6th Fusion HPC Workshop

Optimizing Stellarators with Hidden Symmetry

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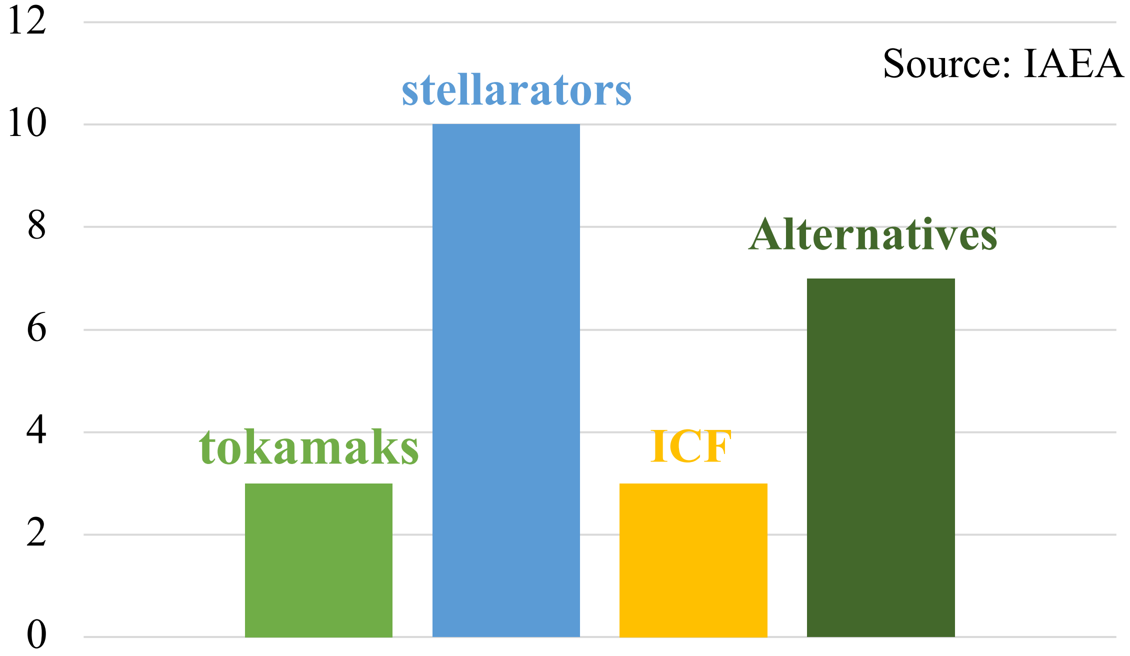
Stellarators have become the rapidest growing fusion concept

- A stellarator is a toroidal plasma confinement configuration that uses external coils to produce a non-axisymmetric magnetic field.

- Stellarators have many advantages.
  - **Steady state operation**
  - **Low recirculating power**
  - **Stable to plasma currents inducing MHD instabilities**
  - **Free of disruptions**
  - **High density operation**

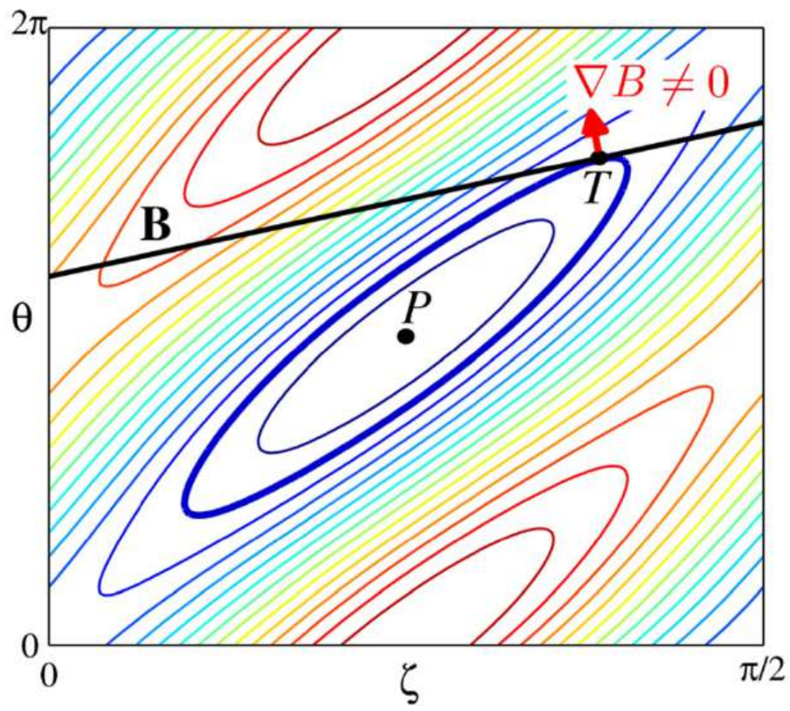
New fusion devices in 2024 (operated/planned)

