



UKAEA

Preconditioners for large geometries

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Overview

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Concepts

Objective

Methodology

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Multi-physics Simulations

Modern science or engineering challenge

Create & control nuclear fusion reactions

Analytical Solution



Exact analysis
Theory, equations

Numerical Solution



Approximate solution
PDE, FEM, Montecarlo

Computational Solution



Framework & libraries
COMSOL, Ansys, MOOSE

PDE problem -> Linear System Problem -> Matrix problem

Direct Solver

Iterative Solver

Preconditioned Iterative Solver



Gauss

Jacobi, Gauss-Seidel, SOR

Krylov subspace methods

Expensive

Approximate results, less cost

appropriate HPC resources

MOOSE

User-Focused

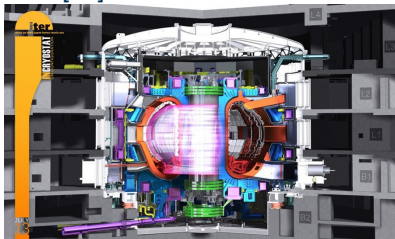
Finite Element Method

Massively parallelisation

HPC libraries:

PETSc * libMesh * Hypra

ITER[01]



Determine a *suitable preconditioner* that helps iterative matrix solutions of multiphysics problems.

There are many factors that affect the preconditioner performance:

- Type of the PDE
- Dimensions
- Solver type
- Boundary Conditions
- Nature of the matrix
- Material used
- Degrees of freedom (DoF)

[01] Requirements for obtaining a good preconditioner (PC):

- The preconditioned matrix should have a clustered eigenspectrum away from 0,
$$n_{iter} \leq \frac{1}{2} \sqrt{\xi(\mathbf{G})} \ln \frac{2}{\epsilon} + 1$$
- The PC should be as cheap to compute as possible,
- Its application to a vector should be cost-effective.

This work considers:

- Execution time (seconds)
- Memory usage (megabytes)
- Accuracy in the value of $|R|$
- Number of linear iterations

Preconditioners (PCs): (Based on PETSc [02] - pc_type)

asm , bjacobi , cholesky , eisenstat , exotic , gamg , hypre , hypre_boomeramg , hypre_euclid , hypre_parasails , hypre_pilut , icc , ilu , jacobi , ksp , lu , mg , ml , none , pfm , redundant , sor , telescope , tfs , pbjacobi , vpbjacobi , shell , lsc , redistribute , svd , kaczmarz , gasm , composite , nn , mat , fieldsplit , galerkin , cp , patch , syspfm , bddc , lmvm

Physical Characteristics: (Based on MOOSE Modules [03]):

- Thermal2D (Heat Conduction)
- Displacement3D (Finite Strain Elastic Stress)
- Pressure 3D (Finite Strain Elastic Stress)
- Fluid 3D (Navier-Stokes)

Execution mode:

- Serial
 - Parallel
 - MPI (2p, 4p, 8p, 16p, 32p)
 - OpenMP (2t, 4t, 8t, 16t, 32t)
 - Hybrid (1t32p, 2t16p, 4p8t, 8t4p, 16t2p, 32t1p)
- p -> processes, t -> threads*

PROCEDURE:

- 1- Running ten times the following combination:
42 preconditioners * 4 physical characteristics * 17 execution modes.
- 2- If converged:
Calculate the standard deviation of the execution time of each run.
Plot execution time, memory, number of iterations, and $|R|$.
Apply the Pareto efficiency in the data already plotted.

Cumulus cluster at UKAEA

Hardware

The cumulus nodes have 65 general worker nodes in cumulus, and 6 GPU nodes. Each node has 32 CPU cores of spec - Intel(R) Xeon(R) CPU E5-2683 v4 @ 2.10GHz

Each has 512GB of RAM, and are connected to each other via a FDR Infiniband network.

The home file system is Lustre.

