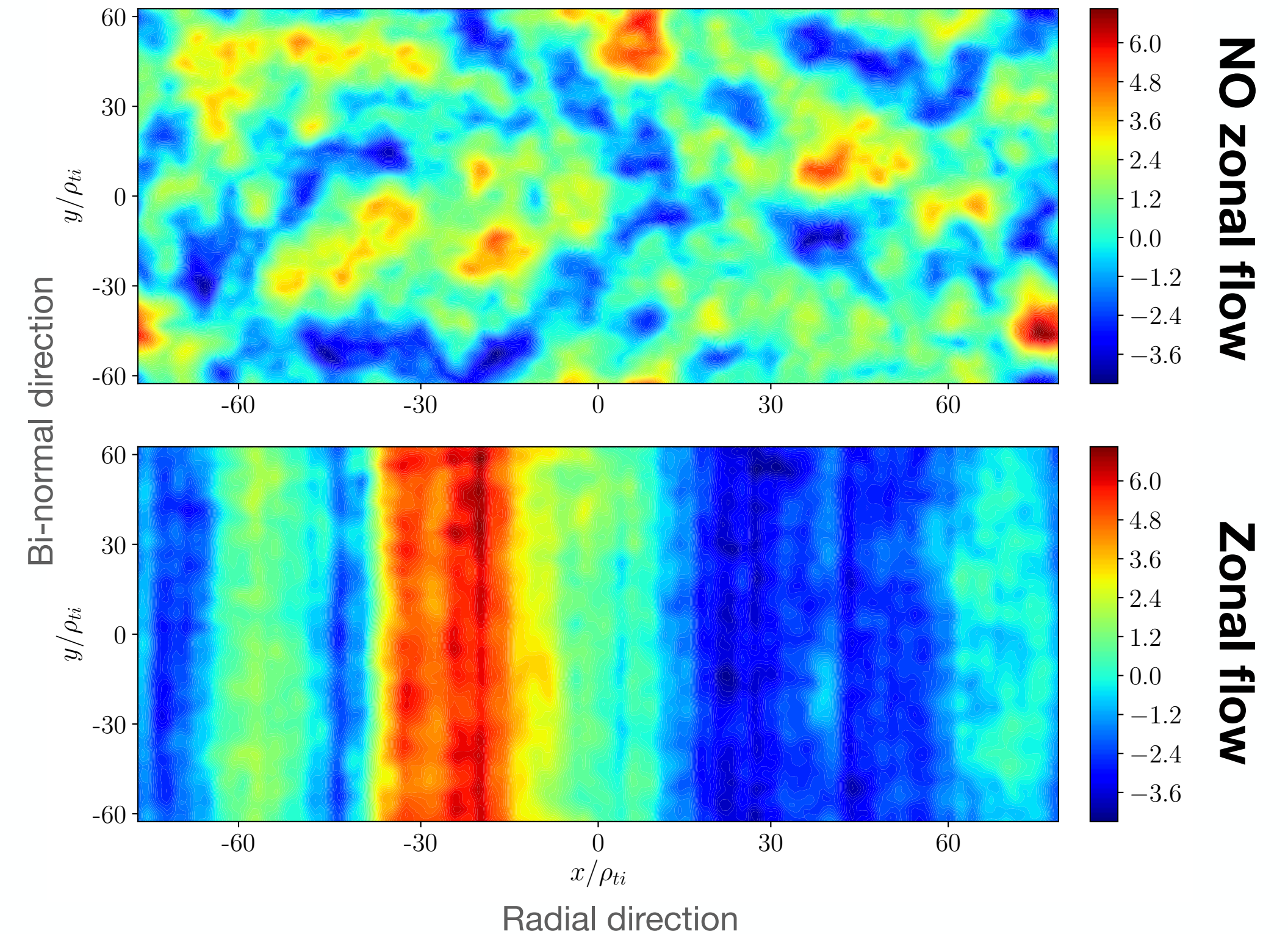
$$\chi_{QL} = C_1 \sum_{k_y} \frac{\gamma_{k_y}}{k_y^2} \times \Delta k_y$$

- Pre-factor  $C_1$  clearly depends on geometry.
- Saturation dynamics to be understood.



# ITG-driven turbulence induces Zonal Flows

- Primary instability: initial eddies.
- Secondary instability:
  - Poloidal elongated structures.
- Saturation through shearing.



# ITG-driven turbulence induces Zonal Flows

- Primary instability: initial eddies.
- Secondary instability:
  - Poloidal elongated structures.
  - Saturation through shearing.
- Curiosity:

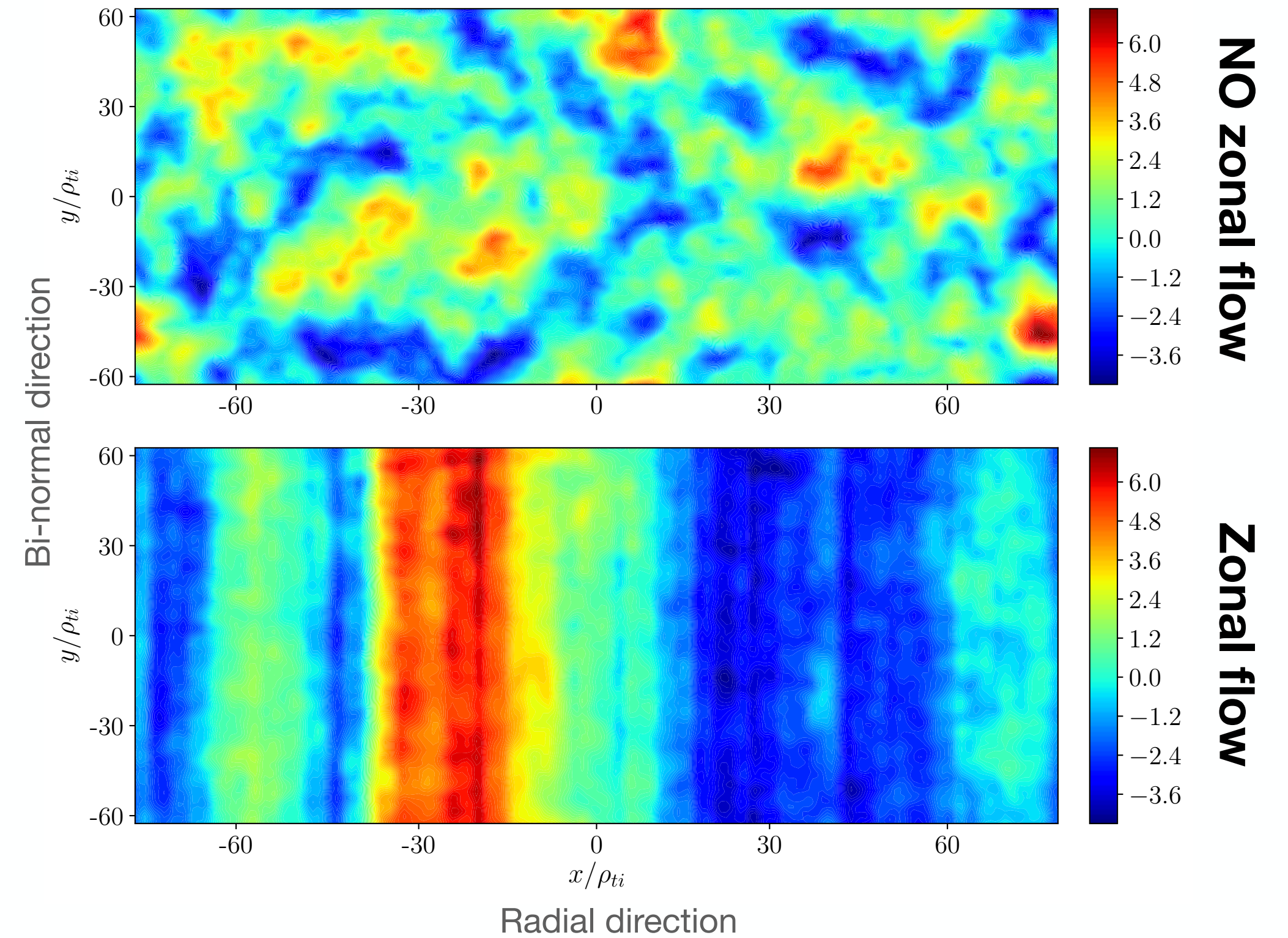
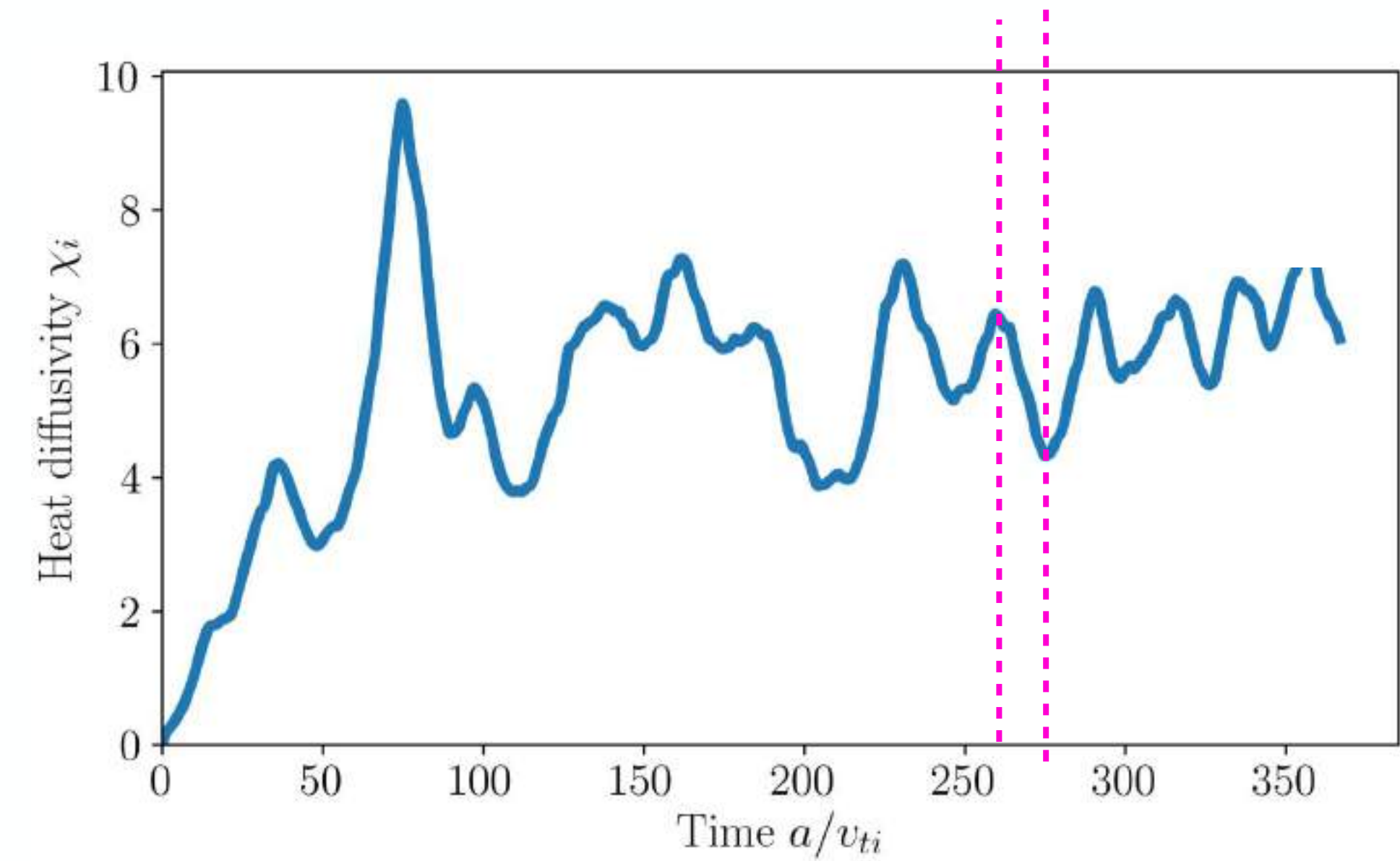


Image credit: NASA/JPL-Caltech/SwRI/MSSS/Kevin M. Gill

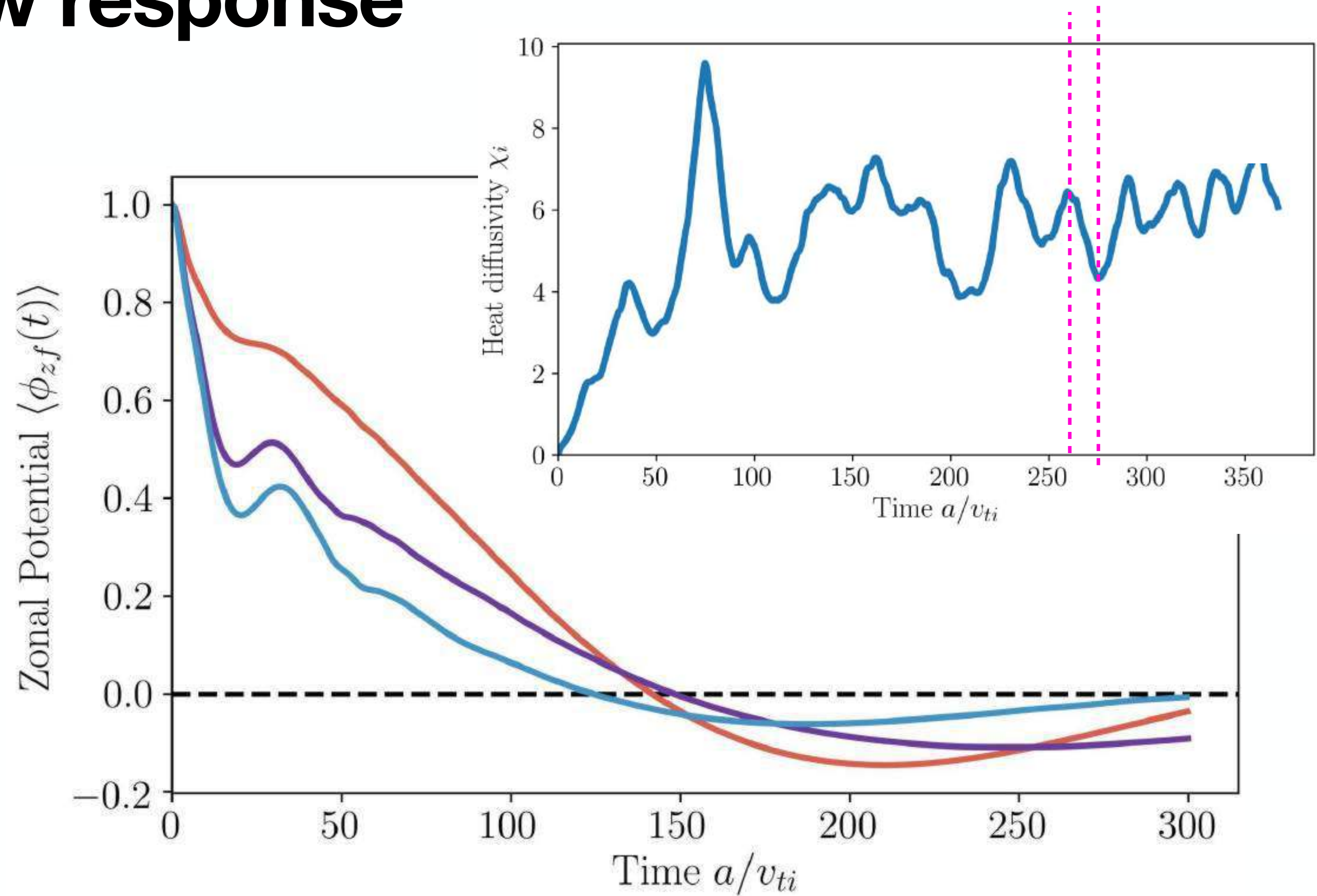
# Saturation time scales dictate dynamics of the linear zonal flow response

- Turbulence saturation time.



