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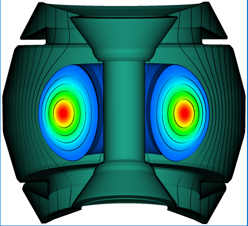
A Computational Multi-physics Approach for Whole-system Fusion Reactor Simulations



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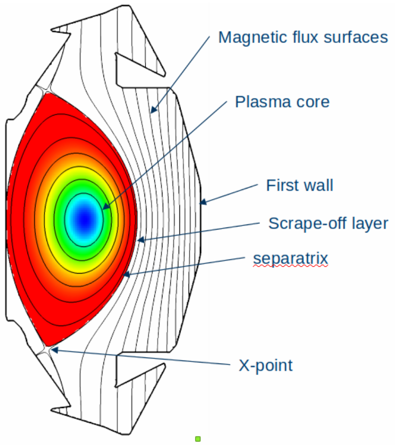
Introduction



Current issues relating to codes raised by community:

- Isolate specific regions of the device.
- Lack shock-capturing capabities.
- Use grid-aligned coordinate systems.
- Do not consider regions of true vacuum.
- Do not consider the wall as an elastoplastic/electromagnetically responsive material.
- Use simple equations of state for plasma.

How do we fit in?

Our goals for this code comprise the following:

- In-house GS solver ofr steady-state configurations.
- Capture all regions of the device in one go through multi-physics, multi-material methods.
- Adaptive mesh refinement in both space **and** time.
- Shock-capturing capabilities.
- Cartesian frame of reference.
- Vacuum interface capturing methodologies.
- Elastoplastic, electromagnetically responsive walls.
- More complex EoS.

Full system of equations

VRMHD system of equations for code development

$$\frac{\delta}{\delta t}\begin{bmatrix}\rho \\ \rho\mathbf{v} \\ \rho E \\ \mathbf{B}\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho\mathbf{v} \\ \rho\mathbf{v}\otimes\mathbf{v} + p_{mag}\mathbf{I} - \mathbf{B}\otimes\mathbf{B} \\ \mathbf{v}^T(\rho E + p_{mag}) - \mathbf{v}^T\mathbf{B}\otimes\mathbf{B} \\ \mathbf{B}\otimes\mathbf{v} - \mathbf{v}\otimes\mathbf{B}\end{bmatrix} = \nabla\cdot\mathbf{F}^{VR} \tag{0.1}$$

where

$$\mathbf{F}^{VR} = \begin{bmatrix}0 \\ \mu(\nabla\mathbf{v} + \nabla\mathbf{v}^T - \frac{2}{3}(\nabla\cdot\mathbf{v})\mathbf{I}) \\ \mu\mathbf{v}^T(\nabla\mathbf{v} + \nabla\mathbf{v}^T - \frac{2}{3}(\nabla\cdot\mathbf{v})\mathbf{I}) + \lambda\nabla T + \eta\mathbf{B}^T(\nabla\mathbf{B} - \nabla\mathbf{B}^T) \\ \eta(\nabla\mathbf{B} - \nabla\mathbf{B}^T)\end{bmatrix} \tag{0.2}$$

OK, so how do we initiate a simulation?

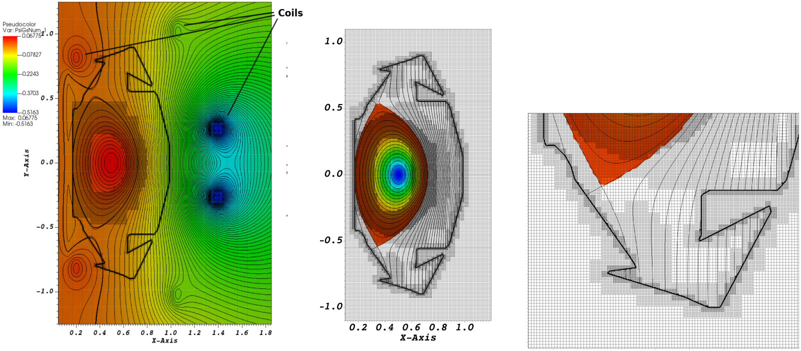
- Steady-state conditions form the basis for unsteady disruption events.
- Assuming force balance, the ideal MHD equations reduce to the Grad-Shafranov (GS) equation.

$$\Delta^* \Psi = r \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Psi}{\partial r}\right) + \frac{\partial^2 \Psi}{\partial z^2} = -\mu_0 r^2 \frac{dp}{d\Psi} - R_0 B_0 \frac{dg}{d\Psi} \tag{0.3}$$

- Ψ is magnetic flux, R_0 is major radius, B_0 is magnitude of B on axis, g is toroidal vacuum field.
- Implement a free-boundary solver which can determine equilibrium solution given constraints such as coil currents, plasma current and pressure.

