Implementation of a Q2D turbulence model and detection of flow instabilities in liquid metal MHD flows

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Outline

1. Introduction
2. Computation Resources
3. The potential of 2D model
4. The 2D model in buoyant flows
5. The potential of Q2D model
6. FFT2: instability detection method
7. Q2D and FFT2 validation cases
8. Q2D and FFT2 application to 3D (Q2D) case
9. Final remarks

Implementation of a Q2D turbulence model and detection of flow instabilities in liquid metal MHD flows

- The study of the liquid metal MHD flows is here applied to the breeding blankets (BB), specially to the Dual Coolant Lead-Lithium (DCLL) design.

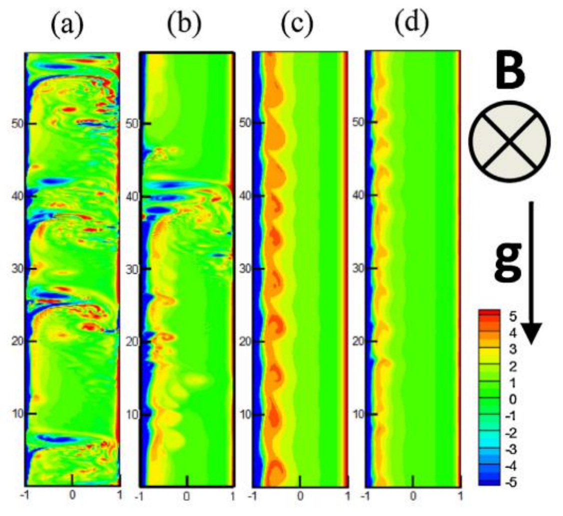

Fig. 2. Vorticity snapshots in a turbulent mixed-convection flow. Strong turbulence: (a) $Ha = 50$, $Gr = 10^8$, $Re = 5000$; (b) $Ha = 75$, $Gr = 10^8$, $Re = 7500$. Weak turbulence: (c) $Ha = 75$, $Gr = 5 \times 10^7$, $Re = 10{,}000$; (d) $Ha = 120$, $Gr = 10^8$, $Re = 5000$.

Smolentsev, S., Vetcha, N., & Abdou, M. (2013). Effect of a magnetic field on stability and transitions in liquid breeder flows in a blanket. *Fusion Engineering and Design*, *88*(6–8), 607–610. https://doi.org/10.1016/j.fusengdes.2013.04.001

Introduction

- 2D fully-developed flow

  vs

- 3D unstable flow

  vs

- Q2D unstable flow

Prototypical Shercliff Case (insulated walls MHD flow)

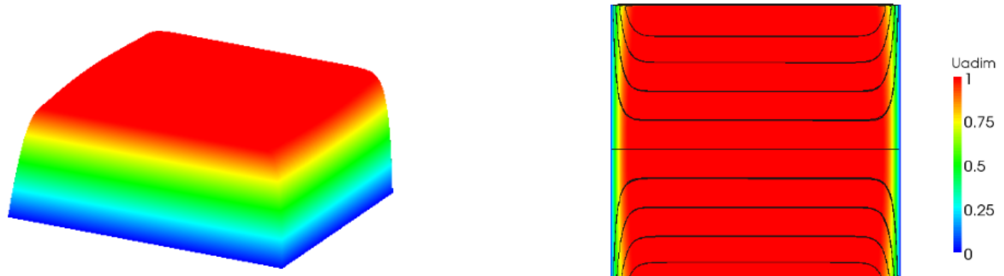

Figure 3.3: $Ha = 300$, $N = 9 \cdot 10^3$, $C_{w,side} = C_{w,Ha} = 0$, velocity profile on the left and electric current stream lines on velocity on the right.

Elisabet Mas de les Valls. *Development of a simulation tool for MHD flows under nuclear fusion conditions.* PhD dissertation, Dept. of Physics and Nuclear Engineering, Universitat Politècnica de Catalunya, 2011.

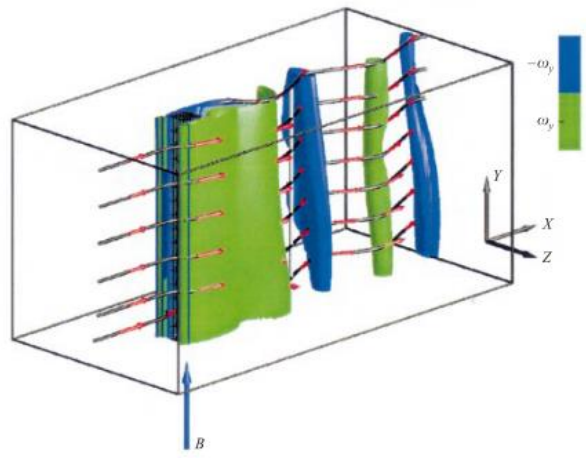

FIGURE 6. Isovorticity surface plot at $Re = 200$, $M = 14.2$, $N = 1$, $\omega_y = \pm 6$; surfaces $\omega_x = \pm 3$ are not present.

J. Fluid Mech. (2000), vol. 418, pp. 265–295. Printed in the United Kingdom
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Three-dimensional MHD flows in rectangular ducts with internal obstacles

By B. MÜCK[1], C. GÜNTHER[1], U. MÜLLER[1] AND L. BÜHLER[1]

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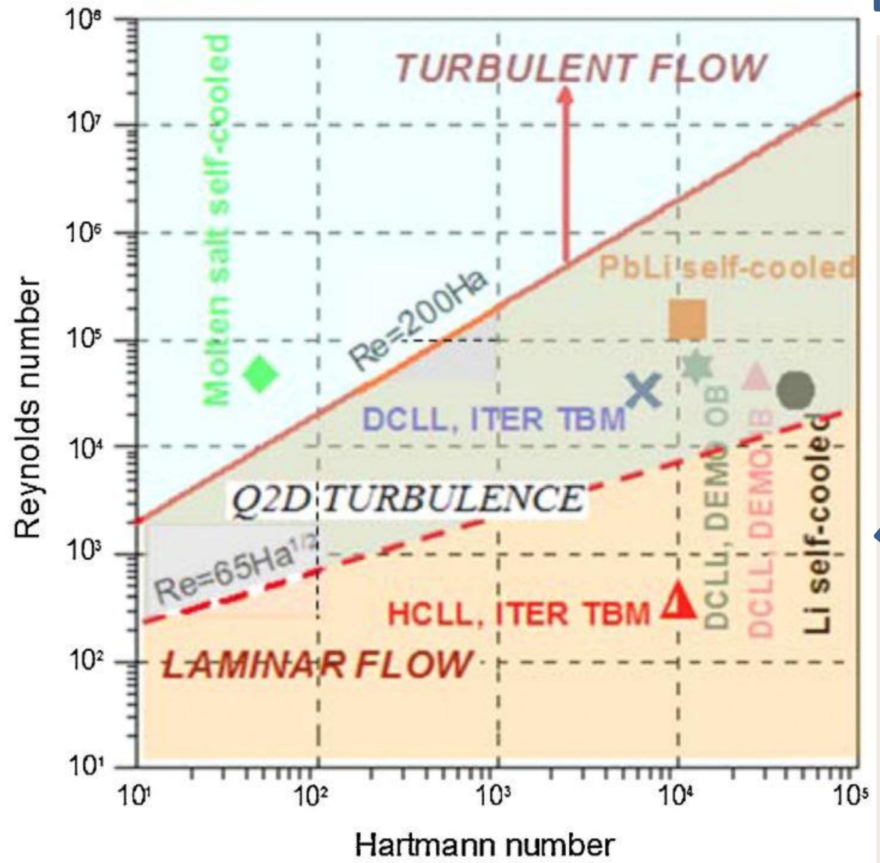


S. Smolentsev, S. Badia, R. Bhattacharyay, L. Buhler, L. Chen, Q. Huang, H. G. Jin, D. Krasnov, D. W. Lee, E. Mas De Les Valls, C. Mistrangelo, R. Munipalli, M. J. Ni, D. Pashkevich, A. Patel, G. Pulugundla, P. Satyamurthy, A. Snegirev, V. Sviridov, P. Swain, T. Zhou, and O. Zikanov. An approach to verification and validation of MHD codes for fusion applications. *Fusion Engineering and Design*, 100, 2015.

Computation Resources

- Computationally very expensive

  (unsteady, 3D, turbulent or Q2D)

  ~10^7 cells

  (and needs to solve each time step!)