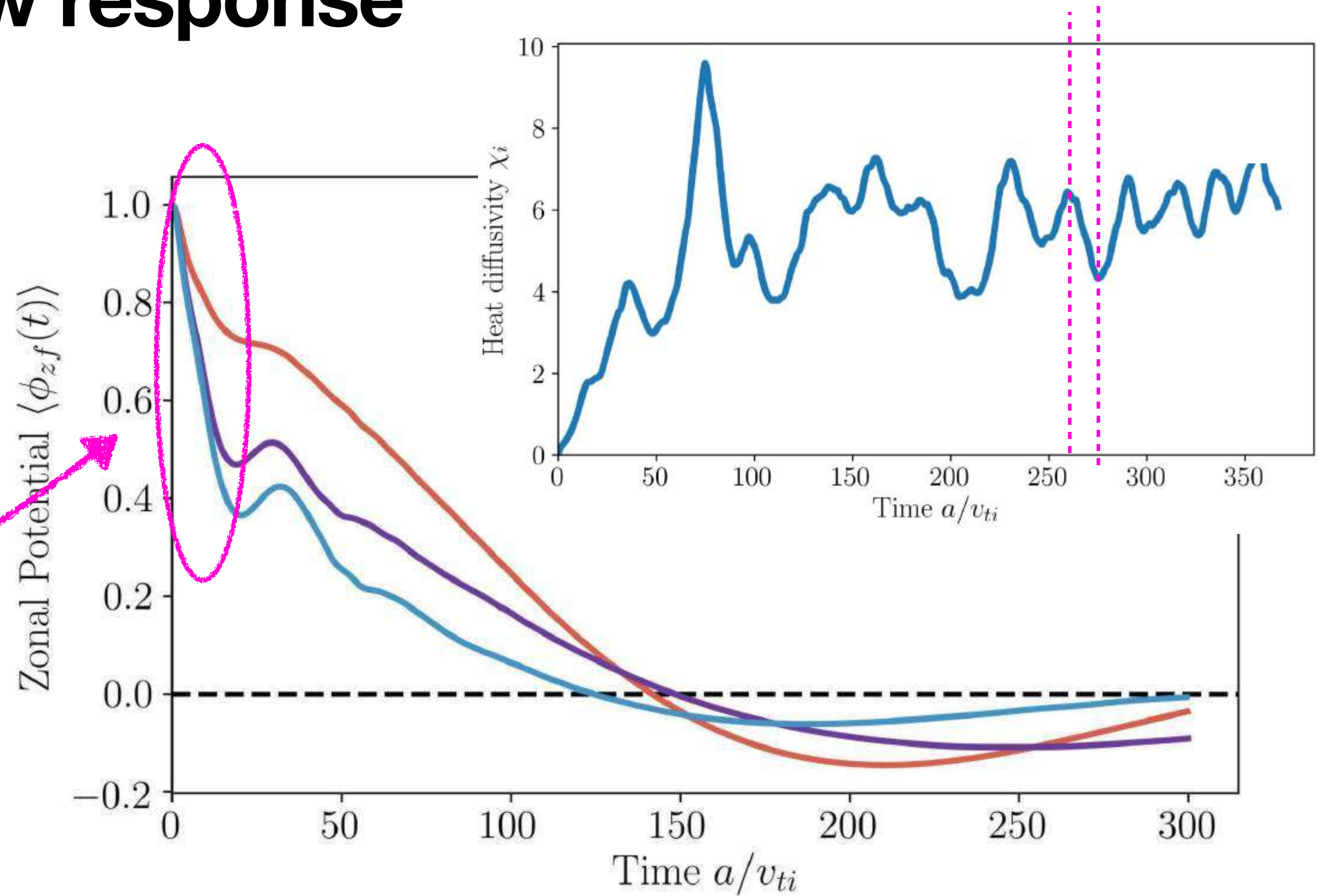
# Saturation time scales dictate dynamics of the linear zonal flow response

- Turbulence saturation time.
- Linear potential response.
- Characteristics of interest:
  - Residual potential.
  - Transient damping.



# Saturation time scales dictate dynamics of the linear zonal flow response

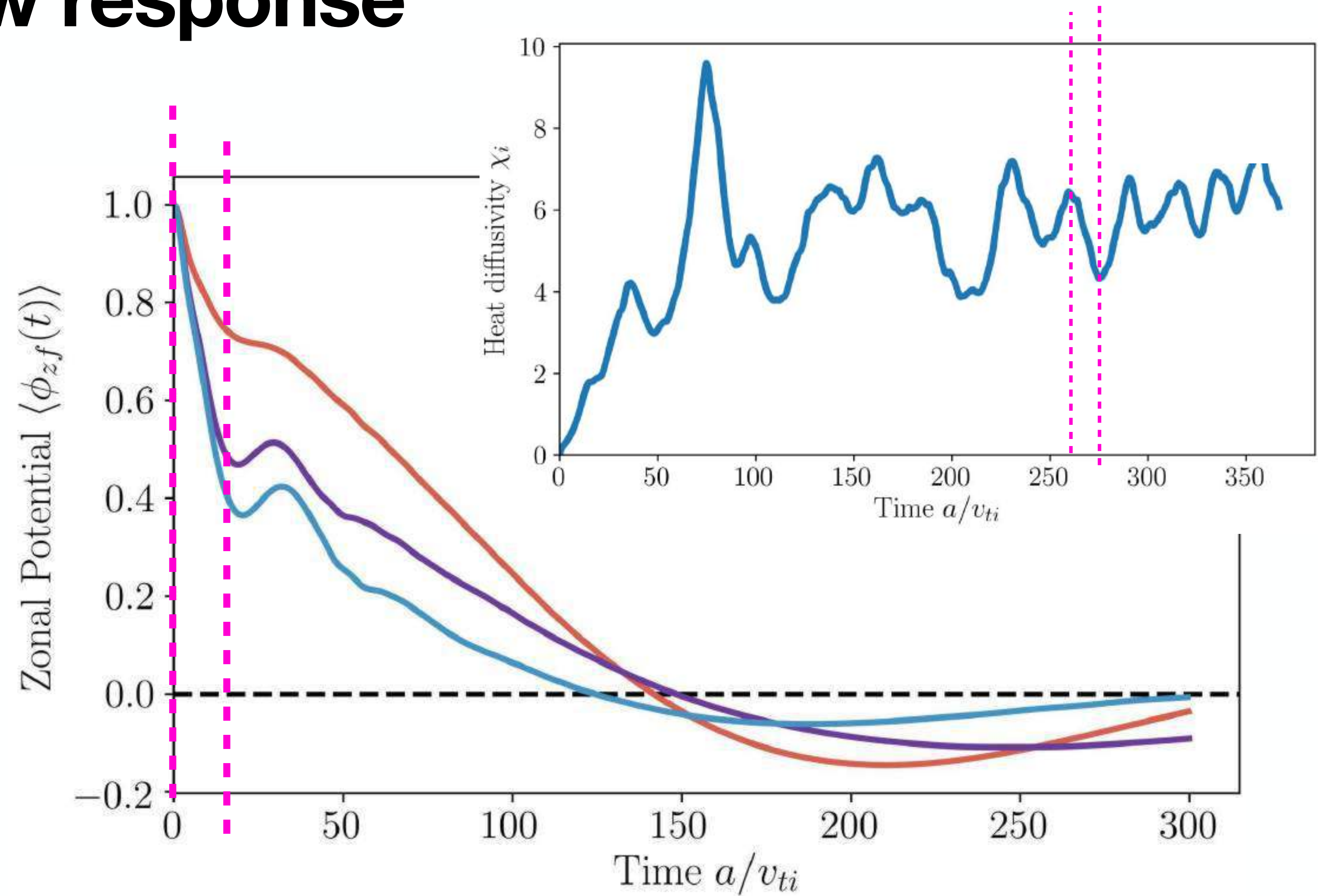
- Turbulence saturation time.
- Linear potential response.
- Characteristics of interest:
  - Residual potential.
  - Transient damping.





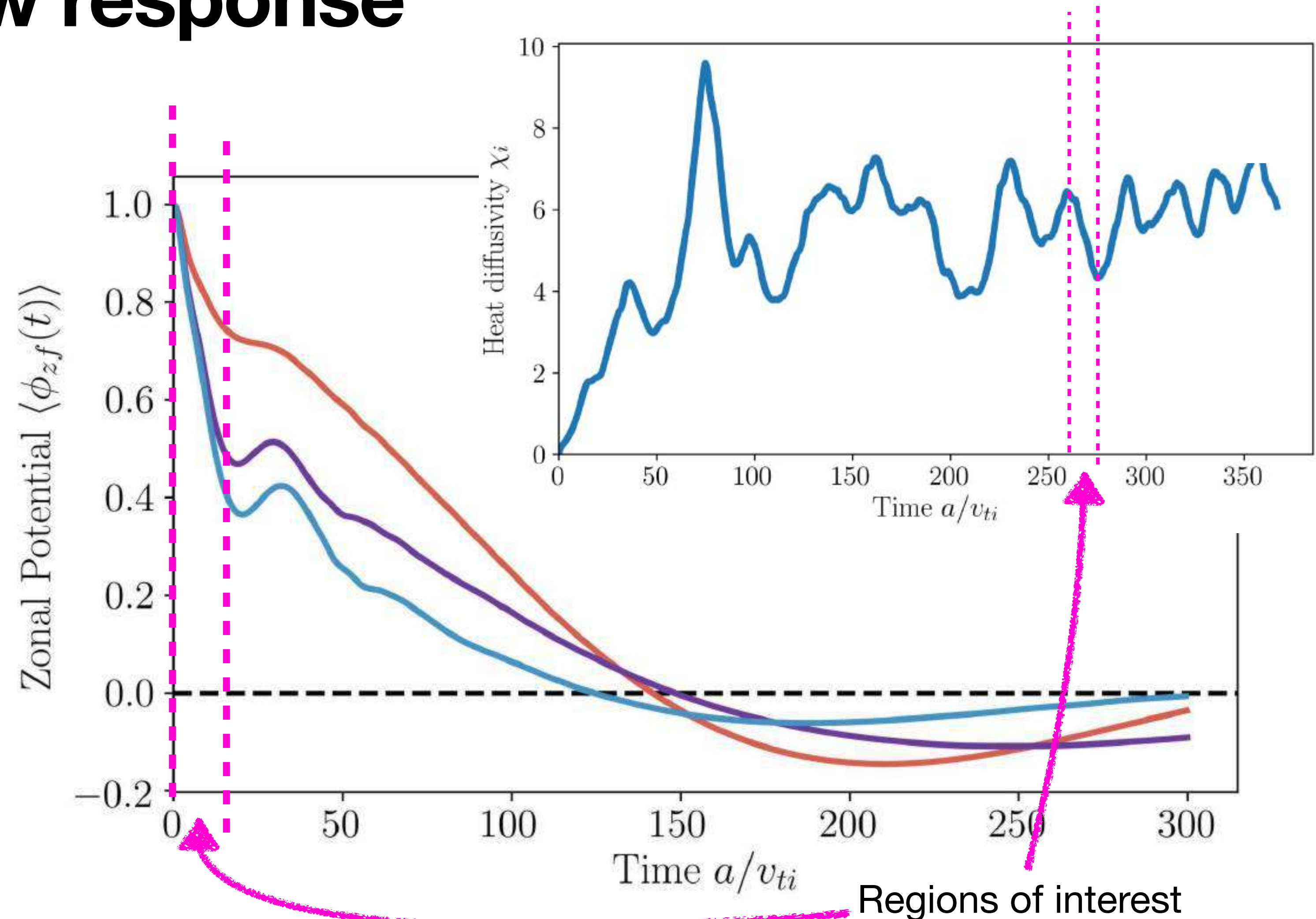
# Saturation time scales dictate dynamics of the linear zonal flow response

- Turbulence saturation time.
- Linear potential response.
- Characteristics of interest:
  - Residual potential.
  - Transient damping.
- Residual is very small.
- Transient damping is comparable.