Free boundary solver - applied to ST40

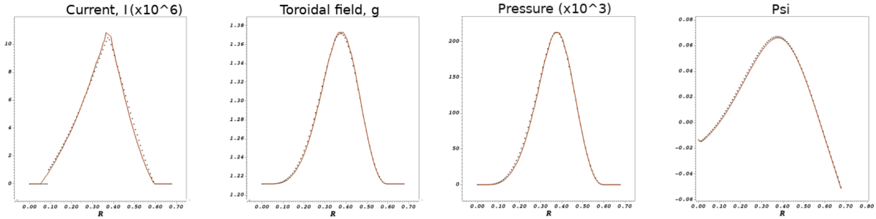
- Used pressure and toroidal field profiles along with coil currents and distributions (provided by TE) as input.



Solution for Ψ in entire domain along with discretized vessel (left), corresponding density profile and mesh (right).

Validation against existing codes

- Excellent agreement observed to existing, validated codes.

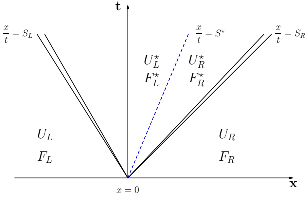


Solutions for I, g, P, and Ψ (points), lines are reference solution.

Unsteady simulations - time dependent evolution

Numerical Methodology

- Use approximate Riemann problem HLLC based methods for unsteady simulations.



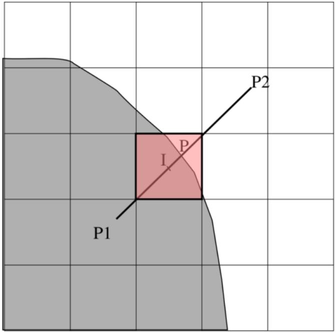
Wave pattern for the HLLC approximate Riemann solver

$$\mathbf{F}^{HLLC}_{i+\frac{1}{2}} = \begin{cases} \mathbf{F}(\mathbf{U}_L) & \text{if} \quad S_L \geq 0\,, \\ \mathbf{F}(\mathbf{U}_L) + S_L(\mathbf{U}^*_L - \mathbf{U}_L) & \text{if} \quad S_L < 0 \leq S^*\,, \\ \mathbf{F}(\mathbf{U}_R) + S_R(\mathbf{U}^*_R - \mathbf{U}_R) & \text{if} \quad S^* < 0 \leq S_R\,, \\ \mathbf{F}(\mathbf{U}_R) & \text{if} \quad S_R < 0\,. \end{cases}$$

- Star states can be found in Li [2005]
- 2nd order achieved via MUSCL reconstruction

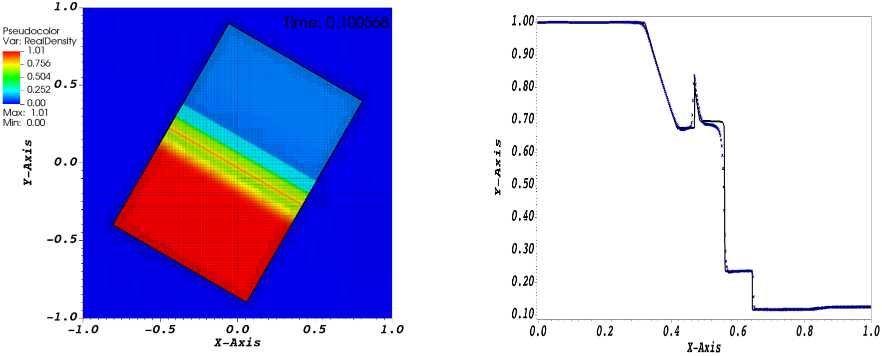
Multi-material simulation

- **Ghost fluid method** to capture non-linear interactions at interface between materials.
- Initially consider **rigid-body** type interactions.
- Fluid states copied to rigid-body, normal velocity reflected.



Multi-material validation

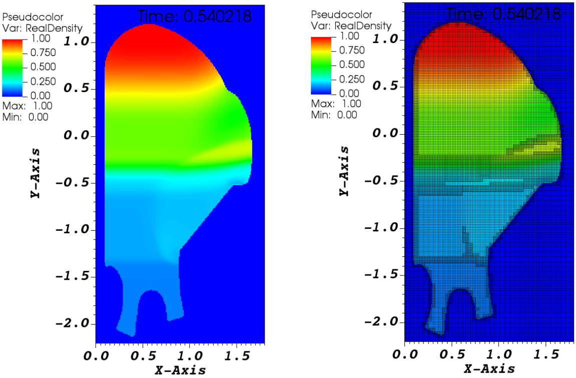
- Rotated Brio-Wu test in rigid container.
- Excellent agreement with standard 1D test.



Solution for density. 2D (left), one-dimensional lineout (right).