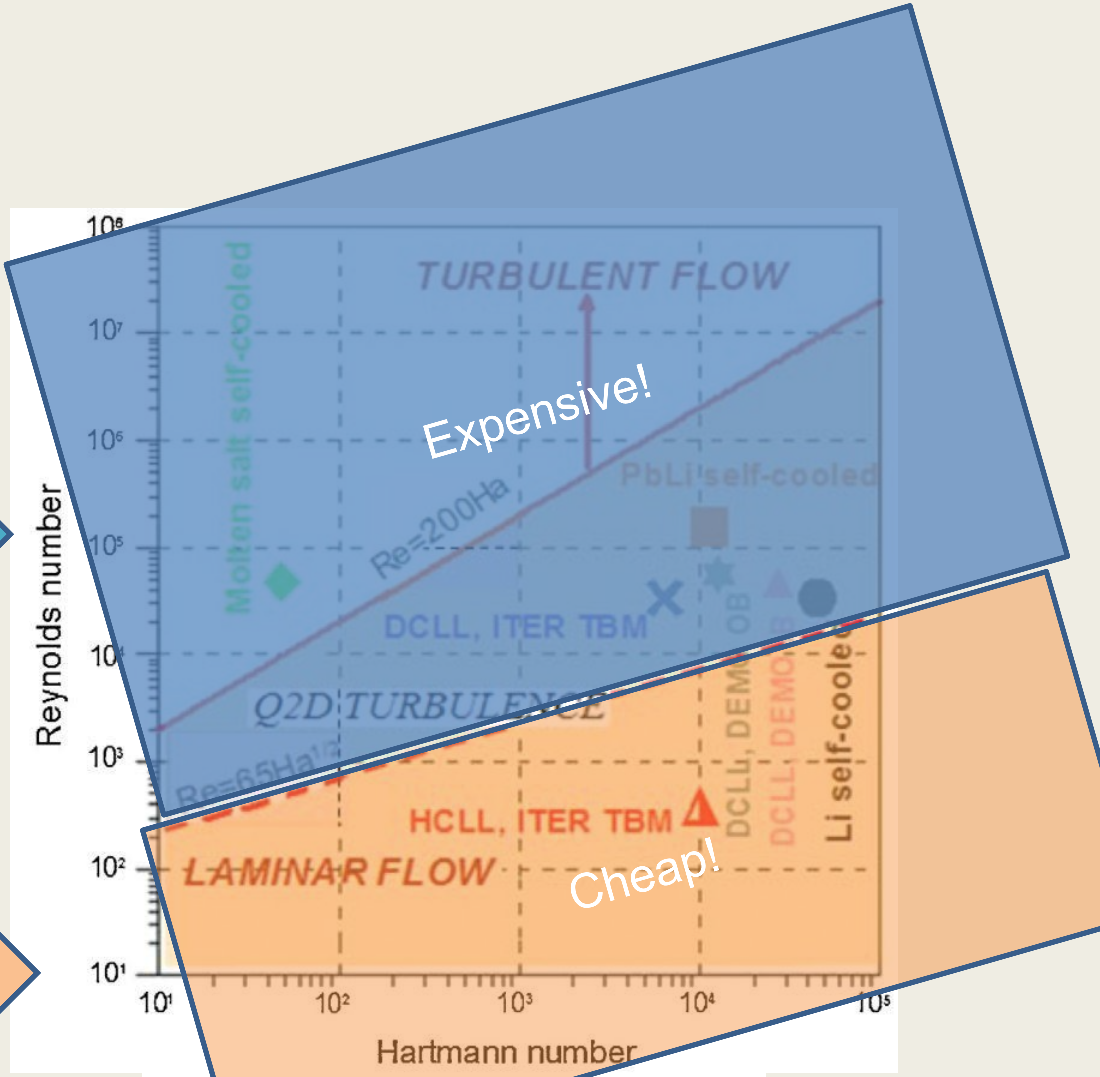
vs

- Computationally cheap

  (2D fully developed)

  ~10^5 cells

  (and steady!)



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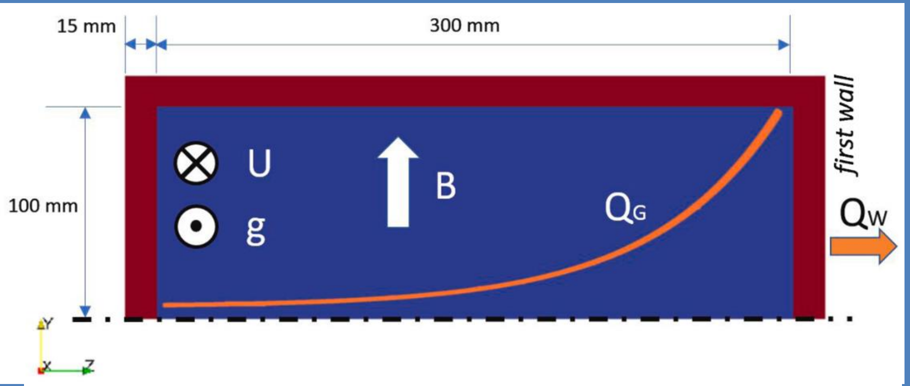
The potential of 2D model



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Liquid metal MHD flow influence on heat transfer phenomena in fusion reactor blankets

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Departament de Física, Universitat Politècnica de Catalunya, Barcelona, Spain

- The combination of HPC with 2D models allow to run hundreds of cases and retrieve correlations!