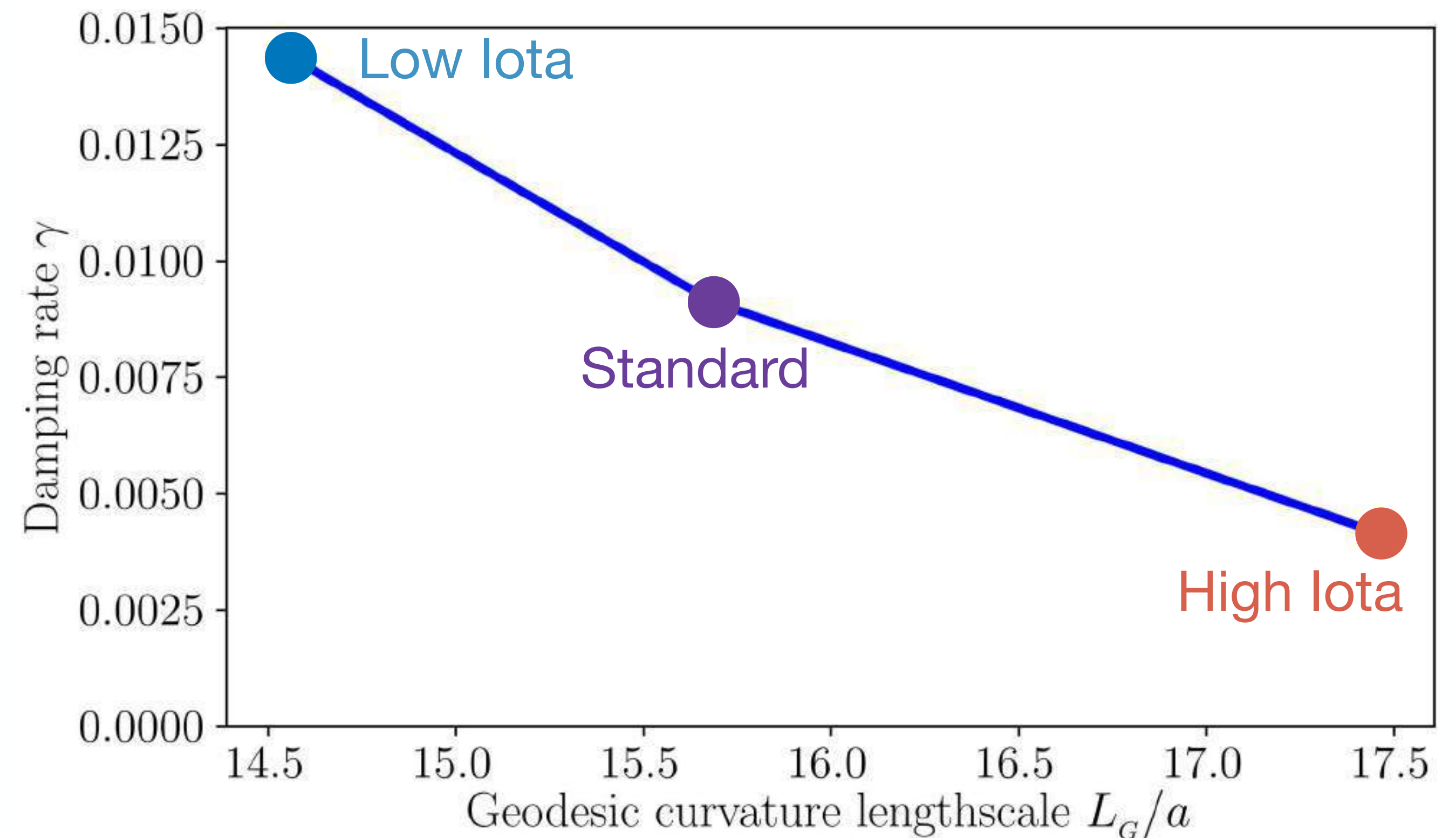
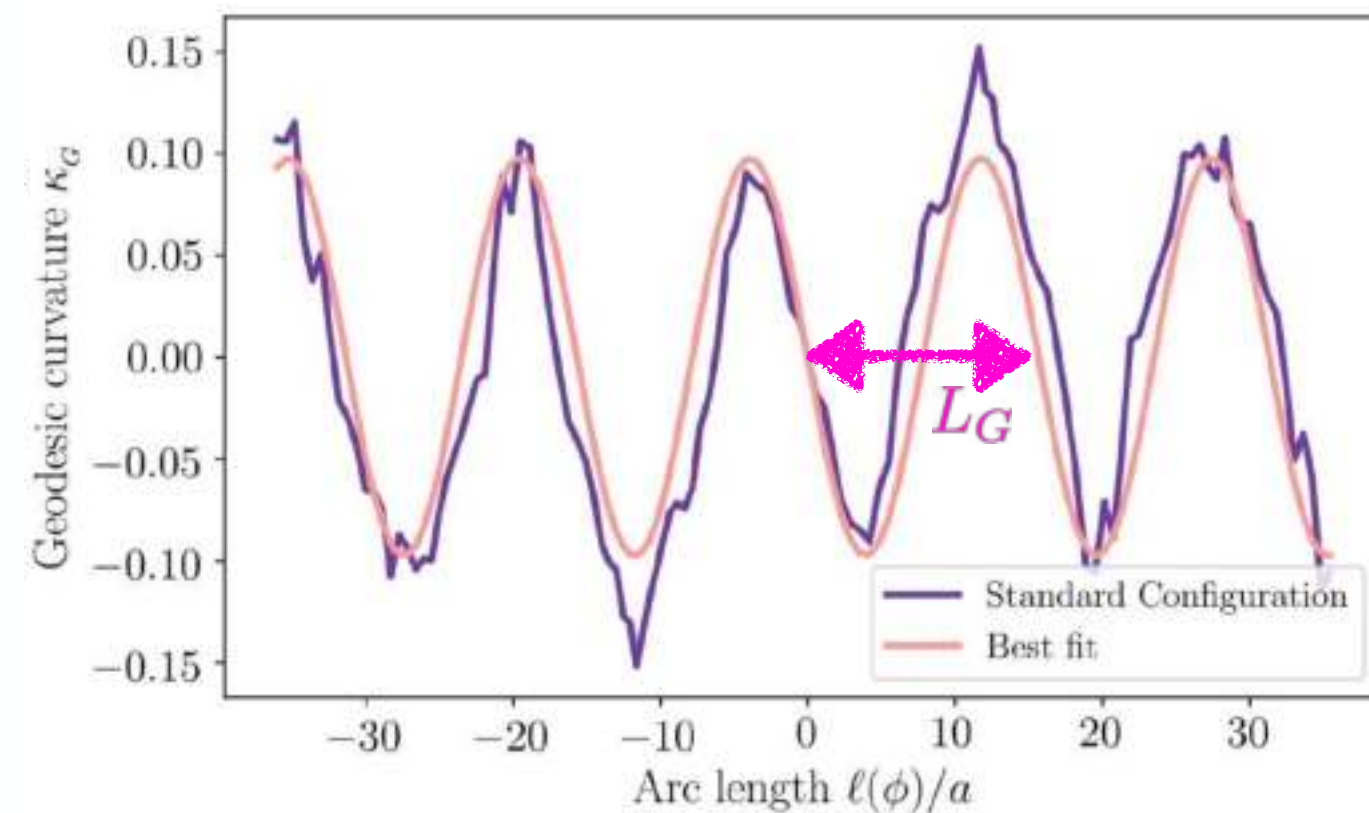
# Saturation time scales dictate dynamics of the linear zonal flow response

- Turbulence saturation time.
- Linear potential response.
- Characteristics of interest:
  - Residual potential.
  - Transient damping.
- Residual is very small.
- Transient damping is comparable.



# Damping is determined by geodesic curvature

- Weaker damping  $\rightarrow$  Longer-lived Zonal Flows.
- **Low  $\iota$  configuration might see lower zonal flow activity.**



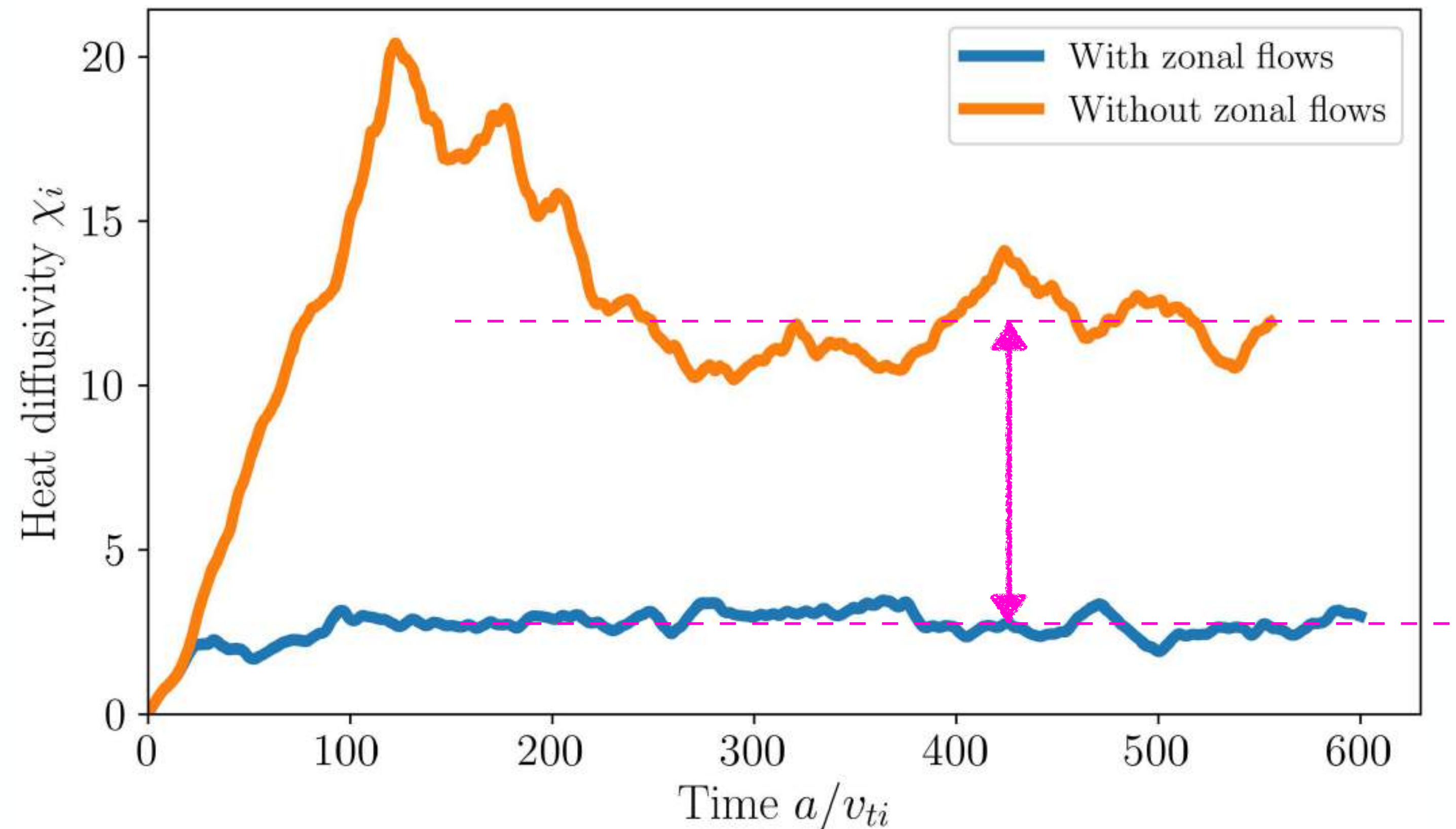
# How do zonal flows impact saturation?

## A numerical experiment

- Zero zonal flow contrib. at each time step.
- Transport delta:

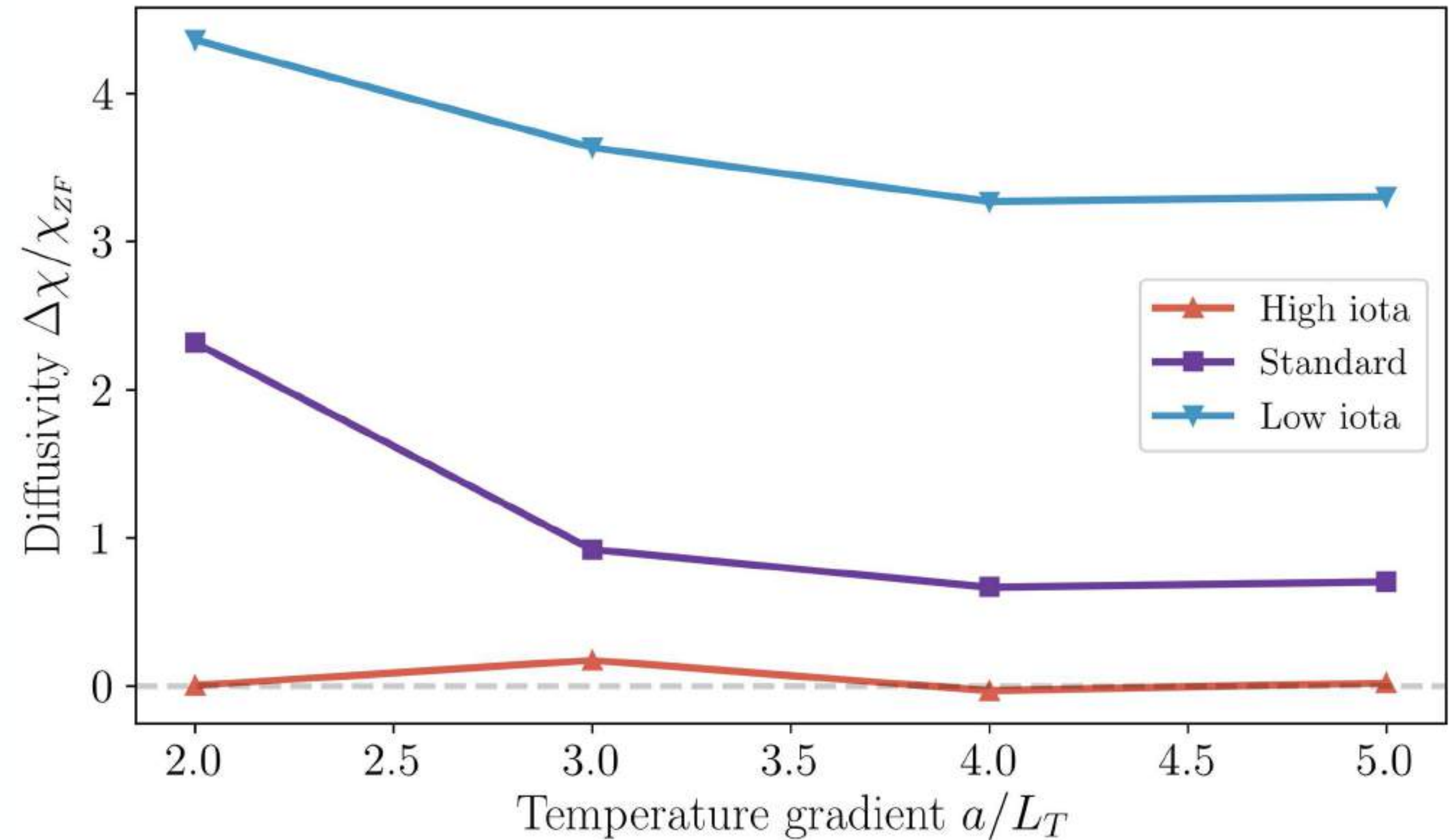
$$\frac{\Delta\chi}{\chi_{ZF}} = \frac{\chi_{no-ZF} - \chi_{ZF}}{\chi_{ZF}}$$

- Configuration impact?