Software

Python 3.6.4

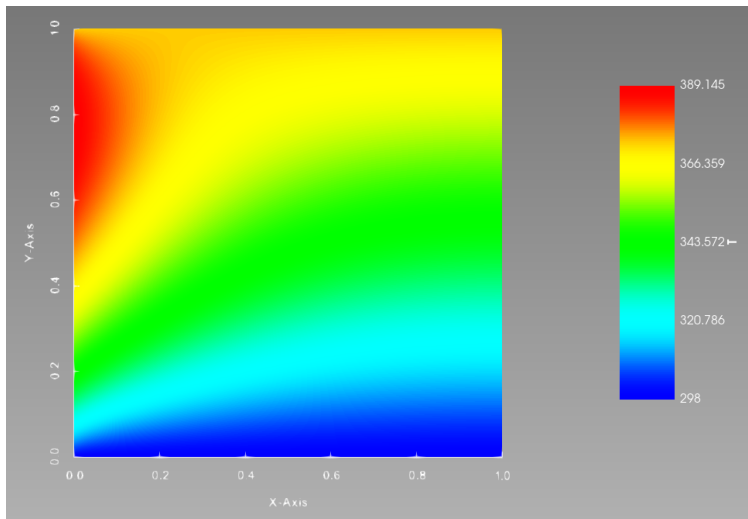
cmake 3.16.4

GCC 7.3.0

Openmpi 3.1.1

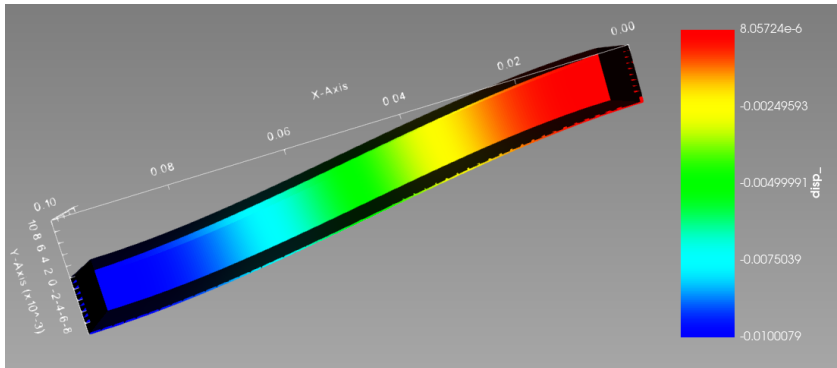
Heat Conduction 2D

Steady state on steel 1cm x 1cm - Mesh 128 x 128

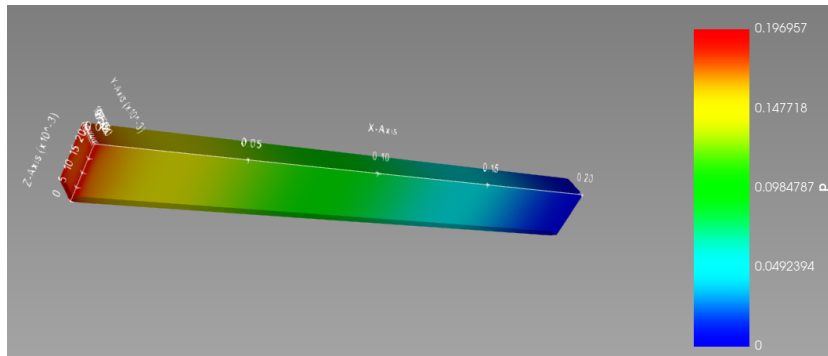


Compute Finite Strain Elastic Stress

Displacement in 3D ($x = 0.10\text{cm}$, $y = 0.01\text{cm}$, $z = 0.02\text{cm}$)



Fluid in 3D ($x = 0.20\text{cm}$, $y = 0.01\text{cm}$, $z = 0.02\text{cm}$)

