# The numerical flow iteration for the Vlasov-Poisson equation

R.Paul Wilhelm[1] and Matthias Kirchhart[2]

[1] *Applied and Computational Mathematics, RWTH Aachen University, Schinkelstraße 2, 52062 Aachen, Germany*

The Vlasov-Poisson equation is a high-dimensional partial differential equation used to model the behaviour of plasmas in the collision-less limit. The six-dimensional phase-space and the turbulent motion of plasmas together with formation of strong filamentation in the solution lead to significant complications in the implementation of solvers. While purely Lagrangian schemes are in theory capable to capture the dynamics correctly, the inherent numerical noise makes it hard to run long-time accurate simulations, requires large numbers of particles and remeshing schemes [1]. Eulerian and Semi-Lagrangian schemes produce more accurate results, however, generation as well as management of meshes can become prohibitively expensive for simulations in the high-dimensioanl phase-space and in particular higher-order schemes often lack conservation properties of the exact solution [2].

Additional usage of the information provided by the method of characteristics can lead to less noise and better conservation properties [3]. Thus we present a novel approach, the numerical flow iteration (NuFI), which iteratively reconstructs the phase flow and then using the method of characteristics can evaluate the distribution function to arbitrary precision, i. e., up to error in time and Poisson discretisation. This approach naturally preserves non-negativity, does not struggle with over-shoots and conserves all $L^p$ norms exactly as well as energy and entropy up to the chosen numerical precision. From the implementation perspective, a high flop-byte ratio as well as a structure which allows for easy parallelization of this method, lead to efficient portability to modern clusters.

## References

[1] A. Myers, P. Colella, B. Straalen, A 4th-Order Particle-in-Cell Method with Phase-Space Remapping for the Vlasov–Poisson Equation, SIAM Journal on Scientific Computing, 2016

[2] L. Einkemmer, A performance comparison of semi-Lagrangian discontinuous Galerkin and spline based Vlasov solvers in four dimensions, Journal of Computational Physics, 2019

[3] M. Campos Pinto, E. Sonnendrücker, A. Friedman, D.P. Grote, S.M. Lund, Noiseless Vlasov–Poisson simulations with linearly transformed particles, Journal of Computational Physics, 2014