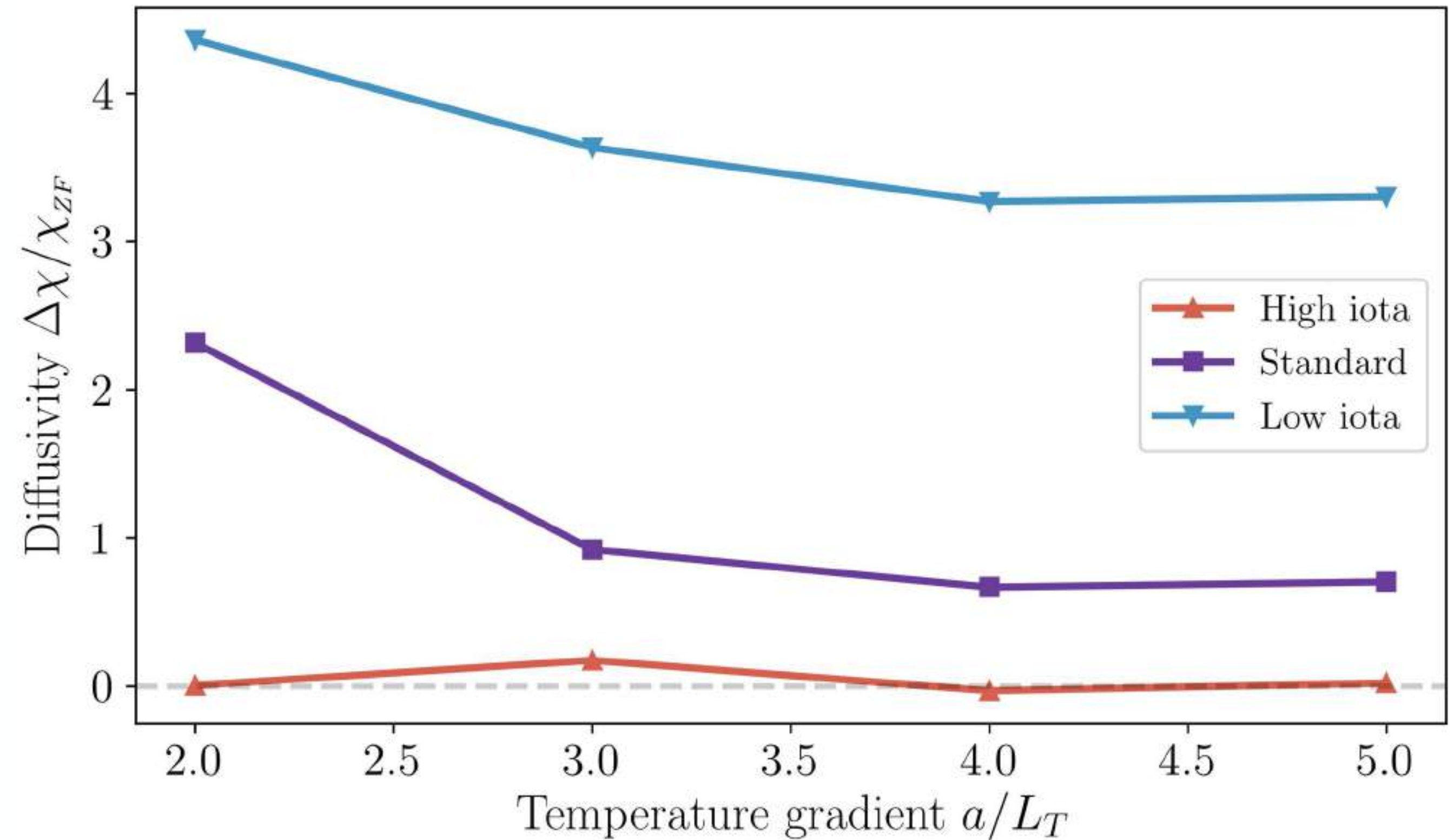
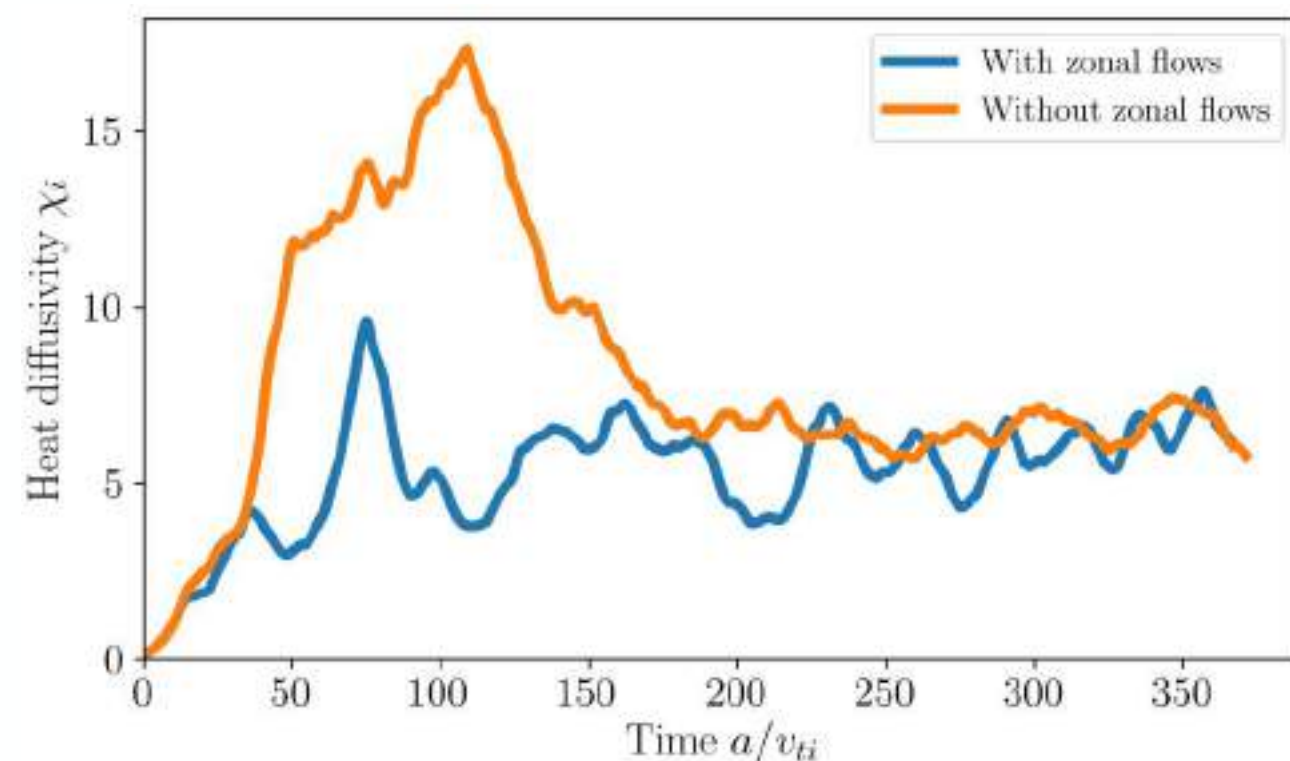
# How do zonal flows impact saturation?

- Zero zonal flow contrib. at each time step.
- Low  $\iota$  depends on zonal flows to reach saturation.



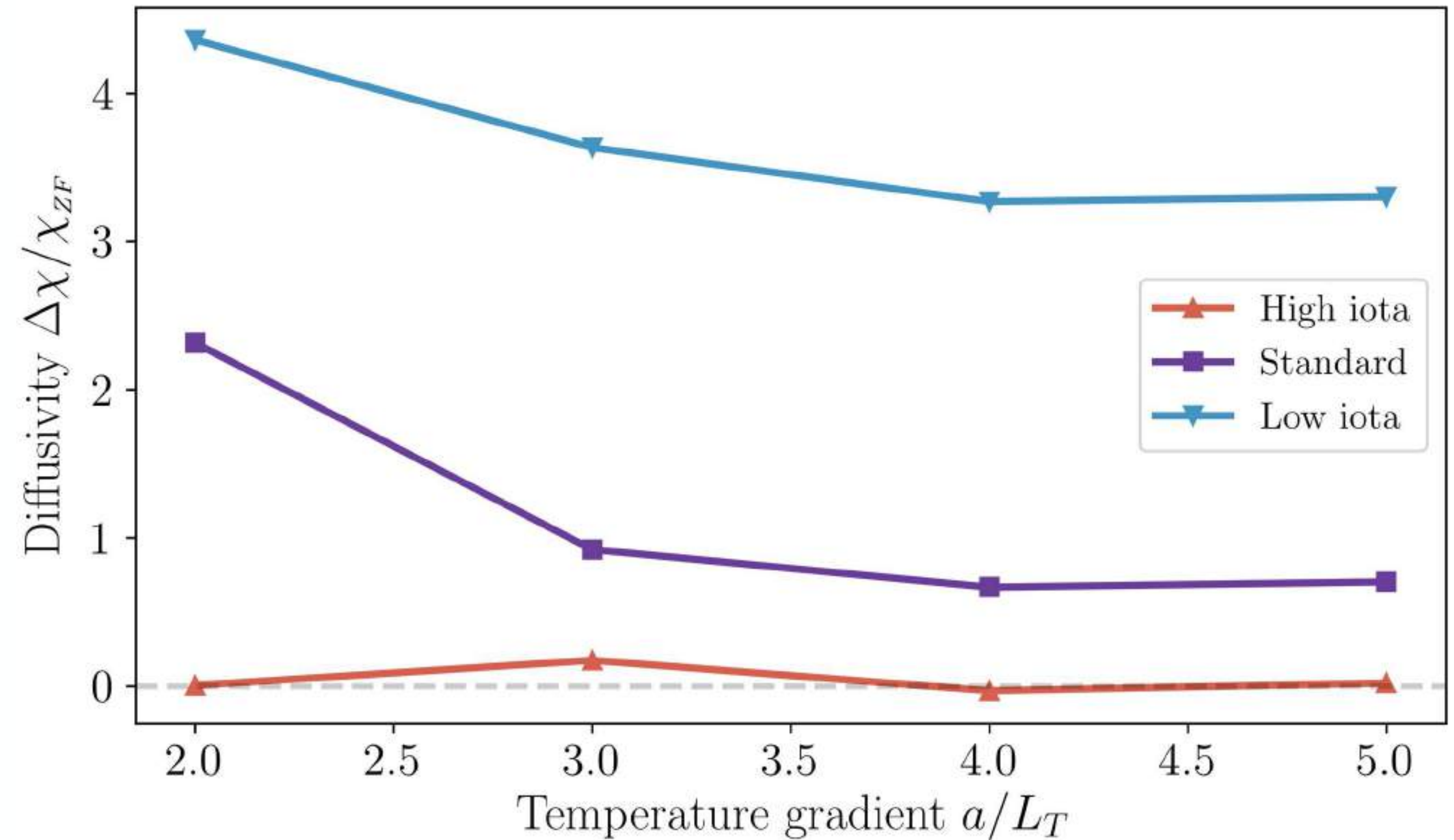
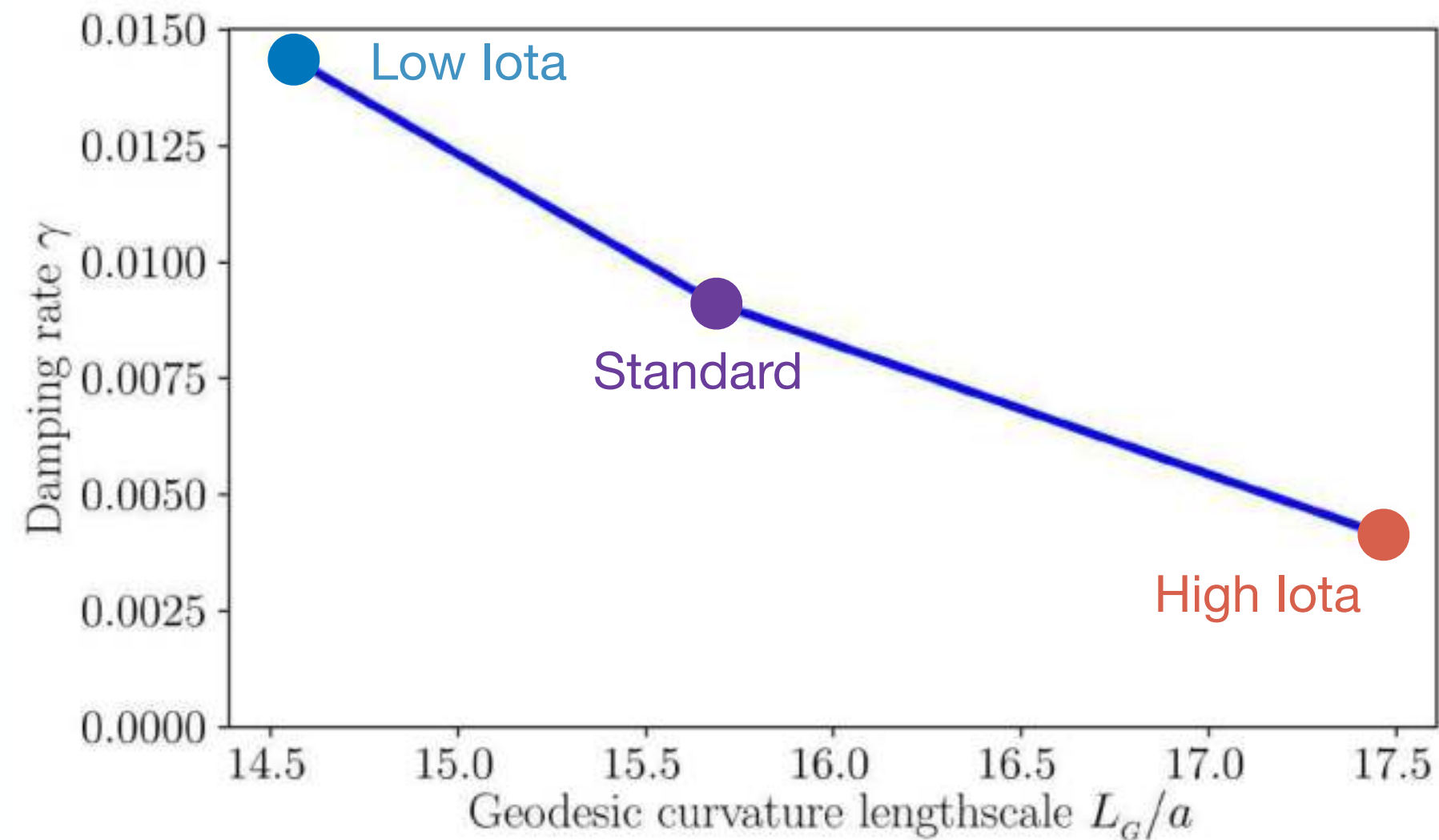
# How do zonal flows impact saturation?

- Zero zonal flow contrib. at each time step.
- Low  $\iota$  depends on zonal flows to reach saturation.
- Zonal Flows make no difference for High  $\iota$ .