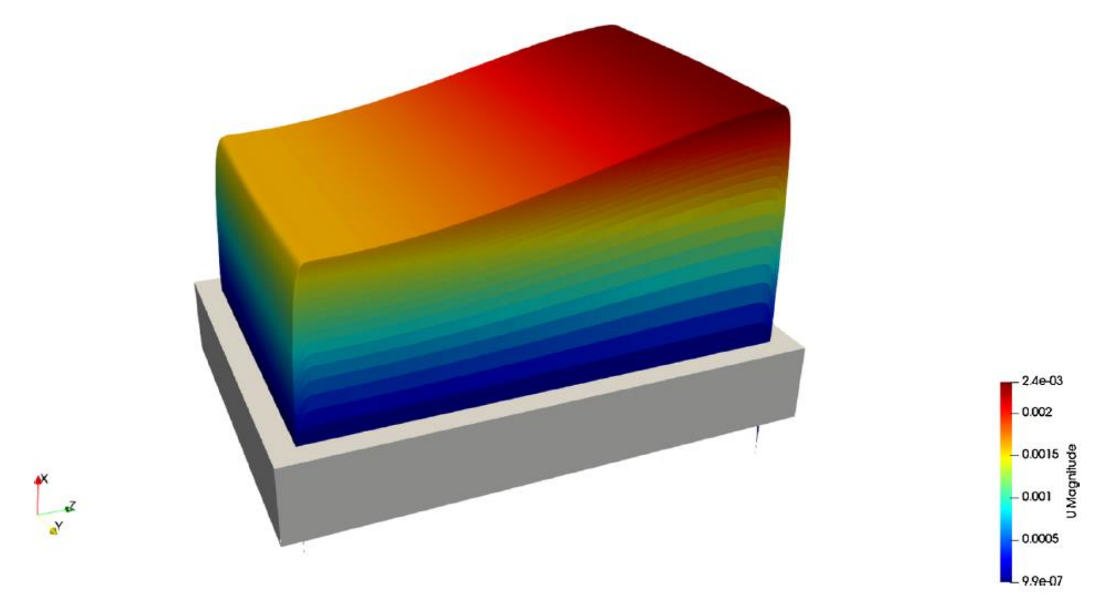
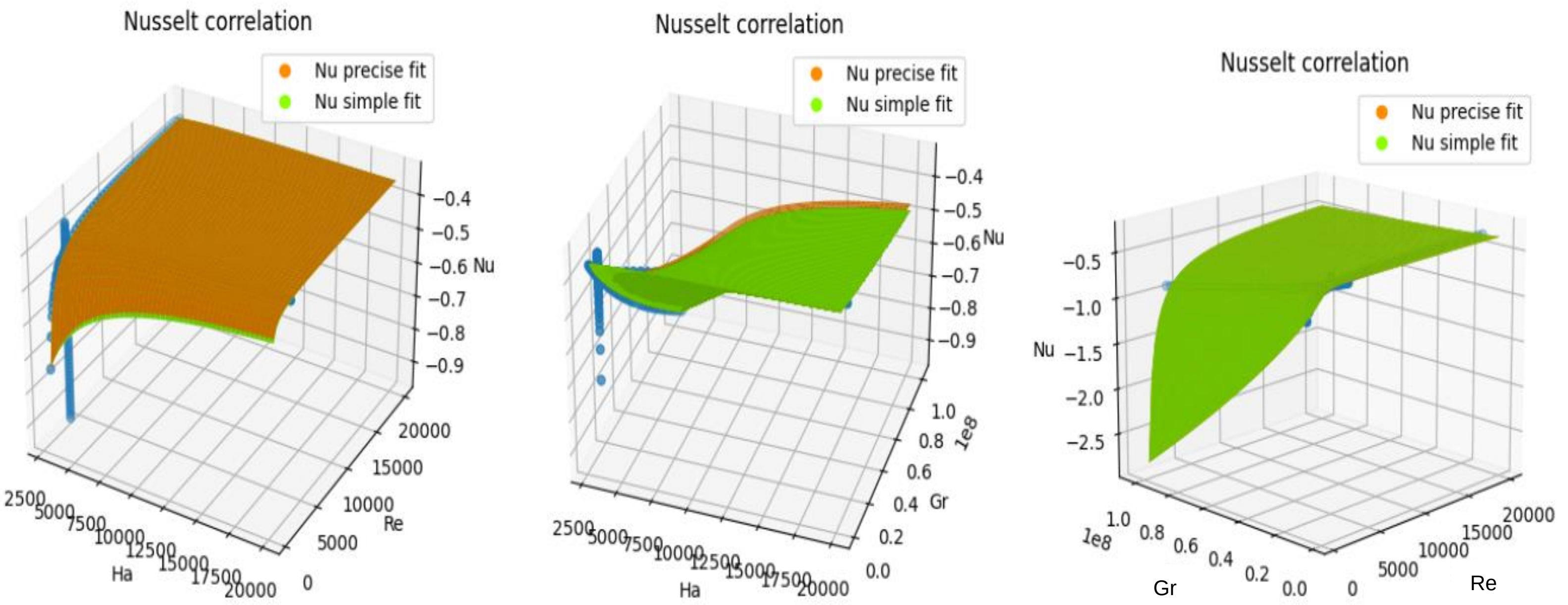
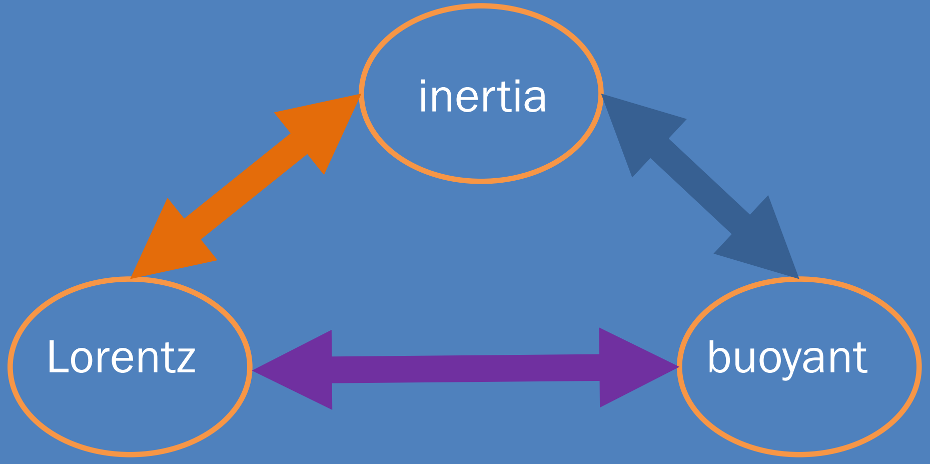


Fig. 8. Velocity profile (m/s) in the case with $Ha = 3000$, $Re = 3000$, $Gr = 1e7$, $GrR = 0.02$ and $c_w = 1e{-}12$.



The 2D model in buoyant flows

- We have the regime map shown in Smolentsev works for insulated walls and non-buoyant cases.
- How is buoyancy going to affect the regime map?



- Some items to analyze it are:

1. The Lykoudis number = $Ly = Ha^2/Gr^{0,5}$

   (Ly_{DCLL} = 200 > 1)

   DCLL STABLE!

   But this does not consider inertia forces ...

2. The transition from force to mixed hydro-convection

   Gr/Re^2>0,3 (aiding flows)

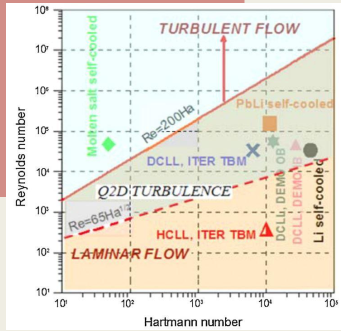
   DCLL MIXED CONVECTION!

   But this does not consider Lorentz forces...

3. The transition from laminar to Q2D MHD flow regime depends on $Re > 65Ha^{0,5}$

   DCLL UNSTABLE!

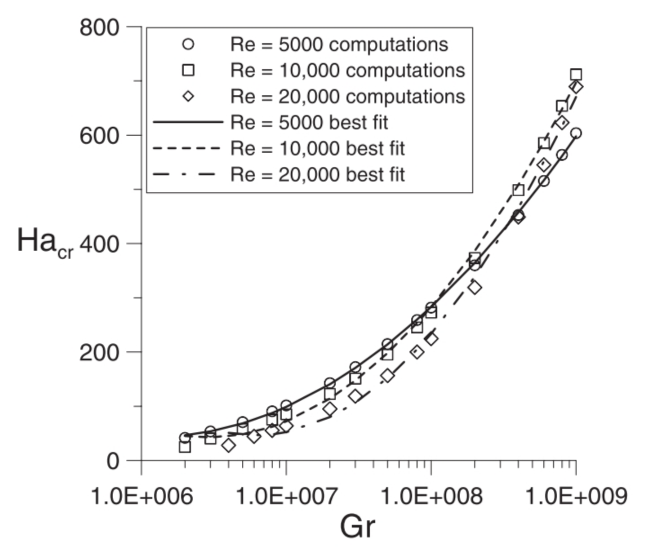
   But this does not consider buoyant forces...



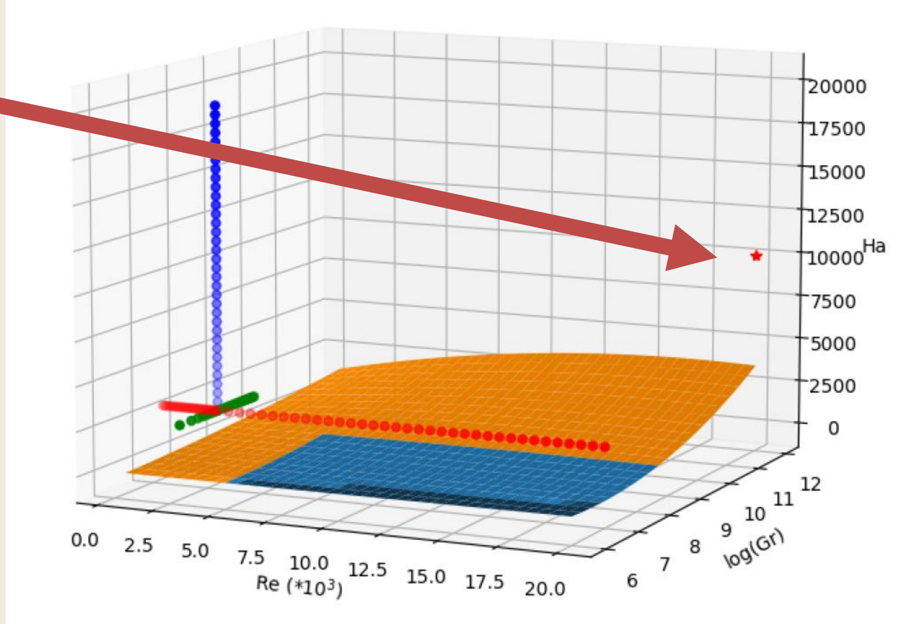
The 2D model in buoyant flows

- Vetcha et. al. developed a solution for the regime map problem using the linear stability analysis for the range of $10^6 < Gr < 10^9$

- The extrapolation of the solution suggests that the DCLL conditions are in the stable region.



N. Vetcha, S. Smolentsev, M. Abdou, and R. Moreau. Study of instabilities and quasi-two-dimensional turbulence in volumetrically heated magnetohydrodynamic flows in a vertical rectangular duct. *Physics of Fluids*, 25(2), 2013. ISSN 10706631. doi: 10.1063/1.4791605.



