Comparison of the Numerical Flow iteration to Lagrangian and Semi-Lagrangian schemes for the Vlasov system

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The Vlasov system is a high-dimensional, non-linear system of partial differential equations required to accurately model the behaviour of strongly non-equilibrium plasmas arising in fusion devices. The up to six-dimensional phase-space, due to the curse of dimensionality, leads to extreme memory requirements when trying to discretize the solution of the Vlasov equation directly even for relatively small numbers of degree's of freedom. Additionally, the turbulent dynamics of the Vlasov equation together with formation of strong filamentation in the solution require us to use fine resolution. While it has been shown that on modern high-performance hardware one is able to run small to medium sized simulations of the Vlasov system with both Particle-In-Cell (PIC) and Semi-Lagrangian (SL) approaches even in the full six-dimensional case, it still remains challenging to run accurate, realistic simulations [2, 3, 4].

Recently, with the Numerical Flow Iteration (NuFI), there has been suggested an alternative method [1]. NuFI promises severely reduced memory complexity by a factor of $\mathcal{O}(\frac{N_t}{N_v^d})$ due to a shift to computation on the fly, which then comes with quadratic complexity in the number of made time-steps. In addition, the algorithm is embarrassingly parallel and conserves L^p-norms and entropy exactly as well as total energy up to time discretization error.

In this talk we want to take a closer look at the relative performances of NuFI compared to other schemes for solving the Vlasov system when looking at instabilities typical for high-temperature plasmas.

References

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