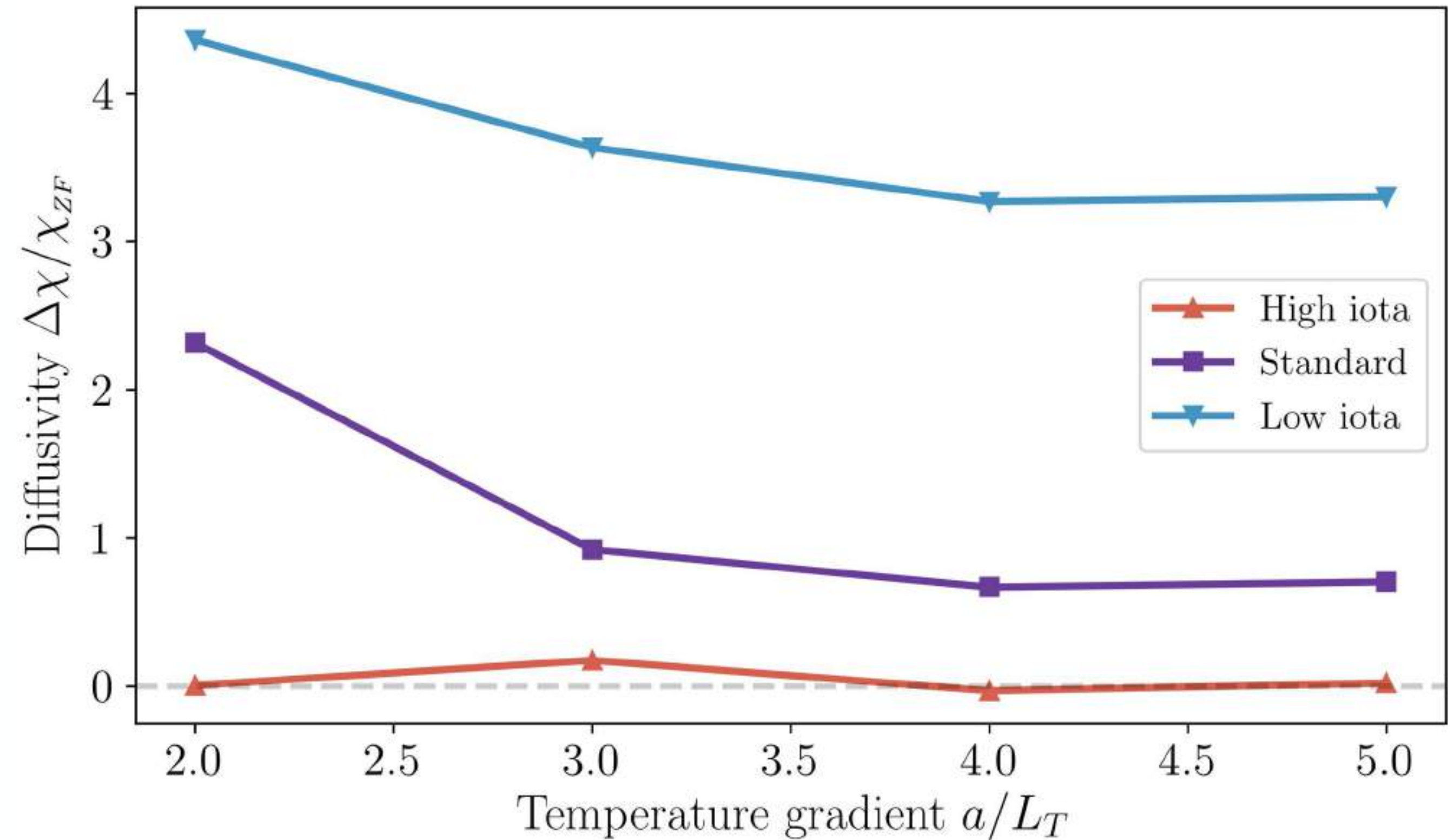
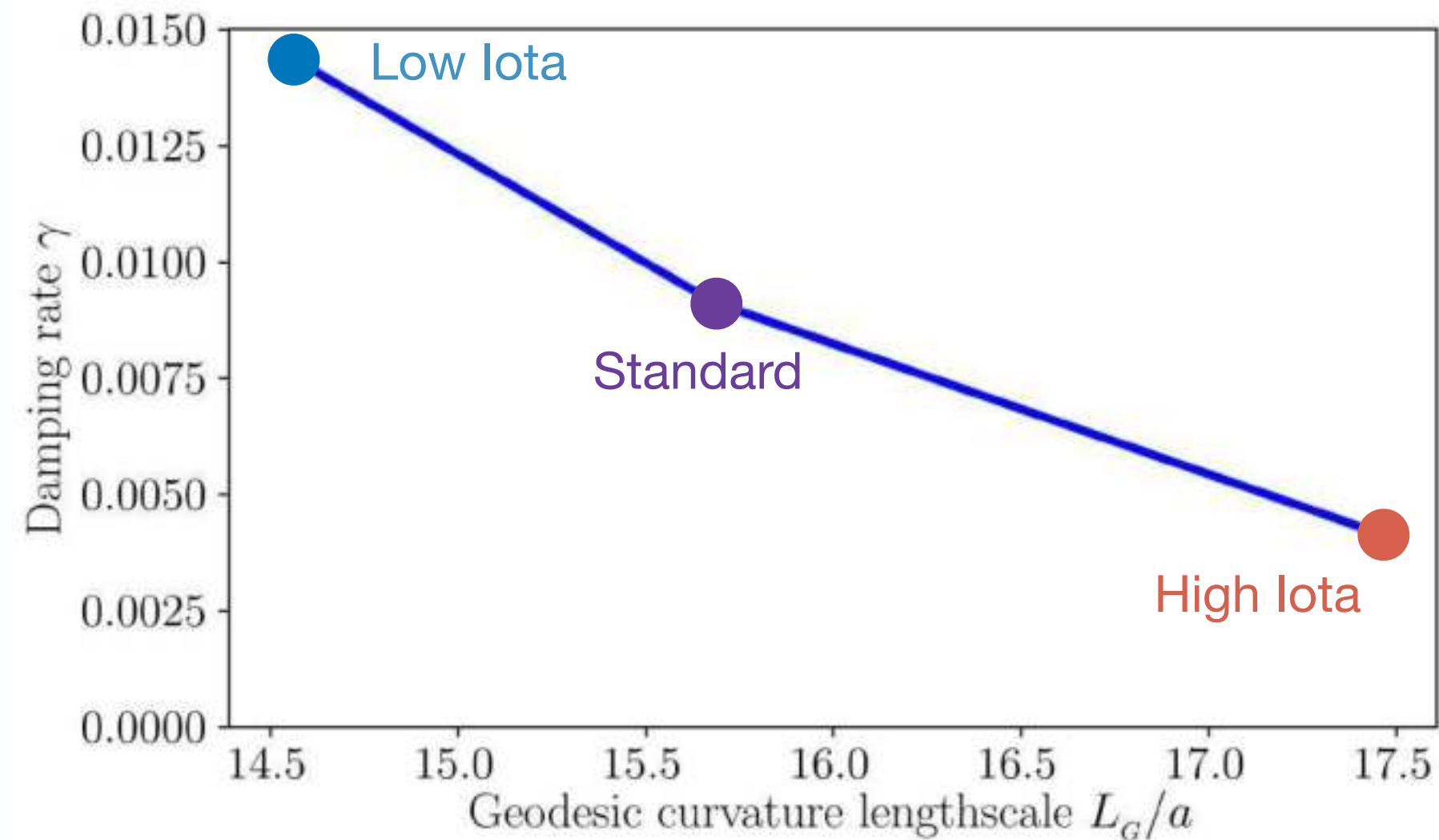
# Linear predictions don't help...

- Low  $\iota$  depends on zonal flows to reach saturation.
- Zonal Flows make no difference for High  $\iota$ .



# Linear predictions don't help...

- Low  $\iota$  depends on zonal flows to reach saturation.
- Zonal Flows make no difference for High  $\iota$ .

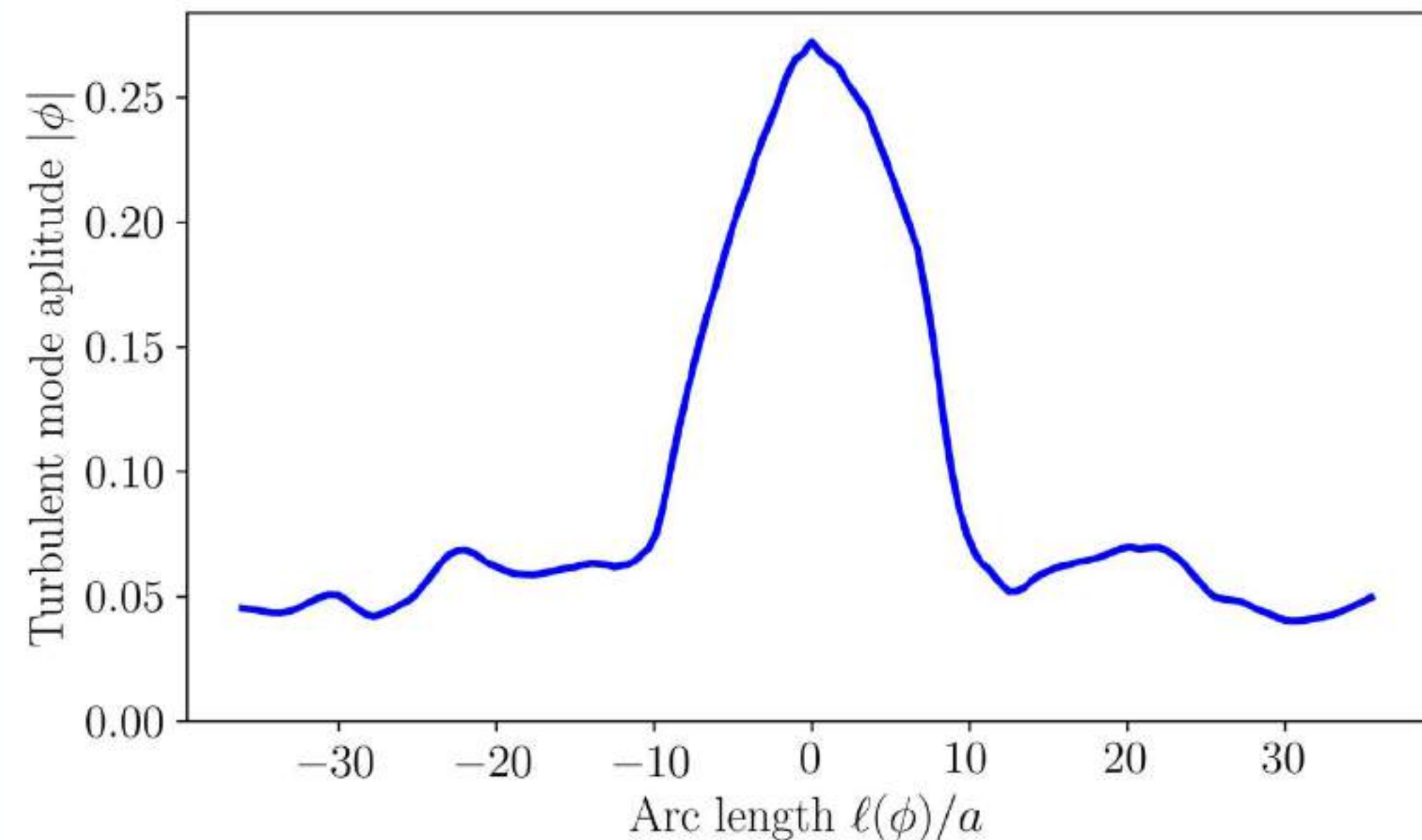


What about zonal flow generation?



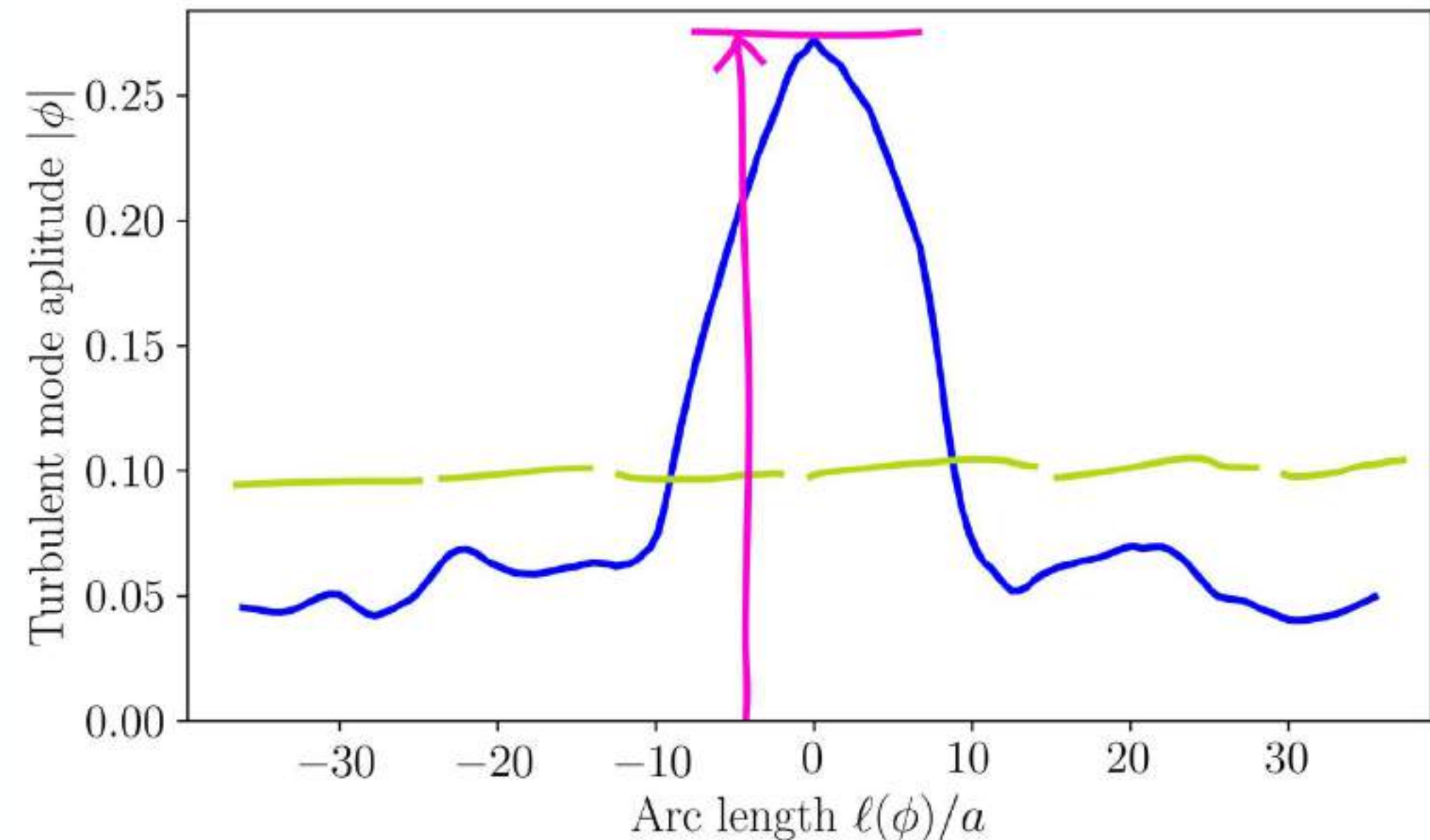
# The mode distribution influences zonal flow generation

- Secondary modes depend on primary.
- Strongest mode.



# The mode distribution influences zonal flow generation

- Secondary modes depend on primary.
- Strongest mode.
- Space filling factor:<sup>1</sup>
  - Estimates toroidal distribution.
- Configuration and gradient-dependent.