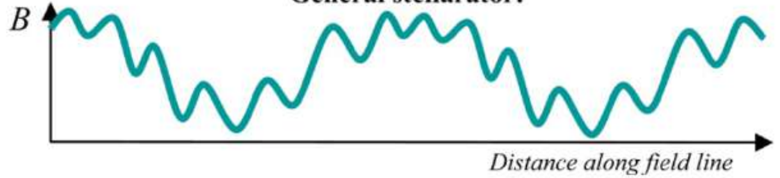


- Conventional stellarators suffered bad neoclassical transport and it can be overcome by carefully **optimized** magnetic field [Beidler Nature 2021].

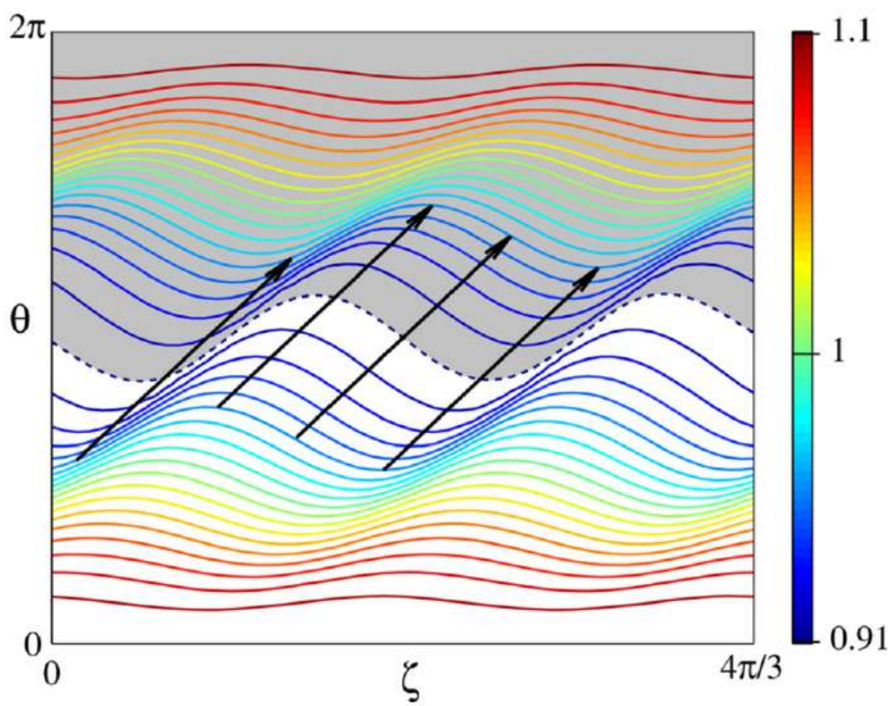
Omnigenity is a primary way to improve confinement