The potential of Q2D model

- Initially developed by Sommeria and Moreau in 1982, named SM82.
- Only valid for insulated walls
- Solves the plane perpendicular to the magnetic field
- Ideal for current DCLL design with ceramic walls (not applicable in sandwich FCI channels)

$$\nabla \cdot \mathbf{v} = 0 \qquad (5)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{v} - \frac{1}{\tau_{Ha}} \mathbf{v} + \beta \mathbf{g}(T - T_0) \qquad (6)$$

where $\mathbf{v}$ is fluid velocity, t time, p pressure, ν kinematic viscosity of the fluid ($\nu = \mu/\rho$), and τ_{Ha} is the *Hartmann braking time* and takes the form of:

$$\tau_{Ha} = \frac{b}{B}\sqrt{\frac{\rho}{\sigma \nu}} \qquad (7)$$

with σ, electric conductivity of the fluid, B external magnetic field and b half width of the channel in the magnetic field direction. Equation 6 includes the buoyant term $\beta \mathbf{g}(T - T_0)$, that is modeled following the Boussinesq hypothesis for incompressible flows, where β is the thermal expansion coefficient, $\mathbf{g}$ is the gravity vector and T is the local temperature, with T_0 the mean temperature of the domain.

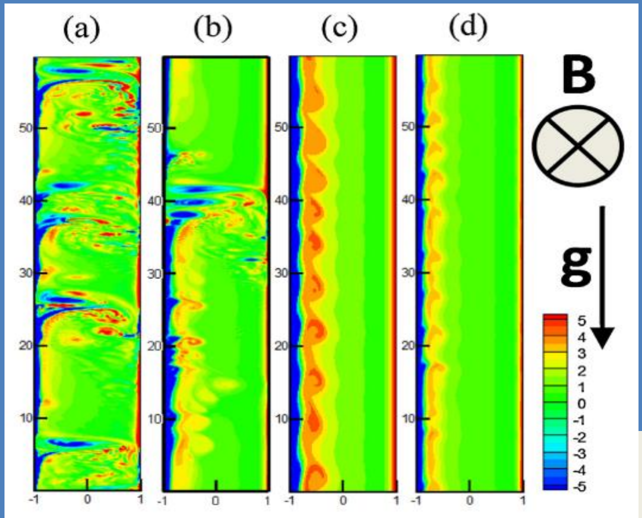


Fig. 2. Vorticity snapshots in a turbulent mixed-convection flow. Strong turbulence: (a) $Ha = 50$, $Gr = 10^8$, $Re = 5000$; (b) $Ha = 75$, $Gr = 10^8$, $Re = 7500$. Weak turbulence: (c) $Ha = 75$, $Gr = 5 \times 10^7$, $Re = 10{,}000$; (d) $Ha = 120$, $Gr = 10^8$, $Re = 5000$.

- The large number of cases to be run needs a method to automatically detect instabilities.
- The selected method has been FFT2

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Fast Fourier Transform 2 (FFT2): instability detection method

- Validation of the FFT2 method:

$$\text{FFT2:} \quad A_{kl} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} a_{mn} \exp\left\{-2\pi i\left(\frac{mk}{M}+\frac{nl}{N}\right)\right\}$$

$$k \in \{0,\dots,M/2-1\} \quad l \in \{0,\dots,N/2-1\}$$

$$\text{Wavenumber in y and z directions:} \quad \nu_k = \frac{k}{L} \quad \nu_l = \frac{l}{L}$$

$$\xi = \underbrace{\cos\left(\frac{\pi y}{a}\right)}_{\text{first term}} - \underbrace{\frac{1}{10}\sin\left(\frac{5\pi y}{a}\right)}_{\text{second term}} + \underbrace{\frac{1}{2}\cos\left(\frac{10\pi y}{a}\right)}_{\text{third term}} + \underbrace{2\sin\left(\frac{\pi z}{2b}\right)}_{\text{fourth term}} \quad (10)$$

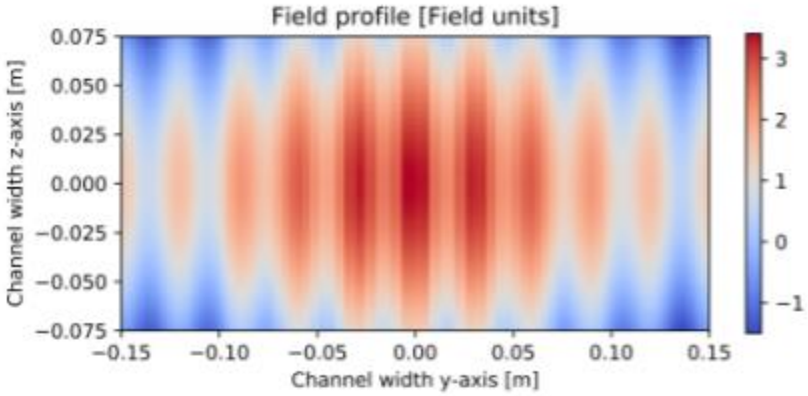


Figure 3: Validation case: ξ scalar 2D field

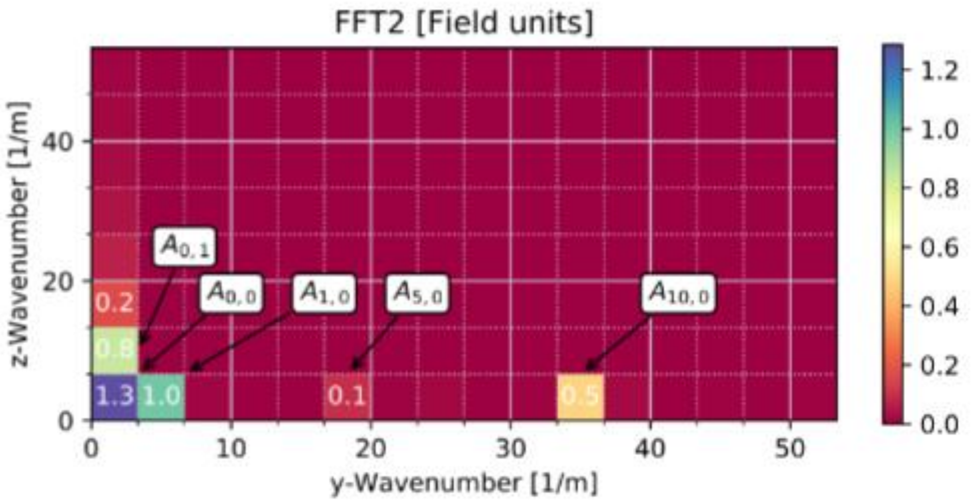


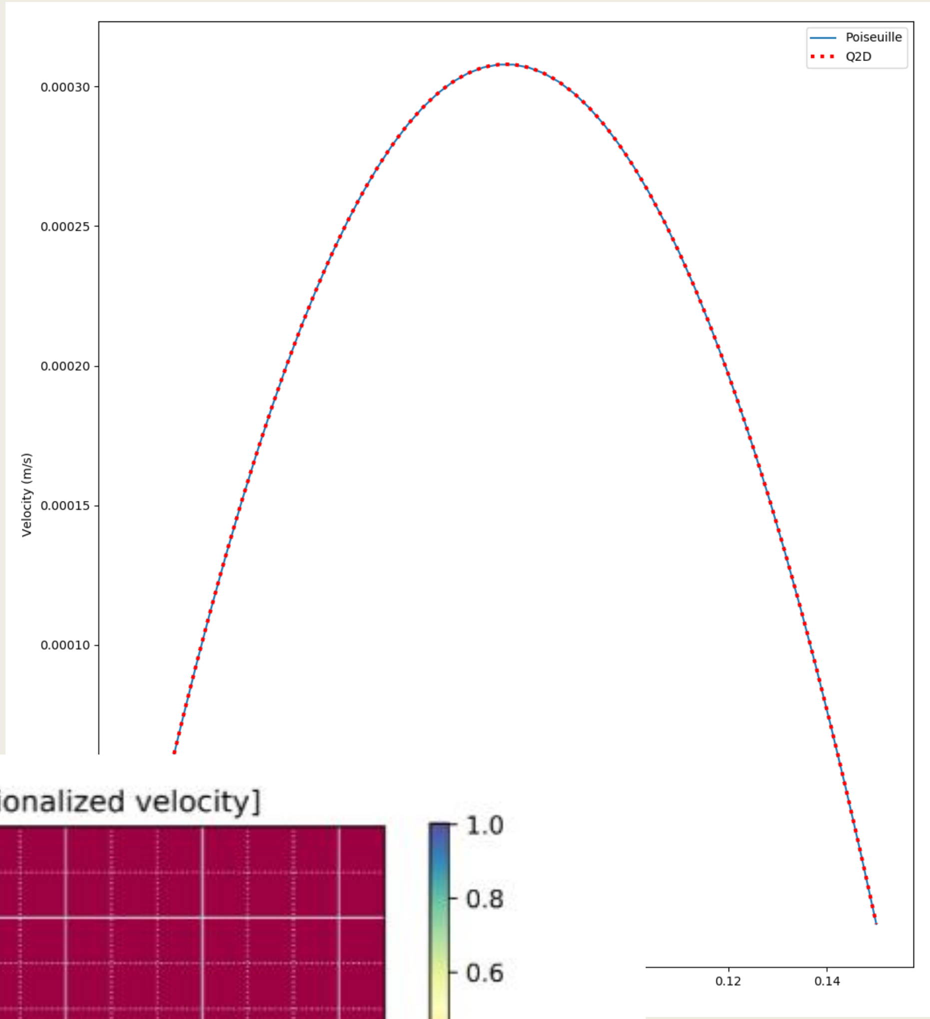
Figure 4: Validation case: FFT2 coefficients