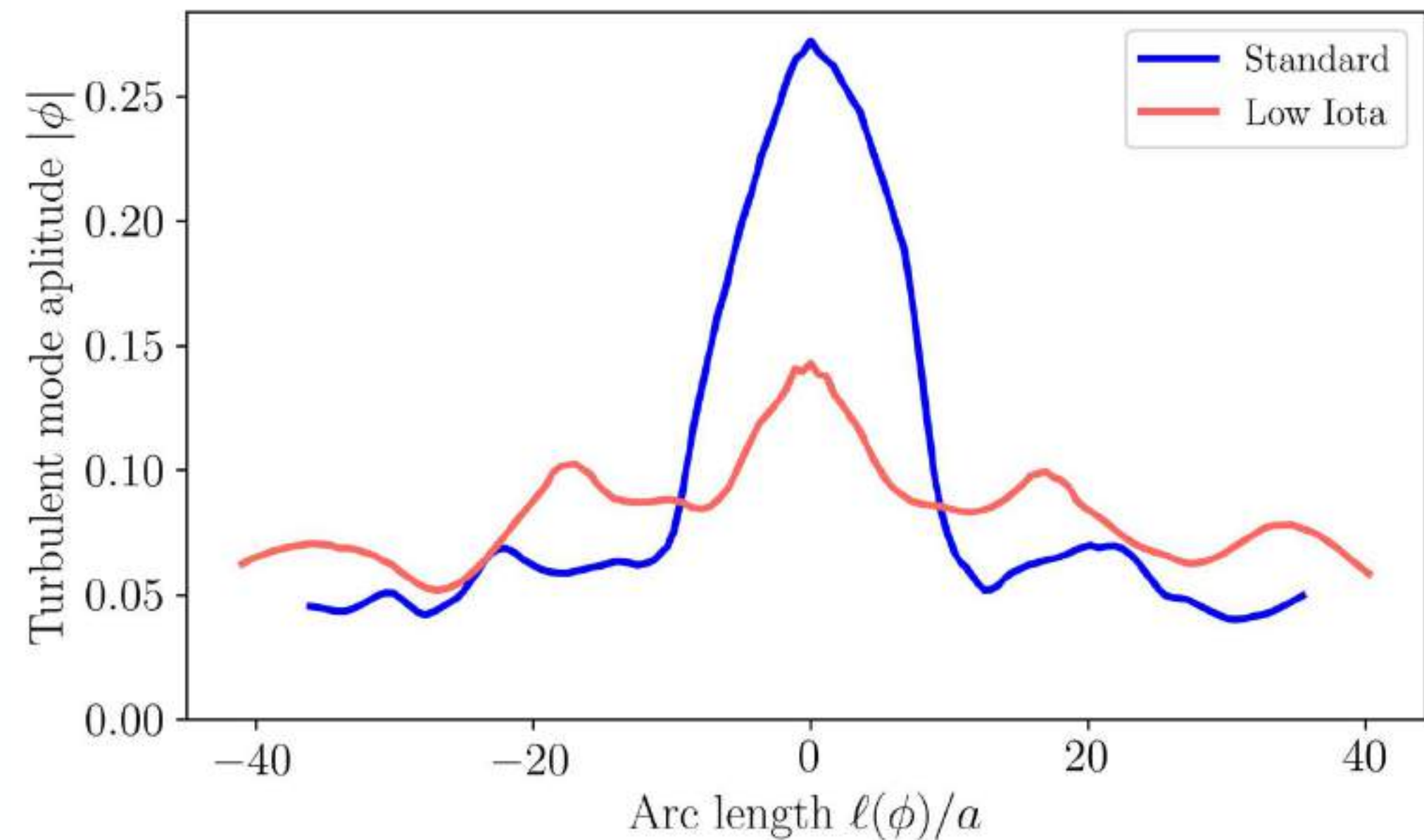


$$\sigma = \sqrt{|\hat{\phi}_p|^2} / \max(|\hat{\phi}_p|)$$

<sup>1</sup> G.G. Plunk et al, Physical Review Letters, 118(10), 105002 (2017).

# The mode distribution influences zonal flow generation

- Secondary modes depend on primary.
- Strongest mode.
- Space filling factor:<sup>1</sup>
  - Estimates toroidal distribution.
- Configuration and gradient-dependent.

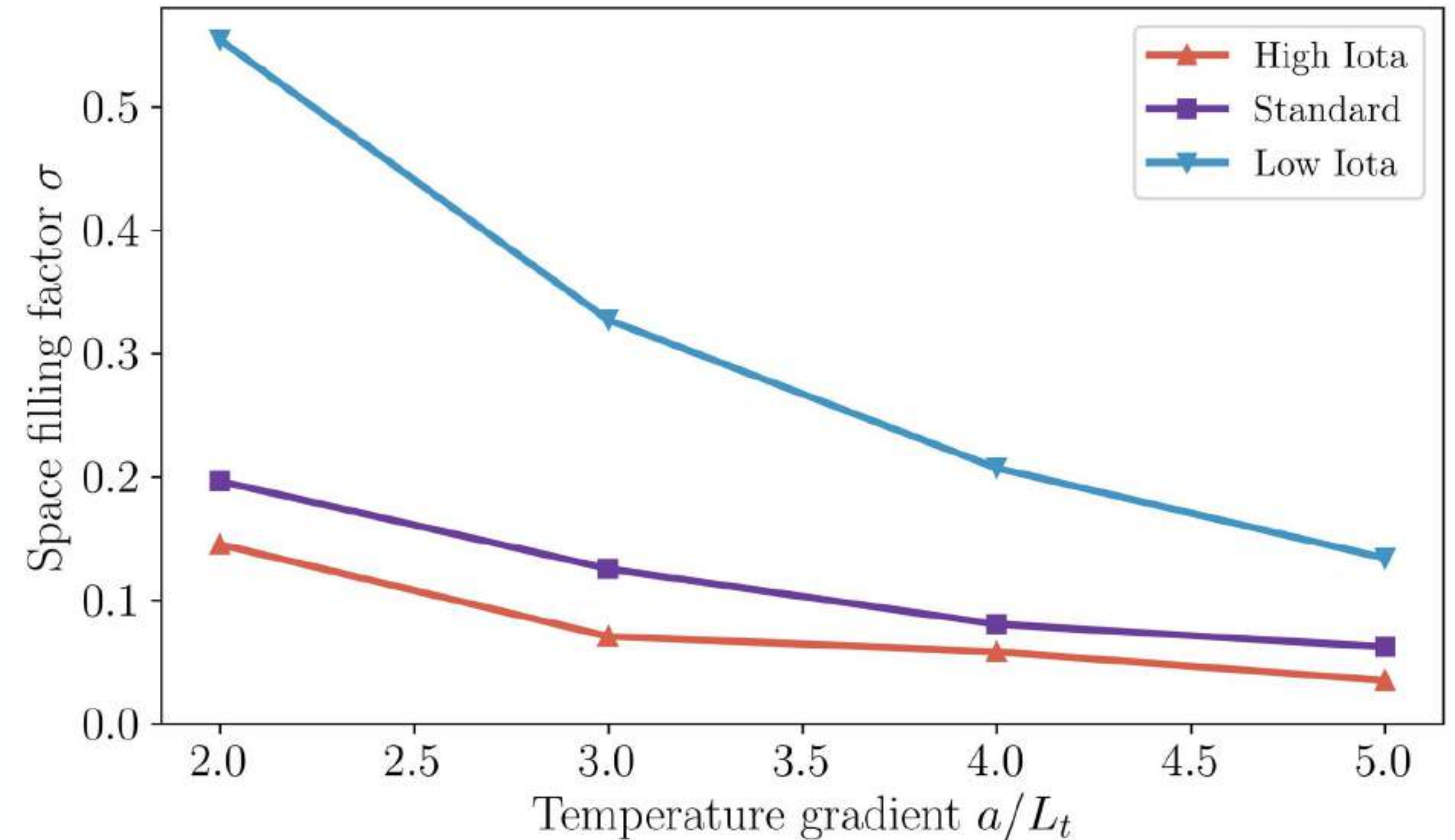


$$\sigma = \sqrt{|\hat{\phi}_p|^2 / \max(|\hat{\phi}_p|)}$$

<sup>1</sup> G.G. Plunk et al, Physical Review Letters, 118(10), 105002 (2017).

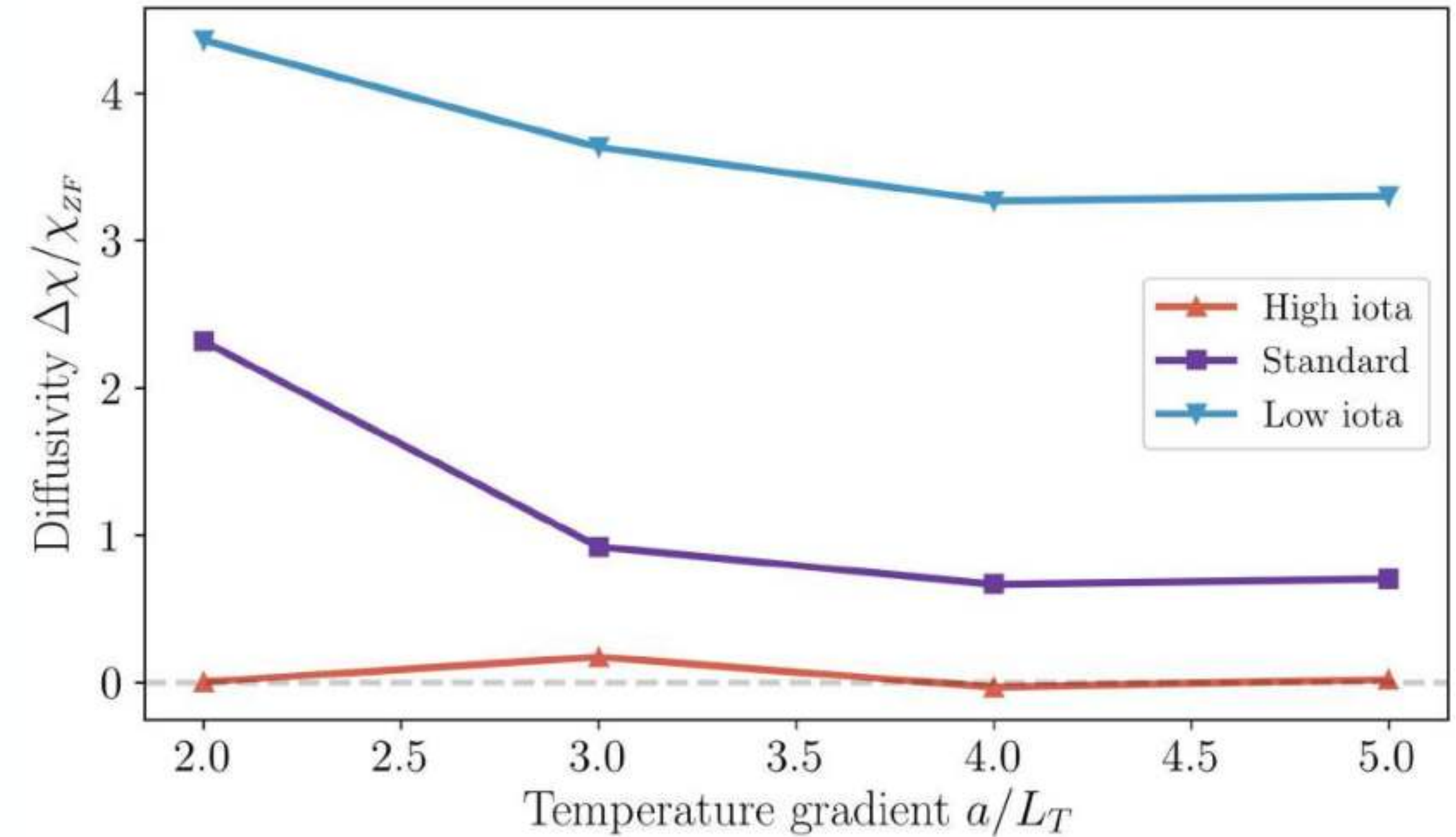
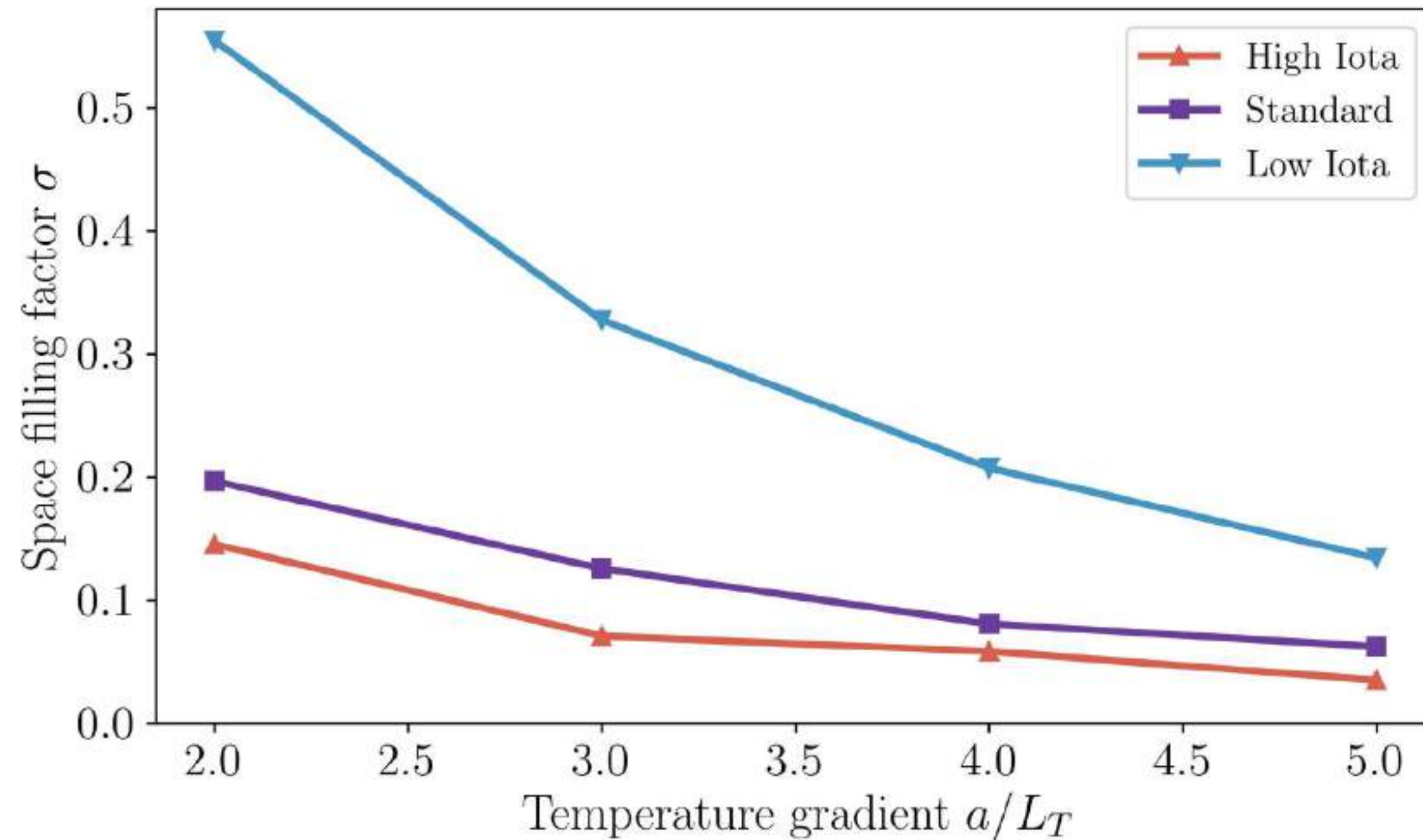
# The mode distribution influences zonal flow generation

- Very peaked modes in High iota.
- Mode is spread more evenly in Low iota.