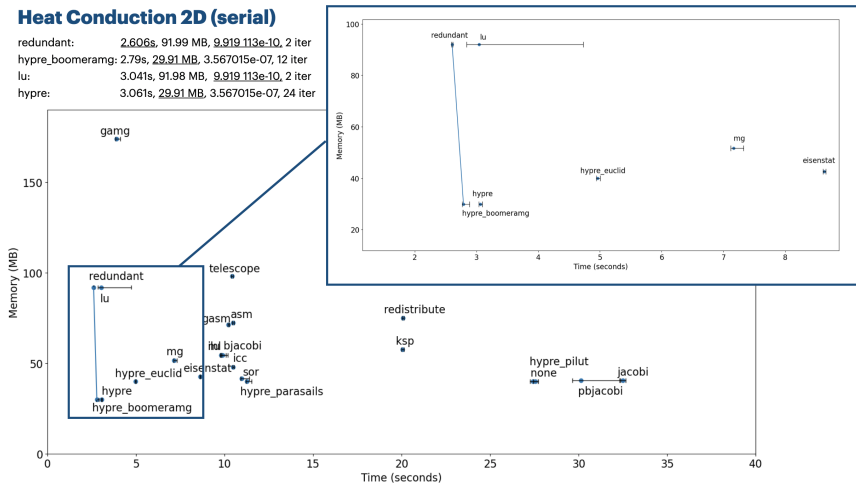


Results

* gamg (3.893 s, 174.026 MB, 1.425356e-06, 23 iterations)

Heat Conduction 2D (serial)

redundant: 2.606s, 91.99 MB, 9.919 113e-10, 2 iter
hypr_boomeramg: 2.79s, 29.91 MB, 3.567015e-07, 12 iter
lu: 3.041s, 91.98 MB, 9.919 113e-10, 2 iter
hypr: 3.061s, 29.91 MB, 3.567015e-07, 24 iter



MPI 16 proc: gamg (1.290s) | OpenMP 16 threads: hypre(1.384s)

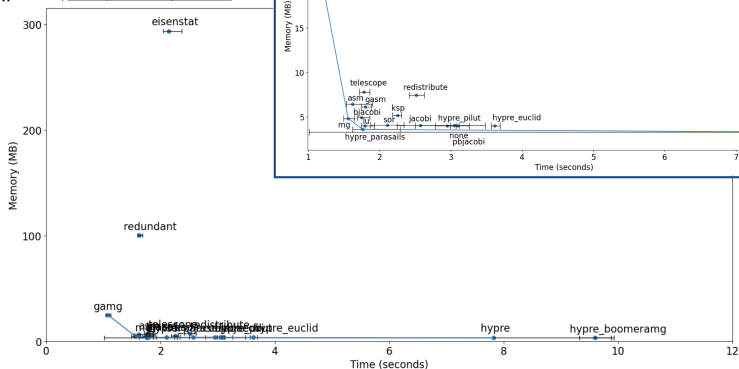
Heat Conduction 2D (Hybrid 2t 16 p)

gamg: 1.083s, 24.89 MB, 4.194201e-06, 23 iter

mg: 1.559s, 4.81 MB, 5.027078e-06, 1092 iter

hypre_parasails: 1.793s, 3.98 MB, 4.943789e-06, 1702 iter

hypre: 7.826s, 3.29 MB, 1.119766e-06, 11 iterations



Heat Conduction 2D

"Good preconditioners"

asm bjacobi eisenstat gamg hypre hypre_boomeramg hypre_euclid
hypre_parasails hypre_pilut icc ilu jacobi ksp lu mg ml none redundant
sor telescope pbjacobi redistribute gasm

"Bad Preconditioners"

cholesky exotic pfmng tfs vpbjacobi shell lsc svd kaczmarz composite nn
mat fieldsplit galerkin cp patch syspfmg bddc lmvm

Compute Finite Strain Elastic - Displacement 3D (serial)