Q2D and FFT2 validation cases:

The implementation of this model will be validated in the following sections. The validation cases will be:

- Poiseuille flow, for the hydrodynamic closure of the pressure-velocity coupling.
- Shercliff flow, for the modelling of the MHD effect.
- Tagawa flow at high Ha, for the buoyant-hydrodynamic and for the buoyant-MHD flow conditions.
- Mixed magneto-convective fully developed flow, for the combination of inertial (pump) with buoyant-MHD phenomena.

- Poiseuille



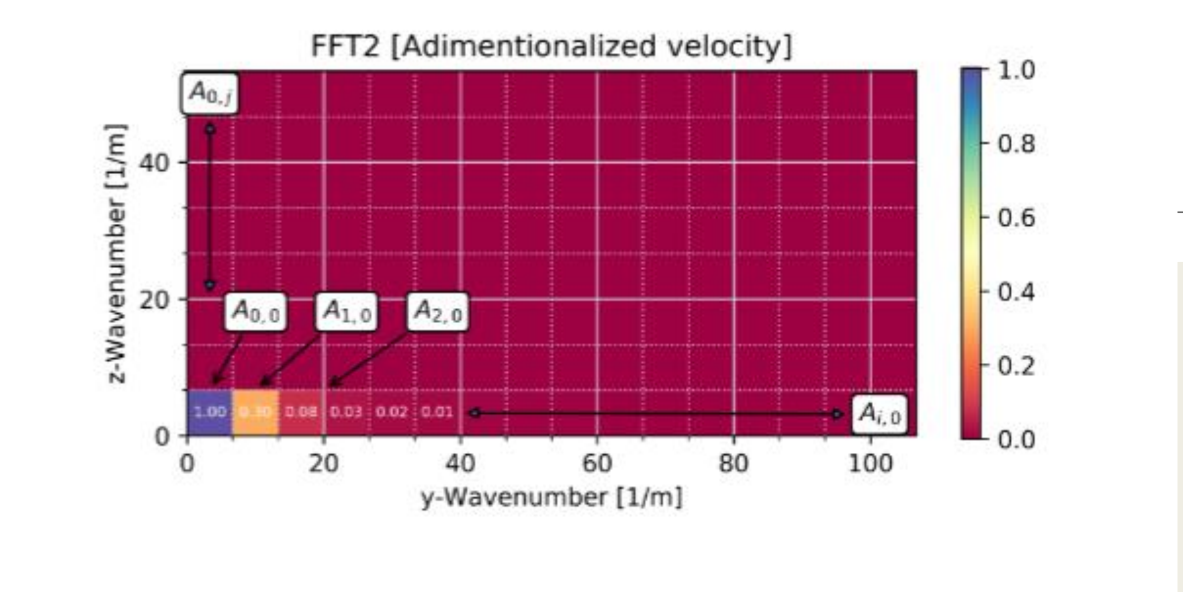


Figure 6: 2D Poiseuille flow spectrum for Re=100 and width of 0.15 m

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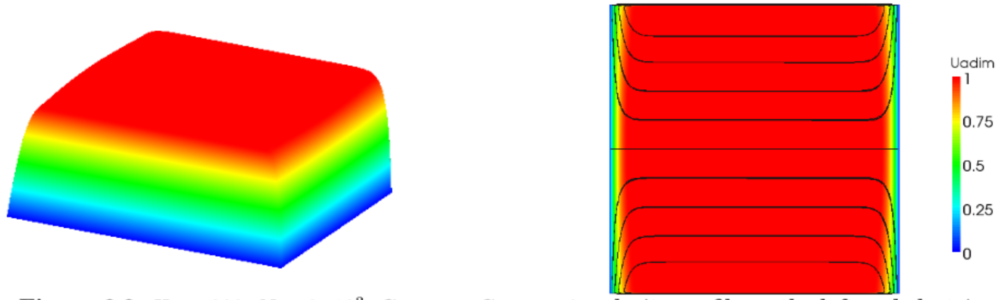

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- Shercliff

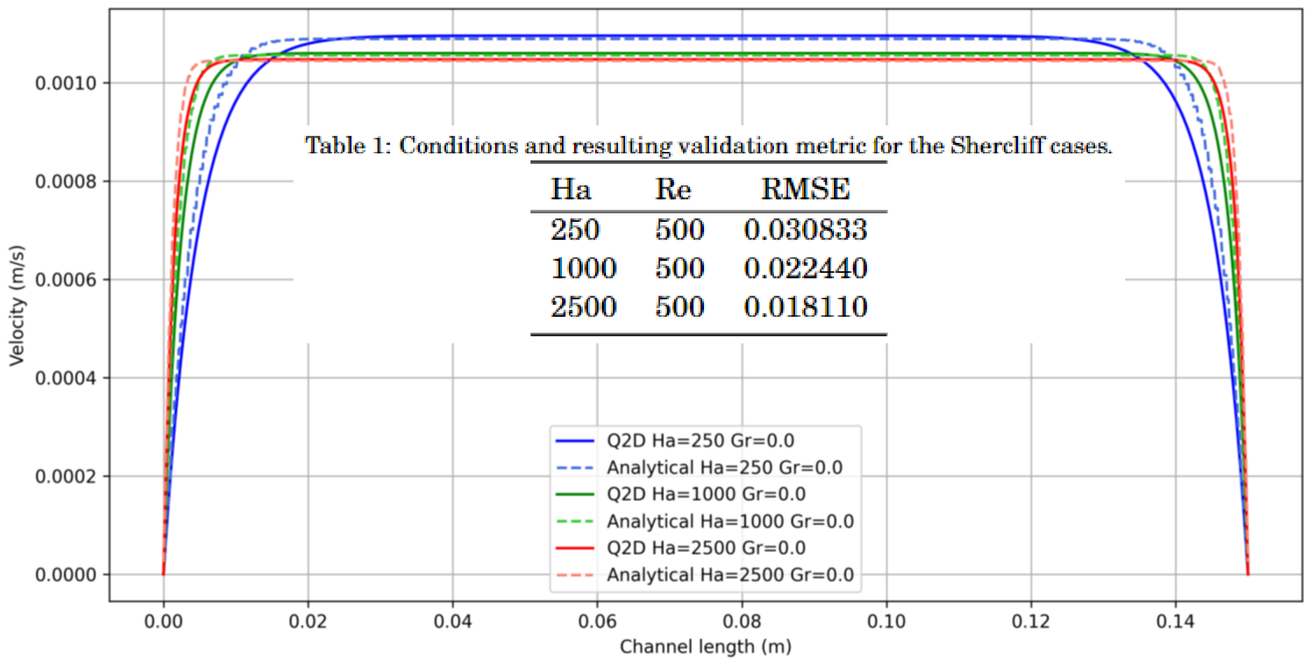
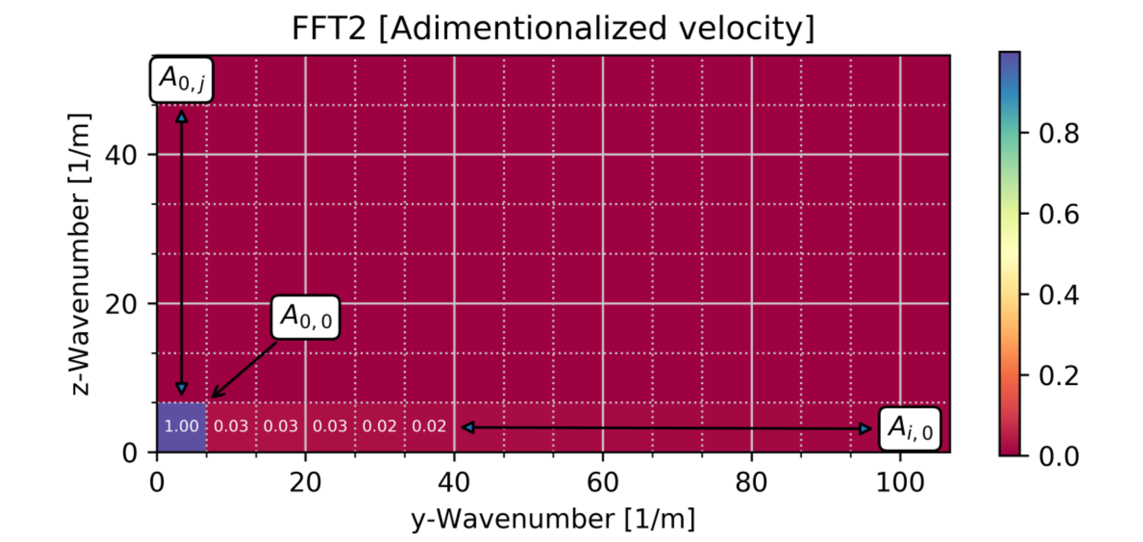