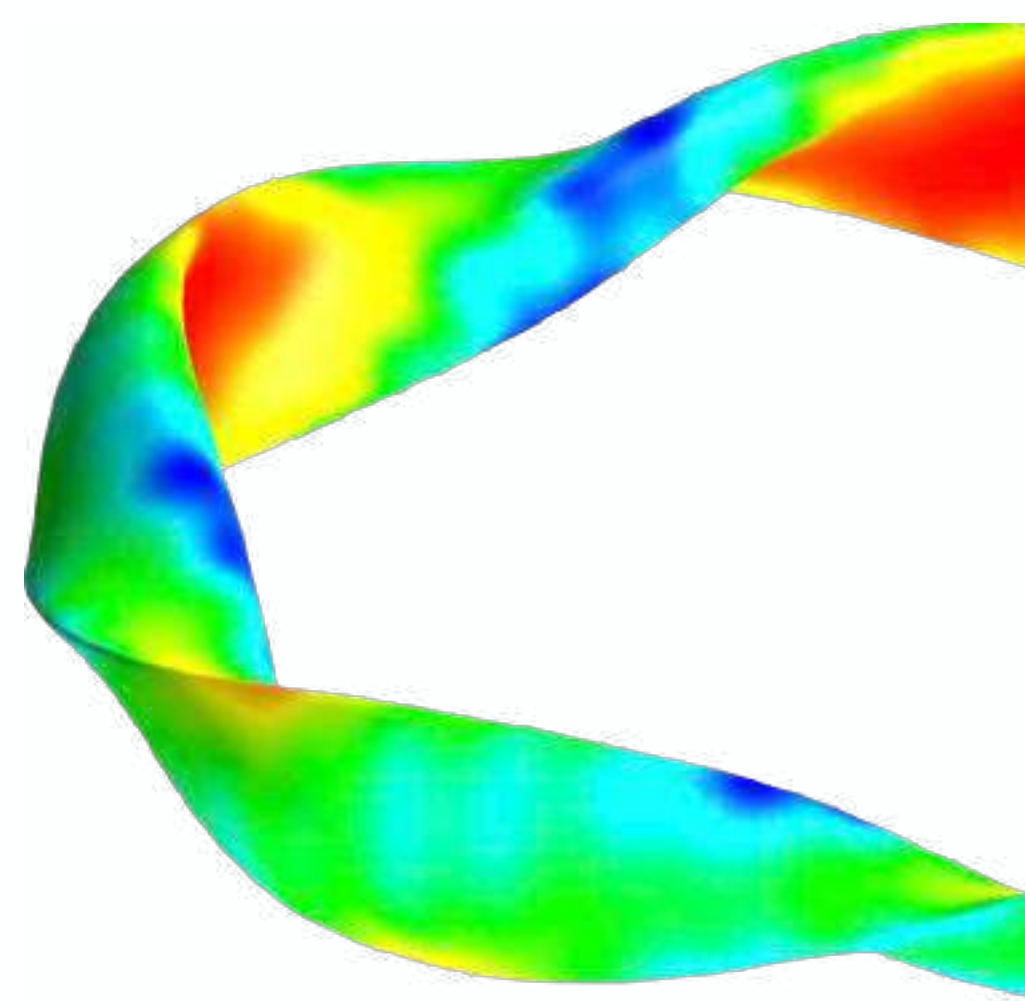


$$\sigma = \sqrt{|\hat{\phi}_p|^2 / \max(|\hat{\phi}_p|)}$$

# Saturation dynamics within configurations stand apart



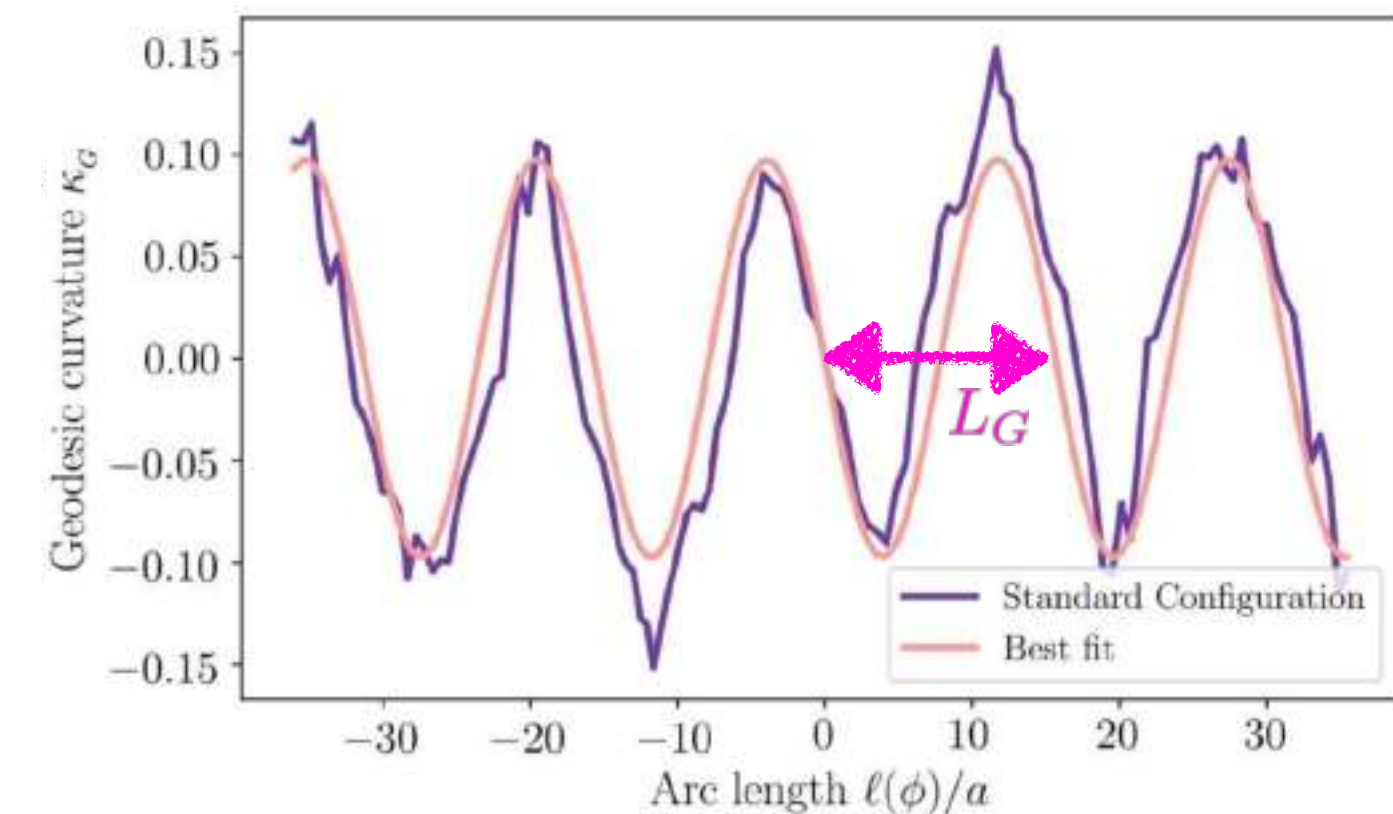
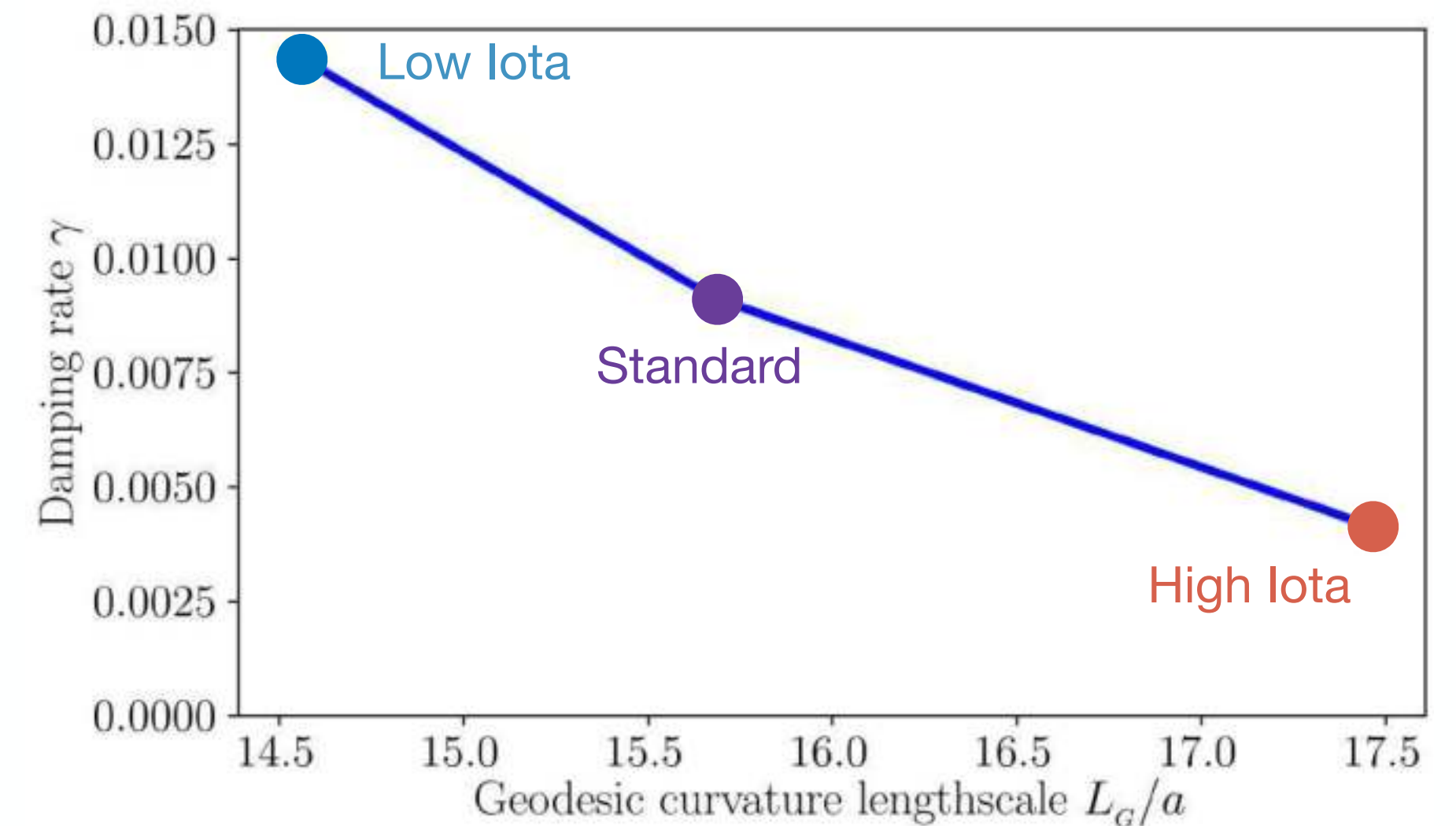
- Evenly spread primary modes: efficient generation of zonal flows.
- Low Iota: Stronger dependence on zonal flows for saturation.



Geometry in Wendelstein 7-X  
Gyrokinetic turbulence modeling  
Reduced transport modeling  
Summary

# What are the implications?

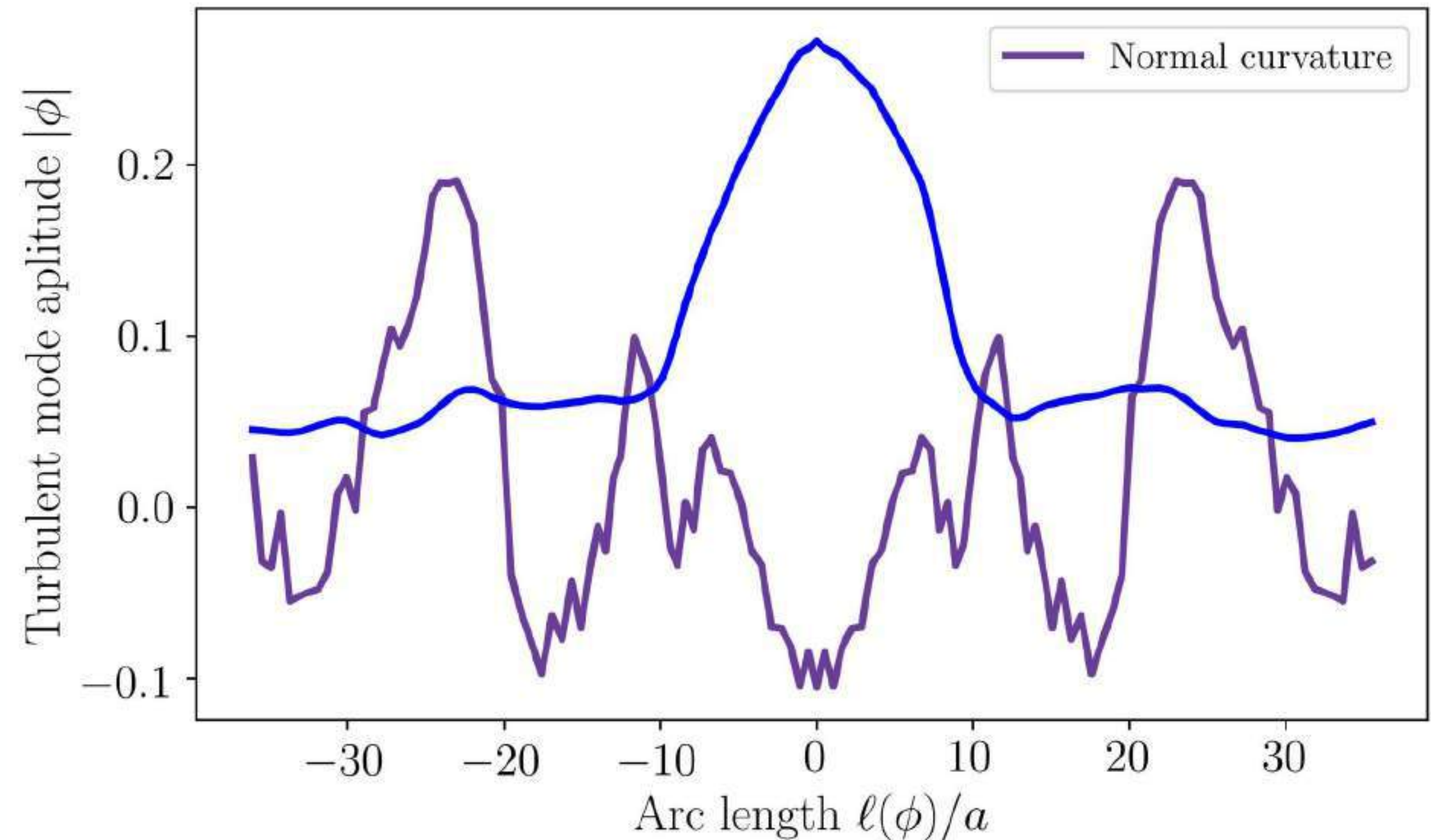
- Zonal flow damping: inverted predictions.
- $L_G$ : Rapid prediction of damping.
- Improve phenomenological transport models.<sup>1</sup>



<sup>1</sup> M. Nunami et al, Physics of Plasmas, 20(9), 092307 (2013).