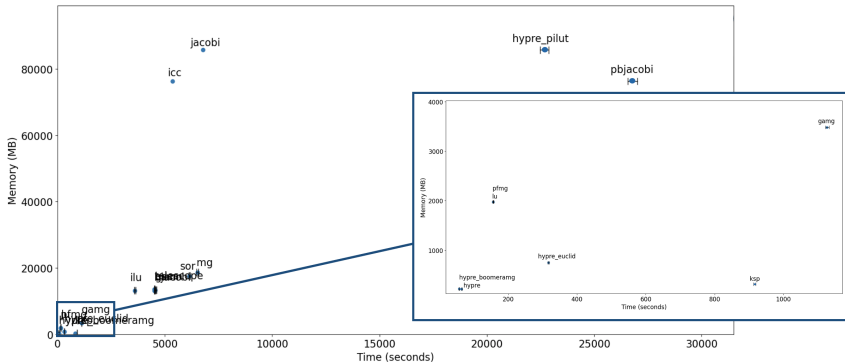
hypr_boomeramg: 57.652 seconds, 219.74 MB, 0.004452565, 50 iterations

hypr: 65.037 seconds, 216.06 MB, 0.005185383, 46 iterations

lu: 156.239 seconds, 1977.42 MB, 0.01921162, 2 iterations

ksp: 822.979 seconds, 316.84 MB, 0.005962132, 6 iterations

gamg: 1126.2 seconds, 3482.23 MB, 0.009540412, 659 iterations

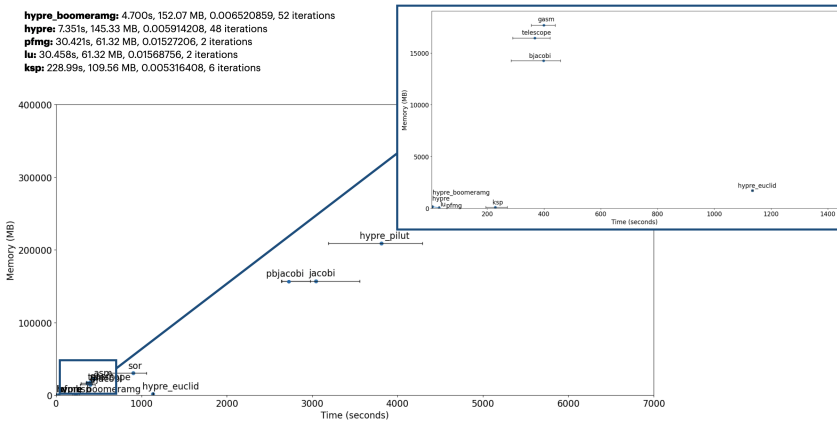


Results

MPI 32 processes: hypr_boomeramg (5.476s)
OpenMP 32 threads: hypr_boomeramg(8.275s)

Compute Finite Strain Elastic - Displacement 3D (Hybrid - 8t4p)

hypr_boomeramg: 4.700s, 152.07 MB, 0.006520859, 52 iterations
hypr: 7.351s, 145.33 MB, 0.005914208, 48 iterations
pfmg: 30.421s, 61.32 MB, 0.01527206, 2 iterations
lu: 30.458s, 61.32 MB, 0.01568756, 2 iterations
ksp: 228.99s, 109.56 MB, 0.005316408, 6 iterations



Finite Strain Elastic Stress 3D

"Good preconditioners"

asm bjacobi cholesky gamg hypre hypre_boomeramg hypre_euclid
hypre_pilut icc ilu jacobi ksp lu mg ml none pfmng sor telescope pbjacobi
gasm mat cp

"Bad Preconditioners"

hypre_parasails redundant tfs vpbjacobi shell lsc redistribute svd
kaczmarz composite nn fieldsplit galerkin

Navier-Stokes 3D (serial)

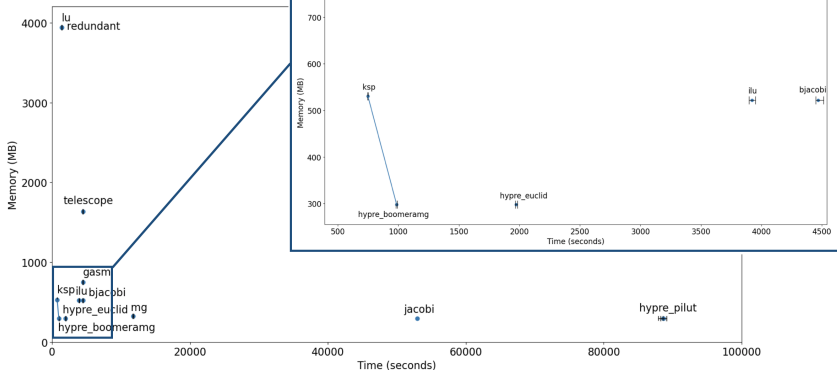
ksp: 748.22s, 530.58 MB, 1.699691e-20, 8 iterations