Table 1: Conditions and resulting validation metric for the Shercliff cases.

Ha	Re	RMSE
250	500	0.030833
1000	500	0.022440
2500	500	0.018110



Q2D and FFT2 validation cases:

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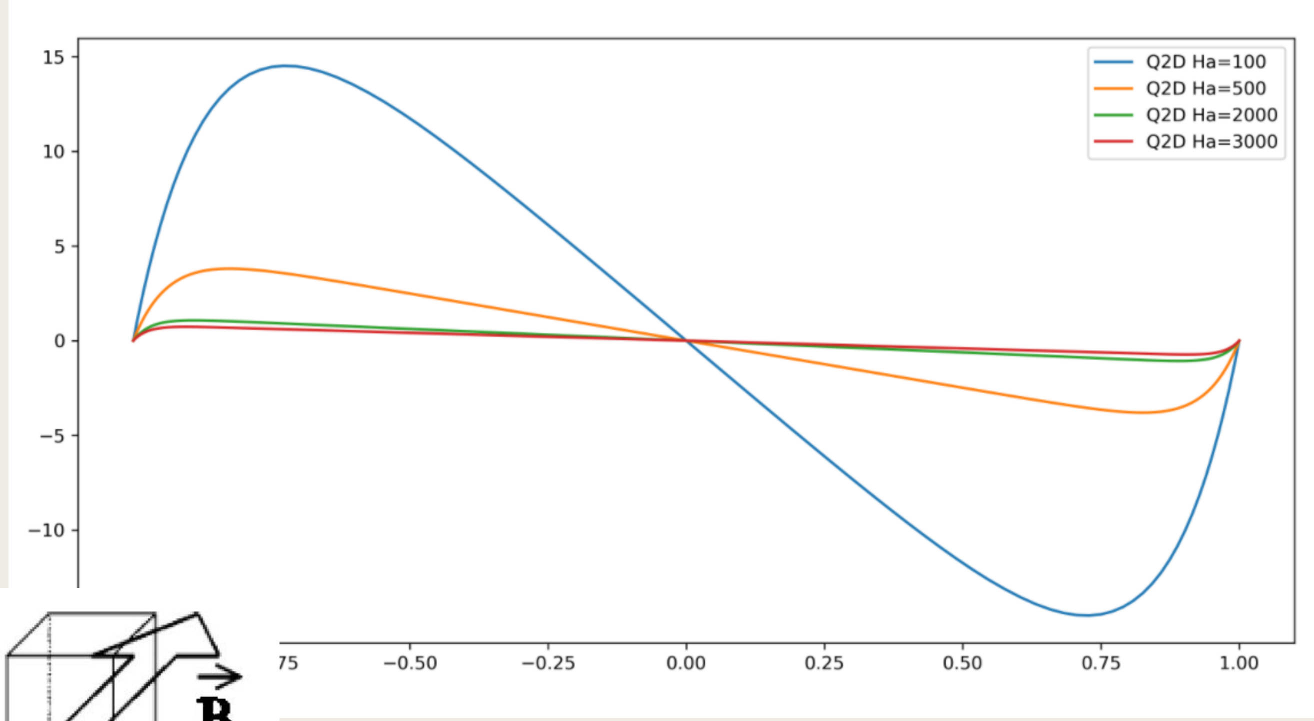
- Tagawa



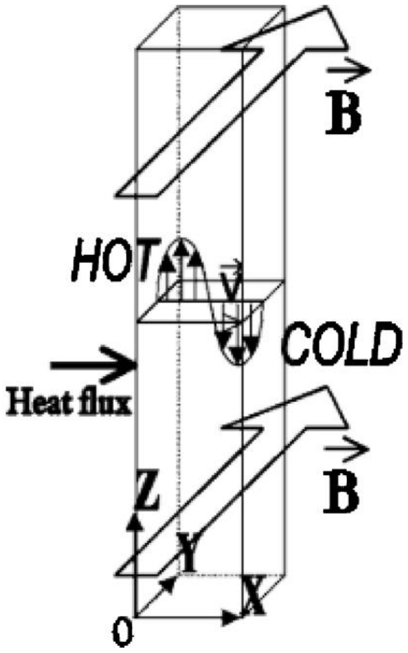


Table 2: Conditions and resulting validation metric for the Tagawa cases.

Ha	U_{max} (Tagawa)	U_{max} (Q2D)	Relative Error
0	73.11	58.51	19.97 %
100	16.43	14.52	11.61 %
500	4.073	3.804	6.601 %
2000	1.123	1.072	4.512 %
3000	0.717	0.732	2.092 %

Q2D and FFT2 validation cases:

The implementation of this model will be validated in the following sections. The validation cases will be:

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- Shercliff flow, for the modelling of the MHD effect.
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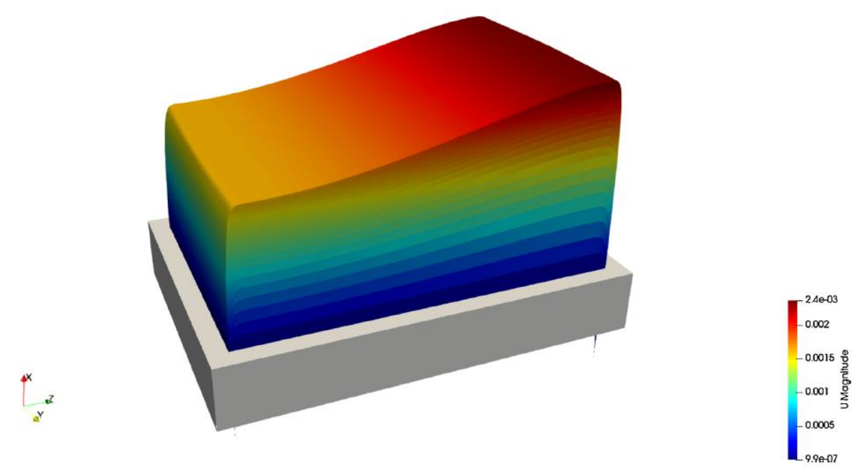
- Mixed magneto-convective fully developed flow, for the combination of inertial (pump) with buoyant-MHD phenomena.