# What are the implications?

- Zonal flow damping: inverted predictions.
- $L_G$ : Rapid prediction of damping.
- Improve phenomenological transport models.<sup>1</sup>
- $K_N \rightarrow \sigma$

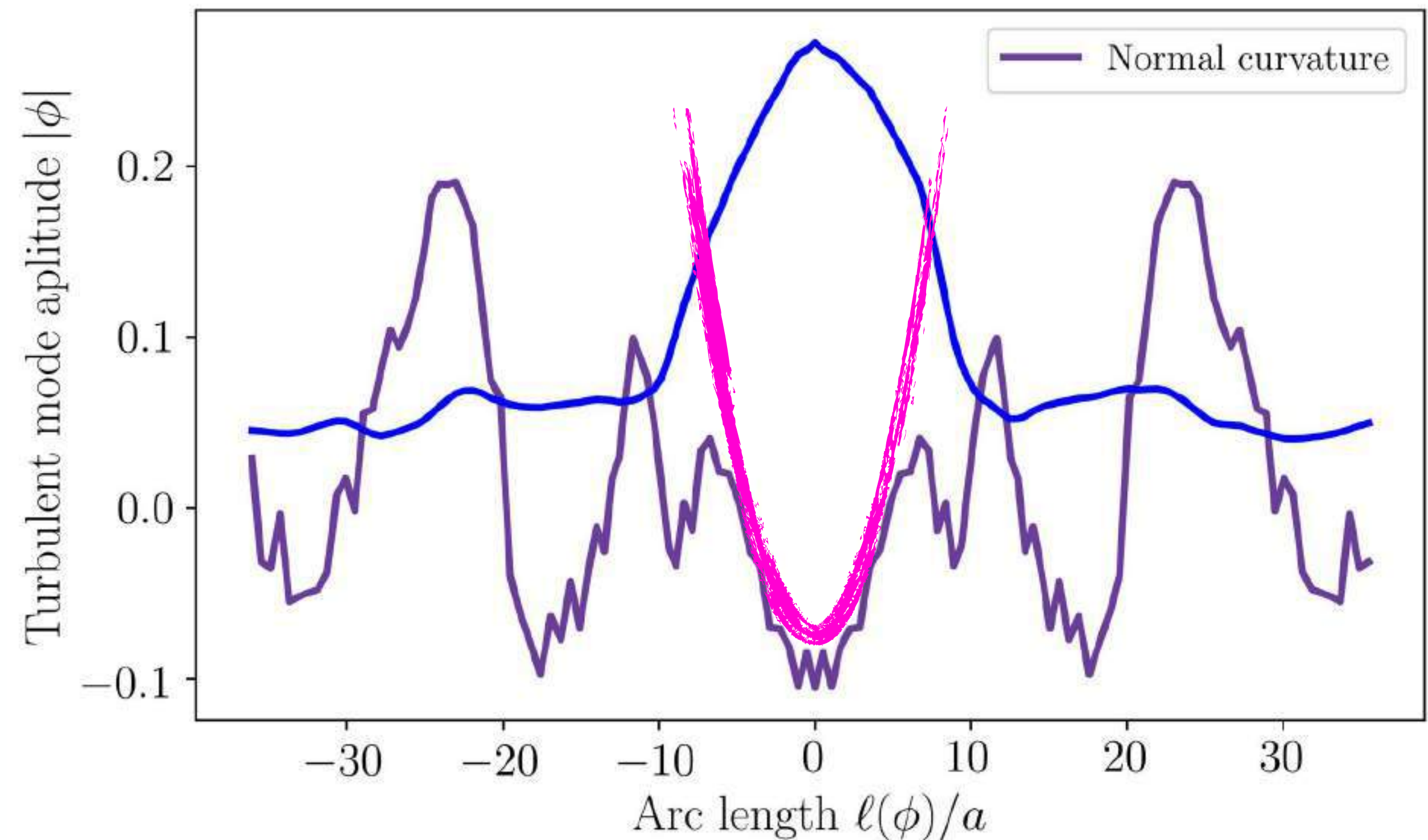


<sup>1</sup> M. Nunami et al, Physics of Plasmas, 20(9), 092307 (2013).



# What are the implications?

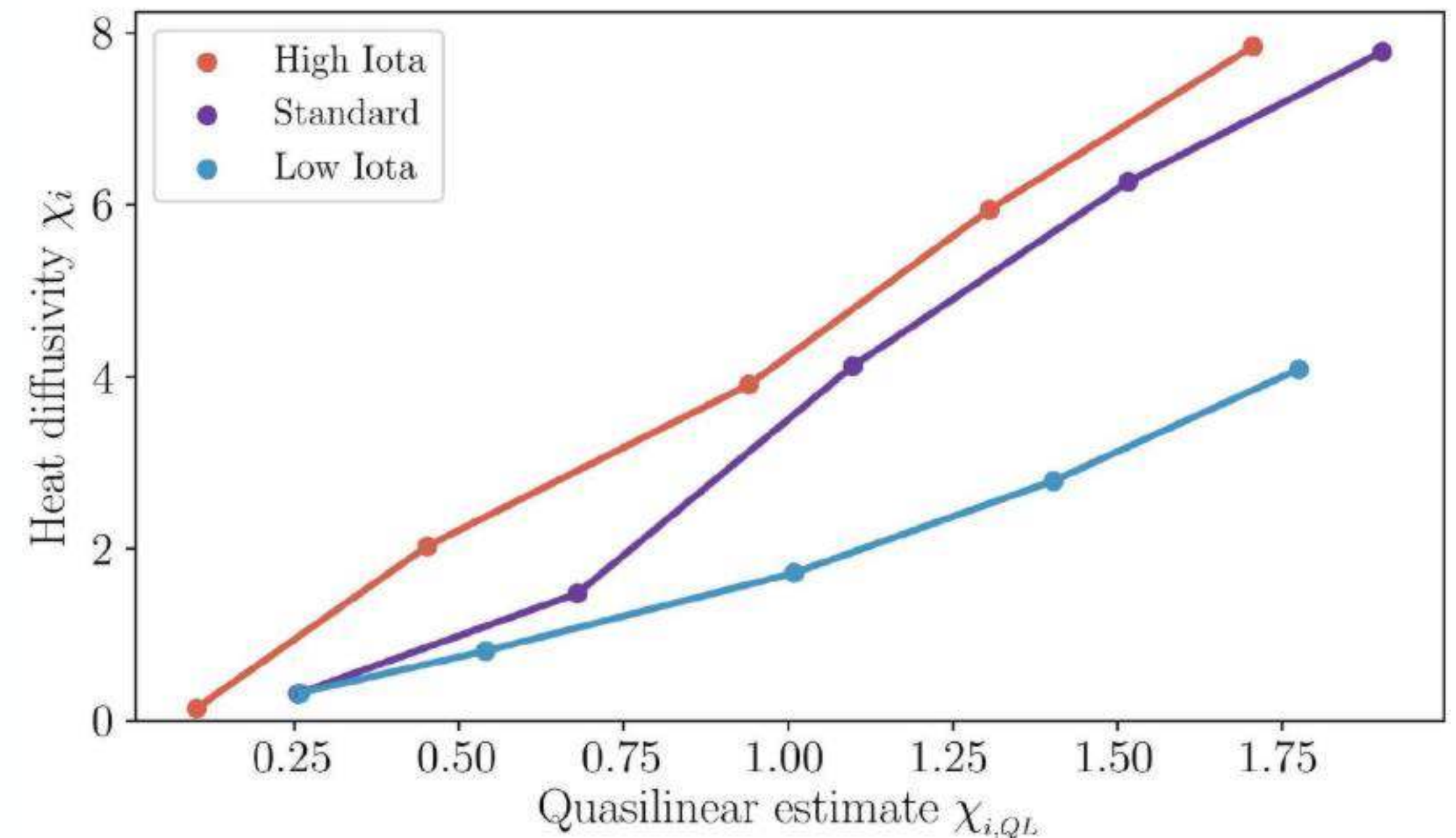
- Zonal flow damping: inverted predictions.
- $L_G$ : Rapid prediction of damping.
- Improve phenomenological transport models.<sup>1</sup>
- $K_N \rightarrow \sigma$
- *Drift well*: proxy for  $\sigma$ .



<sup>1</sup> M. Nunami et al, Physics of Plasmas, 20(9), 092307 (2013).

# What are the implications?

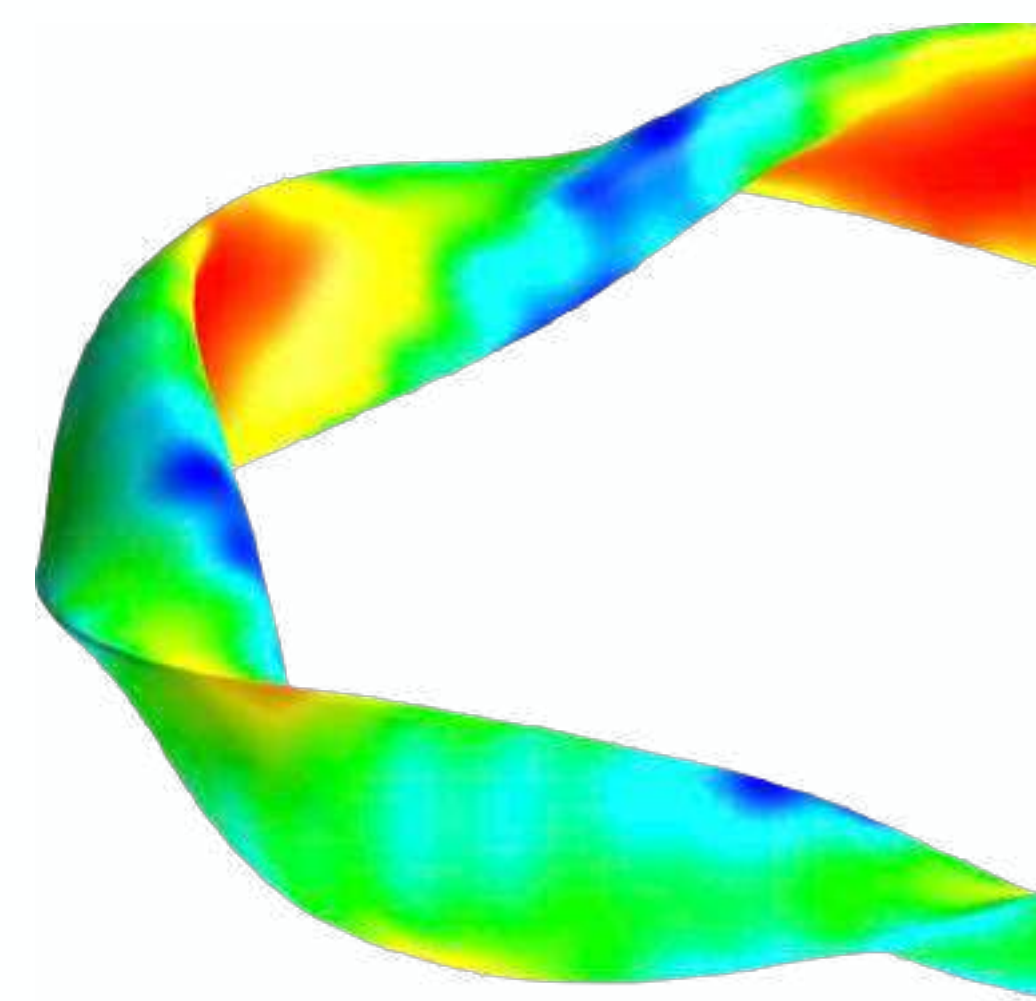
- Zonal flow damping: inverted predictions.
- $L_G$ : Rapid prediction of damping.
- Improve phenomenological transport models.<sup>1</sup>
- $K_G \rightarrow \sigma$
- *Drift well*: proxy for  $\sigma$ .
- Next: Extend QL estimate.



$$\chi_i = f(\textit{Geometry}, \textit{Linear physics})$$

<sup>1</sup> M. Nunami et al, Physics of Plasmas, 20(9), 092307 (2013).

# Summary

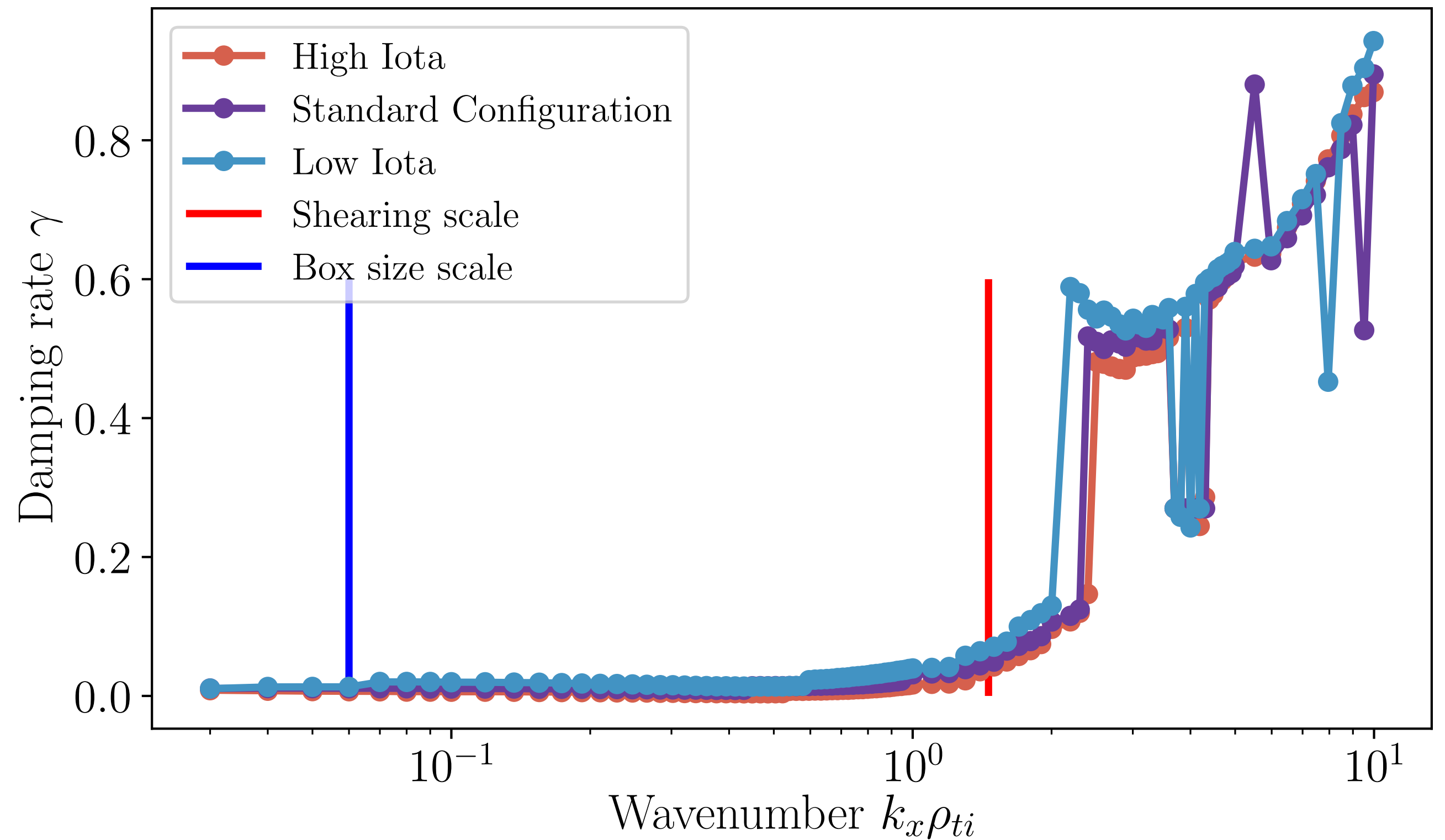


- Basis for improved transport modeling dwells in the field geometry.
- (Linear) Damping predictions are in disagreement with observations.
- Low  $\iota$ : Zonal flows have a bigger impact on transport.
- High  $\iota$ : Highly localized modes saturate through other mechanisms.
- Space-filling factor  $\sigma$ : Zonal flow generation.
- Outlook: Build a reduced model with geometry features and linear estimates.

**Backup slides**

# Transient damping rate as a function of the wavenumber

- Zonal flow damping rate depends on the radial length-scale
- Regions of interest: shearing scales and computational “box” size
- No significant variation observed at large scales
- Small scales reflect the same trend



# Linear zonal flow response and the effect of the upper integration limit