hypr_boomeramg: 985.9s, 297.88 MB, 5.766776e-20, 173 iter

redundant: 1387.3s, 3940.35 MB, 1.843801e-21, 8 iterations

lu: 1391.1s, 3940.35 MB, 1.843801e-21, 8 iterations

hypr_euclid: 1973.8s, 297.88 MB, 5.708037e-20, 575 iter



MPI 32 processes: hypre_boomeramg (30.792 seconds)

OpenMP 32 threads: telescope (191.00 seconds)

Navier-Stokes (Hybrid - 1t32p)

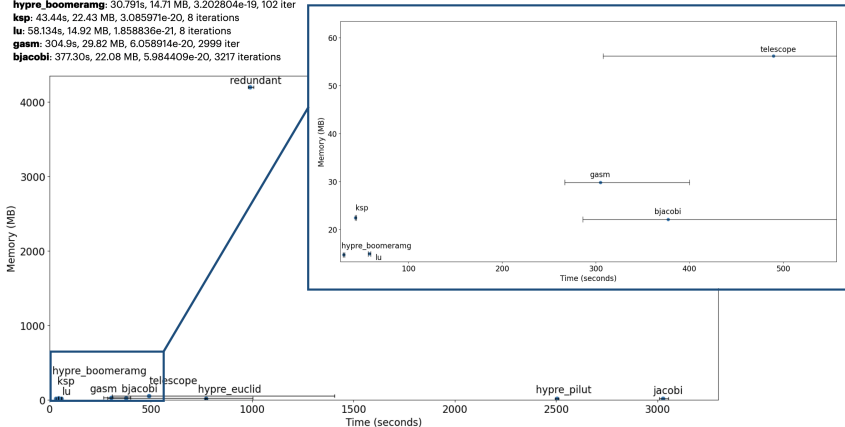
hypre_boomeramg: 30.791s, 14.71 MB, 3.202804e-19, 102 iter

ksp: 43.44s, 22.43 MB, 3.085971e-20, 8 iterations

lu: 58.134s, 14.92 MB, 1.858836e-21, 8 iterations

gasm: 304.9s, 29.82 MB, 6.058914e-20, 2999 iter

bjacobi: 377.30s, 22.08 MB, 5.984409e-20, 3217 iterations



Navier -Stokes 3D

"Good preconditioners"

asm bjacobi cholesky hypre_boomeramg hypre_euclid hypre_pilut icc ilu
jacobi ksp lu mg ml none redundant telescope redistribute gasm
composite cp

"Bad Preconditioners" eisenstat exotic gamg hypre hypre_parasails
pfmtg sor tfs pbjacobi vpbjacobi shell lsc svd kaczmars nn mat fieldsplit
galerkin

Open questions and next steps

- Parallelisation behaviour (communication, synchronization)
- Scalability of each preconditioner
- Other HPC architectures
- Solver types
- Finer grids
- Internode
- Optimization flags
- MultiPDEs simulations

Acknowledgement

- Andrew Davis (Line Manager at UKAEA)
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[01] source: <https://www.diariomovil.info/2020/08/07/iter-el-futuro-de-la-fusion-nuclear-comercial/>

[02] Preconditioning for Sparse Linear Systems at the Dawn of the 21st Century: History, Current Developments, and Future Perspectives, 2012

<http://downloads.hindawi.com/archive/2012/127647.pdf>

[03] KSP: Linear System Solvers y Preconditioners

<https://docs.petsc.org/en/latest/manual/ksp/preconditioners>

[04] Physics Modules on MOOSE

<https://mooseframework.inl.gov/modules/index.html>