- Mixed convection



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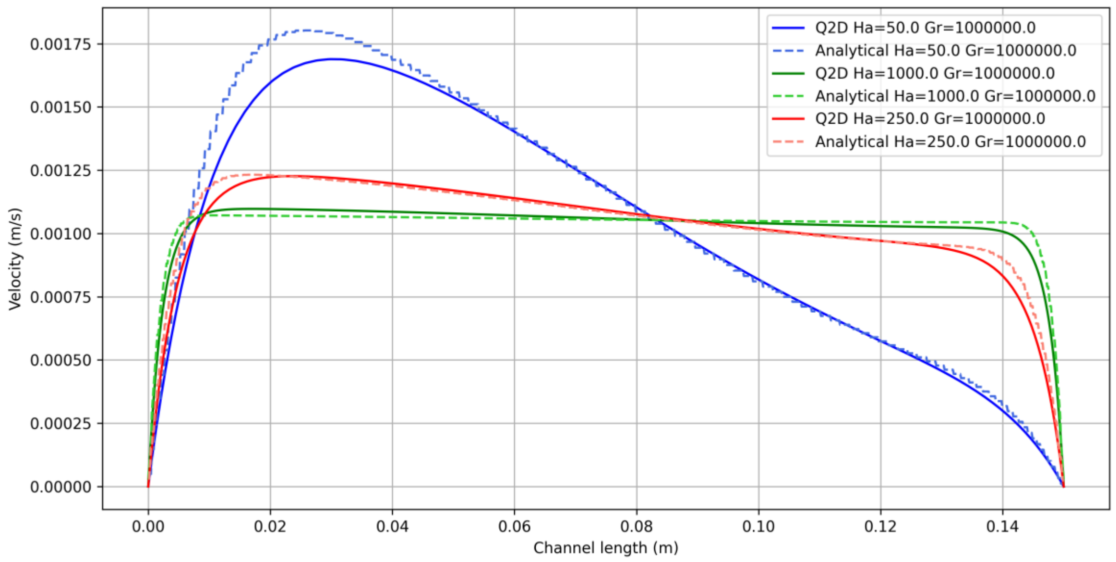
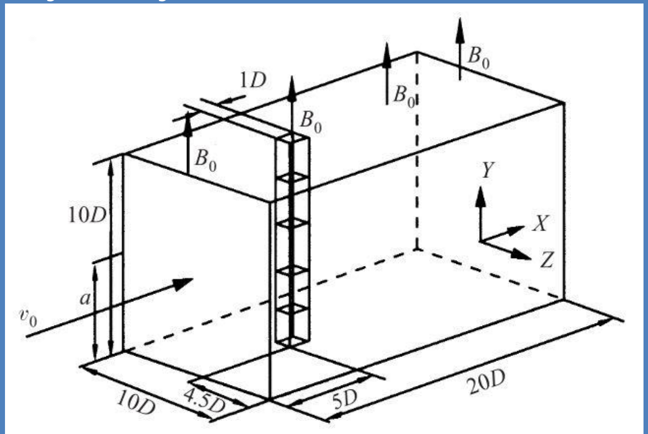


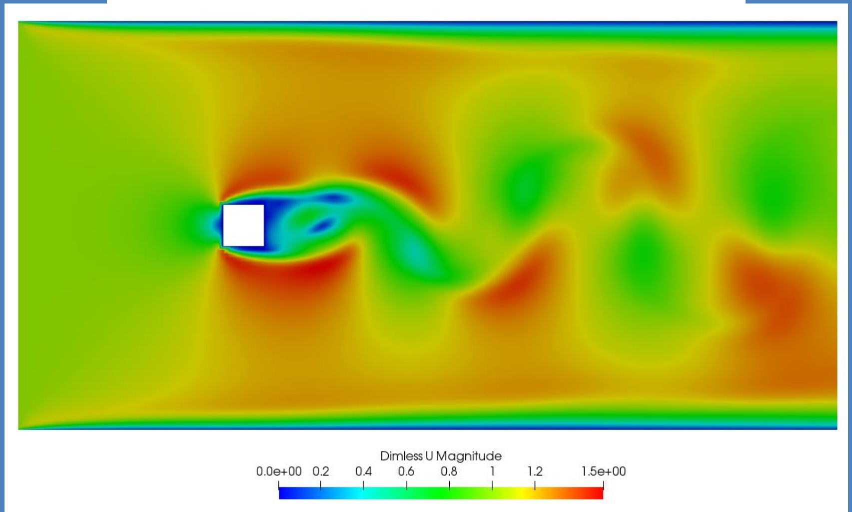
Table 3: Conditions and resulting validation metric for the mixed-magnetoconvective cases. All cases are set to $Re = 500$ and $Gr = 1e6$.

Ha	RMSE
50	0.05084
250	0.03126
1000	0.02801

Q2D and FFT2 application to 3D (Q2D) case:

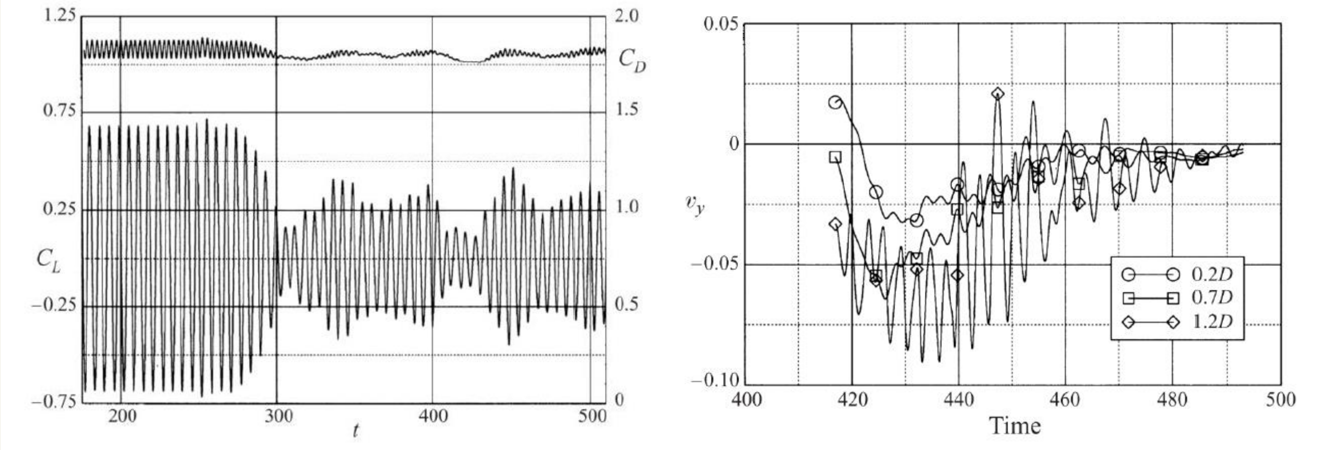
- Starts hydrodynamic, continues MHD



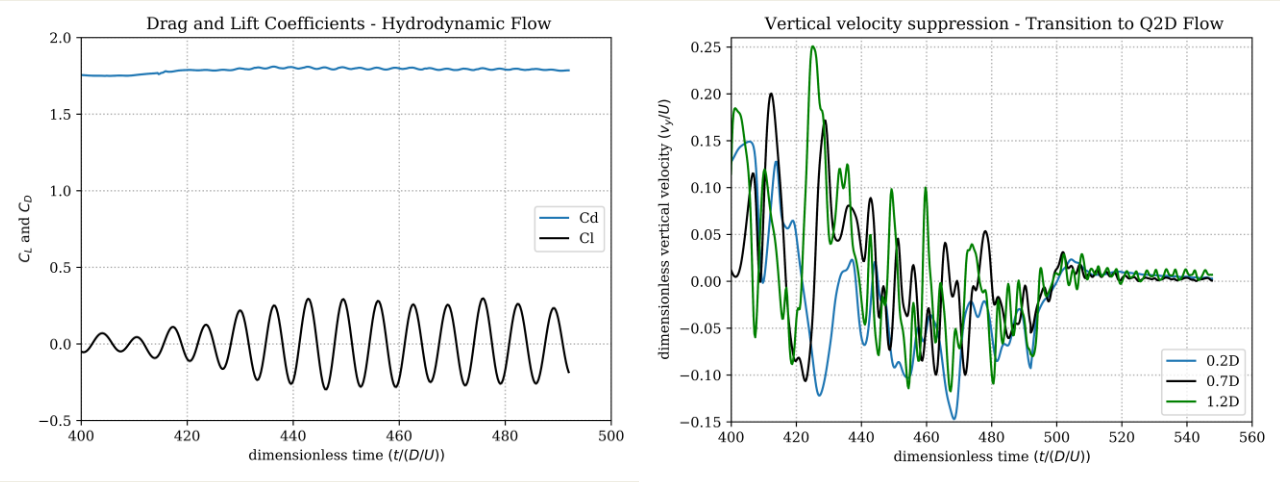


3D solver MHD midplane results

- Mück



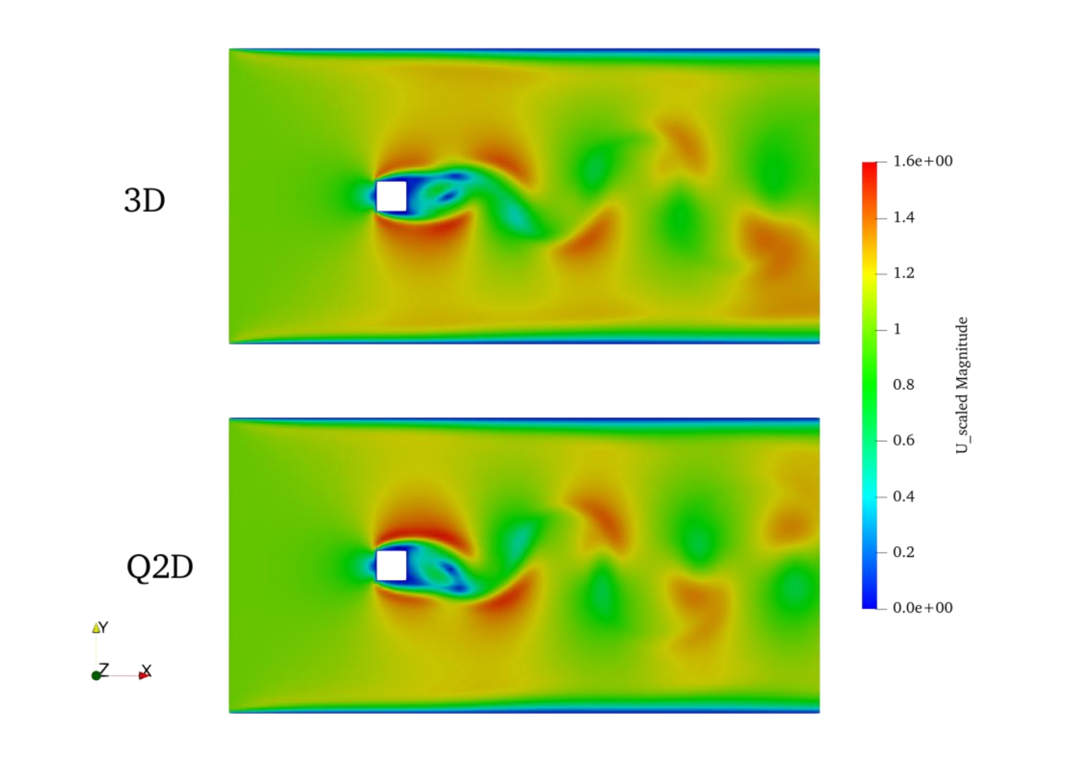
Muck, B., Gunther, C., Muller, U., & Buhler, L. (2000). Three-dimensional MHD flows in rectangular ducts with internal obstacles. *Journal of Fluid Mechanics*, *418*, 265–295.



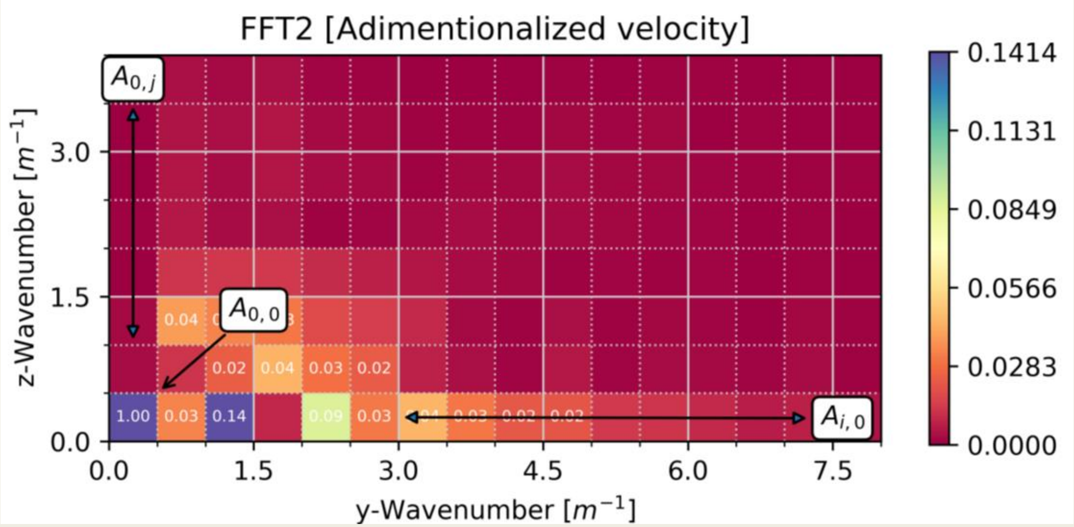
Our results

Q2D and FFT2 application to 3D (Q2D) case:

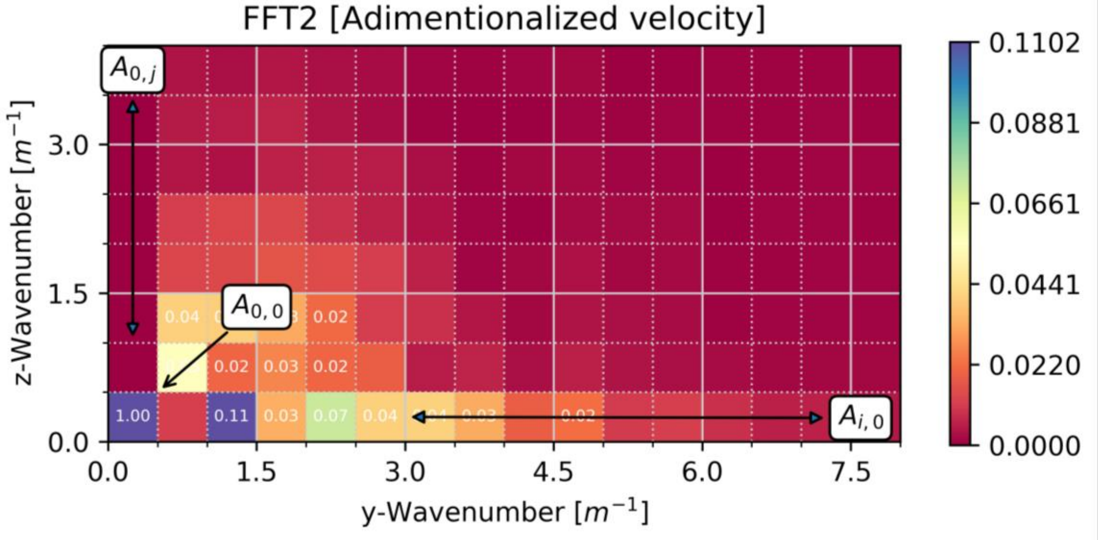
- Starts hydrodynamic, continues MHD



- Mück



3D solver MHD midplane FFT2 results



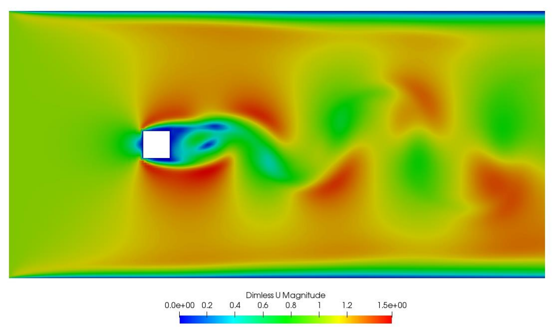
Q2D (2D) solver MHD midplane FFT2 results

Final remarks

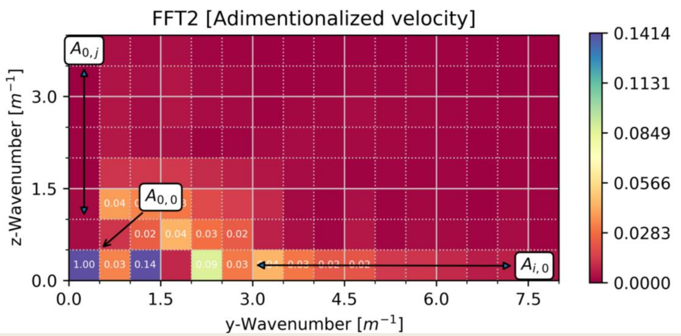
- The Q2D model is very promising to be used in parametric studies where many simulations are needed.
- The implemented Q2D model shows accurate results, specially for high magnetic fields.
- **Our research group is on a quest to retrieve the flow regime map, a work for next year.**
- The use of the FFT2 method may allow for an automatic detection of instabilities and the eventual construction of the regime map.
- Once the stability boundaries are known, the cross sectional 2D model may be applied on solid grounds.

Implementation of a Q2D turbulence model and detection of flow instabilities in liquid metal MHD flows

Q2D turbulence MHD model



FFT2 method



That was all
Thank you

Daniel Suarez, Eduardo Iraola, Joaquim Serrat,
Elisabet Mas de les Valls, Shimpei Futatani, Lluís Batet
Advanced Nuclear Technologies Research Group
Department of Physics (UPC)