Multi-material simulation

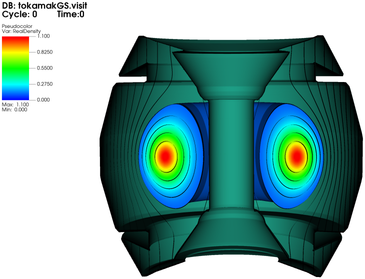
- Brio-Wu test in tokamak geometry.

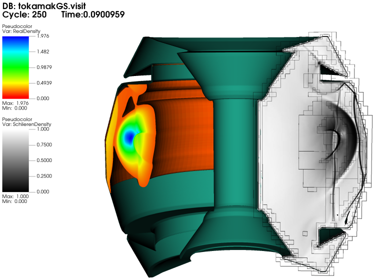


Solution for density. 2D (left), depiction of AMR (right).

Combining the above

- Begin from GS steady state, perturb, evolve unsteady behaviour in multi-material simulation.
- Demonstration of code capabilities in AMR, shock capturing, and multi-material interaction.





Initial conditions given by GS solver (left), results for density (with mock Schlieren) after perturbation (right).

Limitations of explicit schemes

Disruption events typically develop **gradually** from near steady-state behaviour in magnetic pressure dominated **low-Mach** fluid regimes where explicit solvers struggle...

- Unfeasible simulation times due to number of time-steps required.
- Inaccurate solutions due to excessive numerical viscocity.

Fast waves not so important for phenomena of interest $\rightarrow$ some form of implicit treatment can help with this

Numerical strategy

Motivated by Balsara et al., 2016 we use the following flux splitting:

$$\mathbf{F}^{Conv} = u \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho k \\ 0 \\ B_y \\ B_z \end{bmatrix}, \quad \mathbf{F}^{PB} = \begin{bmatrix} 0 \\ p + m - B_x^2 \\ -B_x B_y \\ -B_x B_z \\ \rho u h + 2mu - B_x(\mathbf{v} \cdot \mathbf{B}) \\ 0 \\ -v B_x \\ -w B_x \end{bmatrix} \tag{0.4}$$

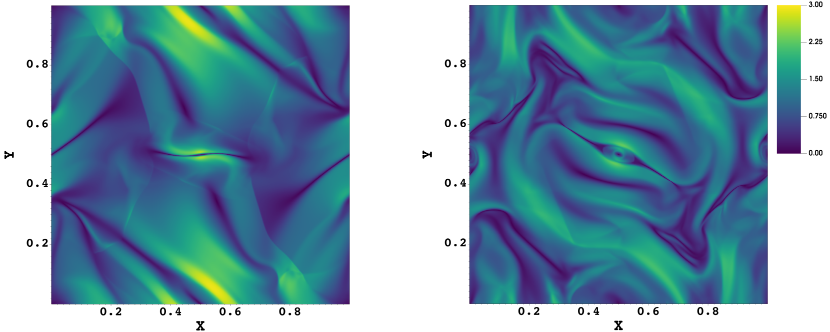
- Convective treated **explicitly**, P&B treated **implicitly**.
- Δt is only driven by the eigenvalues of the convective sub-system, in this case just u:

$$\Delta t \leq C_{cfl} \frac{\Delta x}{\max_i |u_i|} \ . \tag{0.5}$$

Numerical Solutions

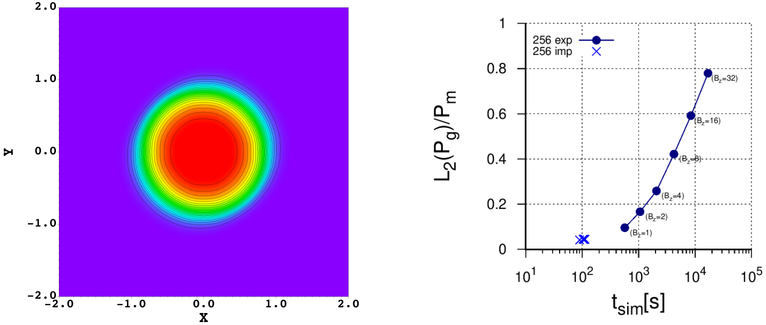
High-Mach

- Orszag-Tang problem, test shock-capturing and transition to supersonic turbulence capabilities of scheme.



Solution for $|B|$ at time 0.5 (left), time 1.0 (right).

- Advected screw-pinch equilibrium, test low-Mach behaviour of scheme.



Solution for p_{gas} at time 100 (left), comparison between semi-implicit and fully explicit scheme (right).

Conclusions and Future work

- Steady-state
  - Include experimental profiles and toroidal rotation for GS solve.
  - Use in optimisation algorithm for device design.
- Unsteady simulation
  - Extention to 3D.
  - Validation of rigid-body interactions in more scenarios.
  - Extension to high order in space and time.
  - Extension of equations (2-fluid) to include more physics.
  - More complex EoS.
- Simulation of ELMs (Edge Localised Modes)!

Thank you for listening!