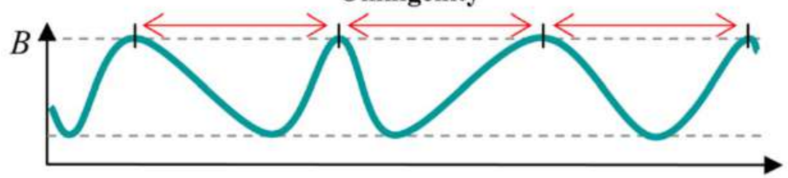
General stellarator





Omnigenity





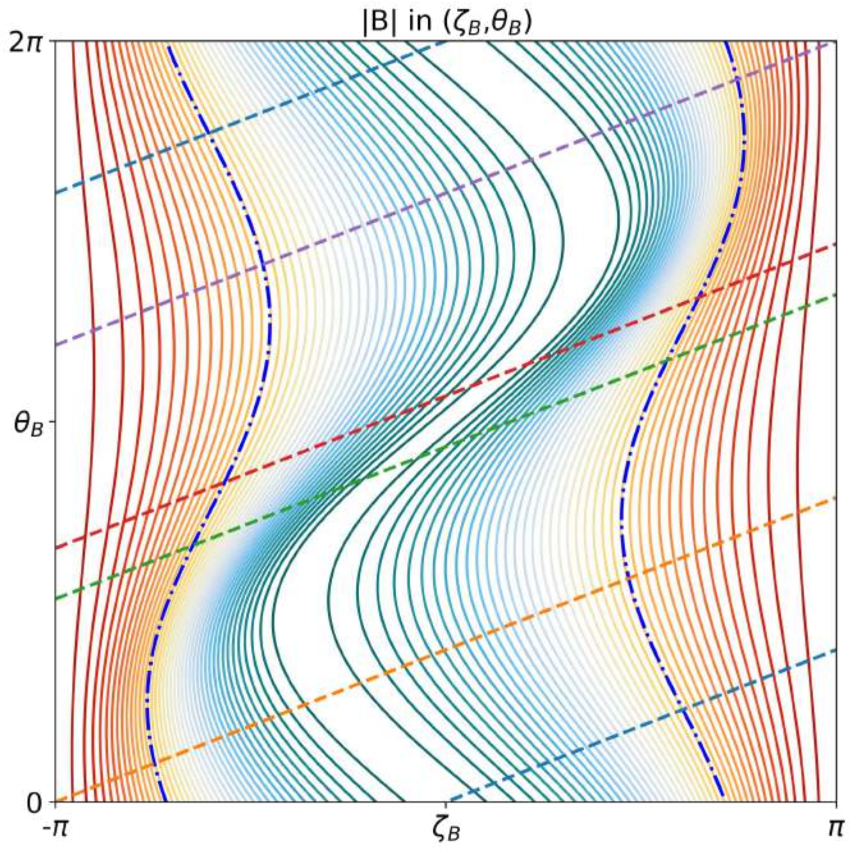
Omnigenity: time-averaged radial drift along each field line vanishes.

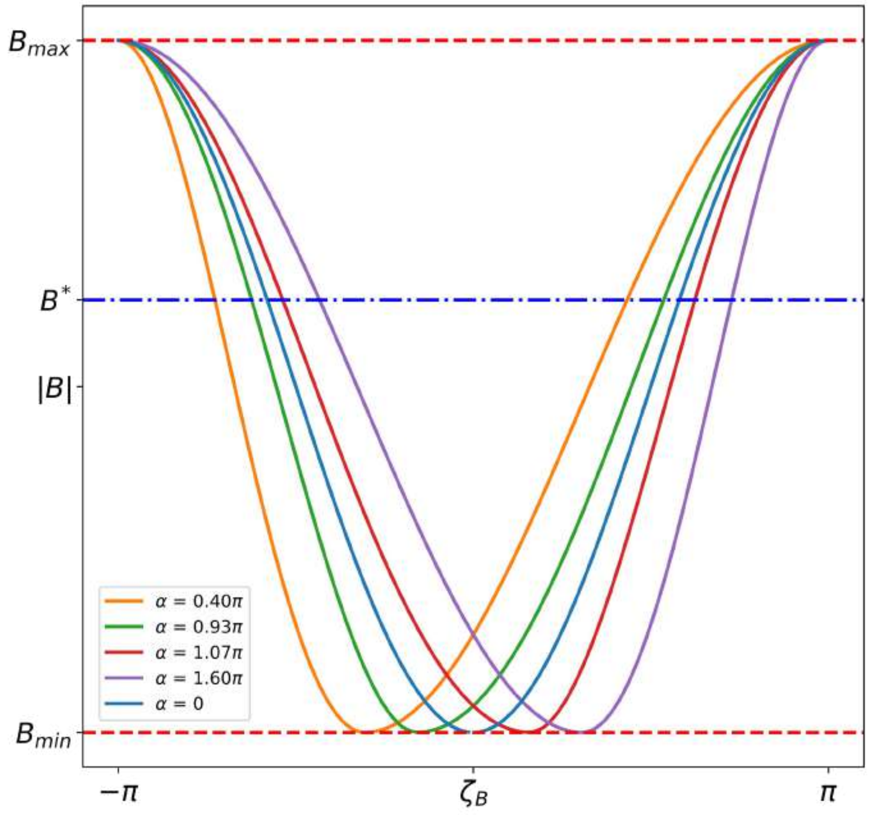
$$\int (\boldsymbol{v}_{\mathrm{d}} \cdot \nabla \psi) \frac{\mathrm{d}l}{v_{\parallel}} = 0$$

Landreman & Catto PoP 2012

Properties of omnigenity

Cary & Shasharina PoP 1997





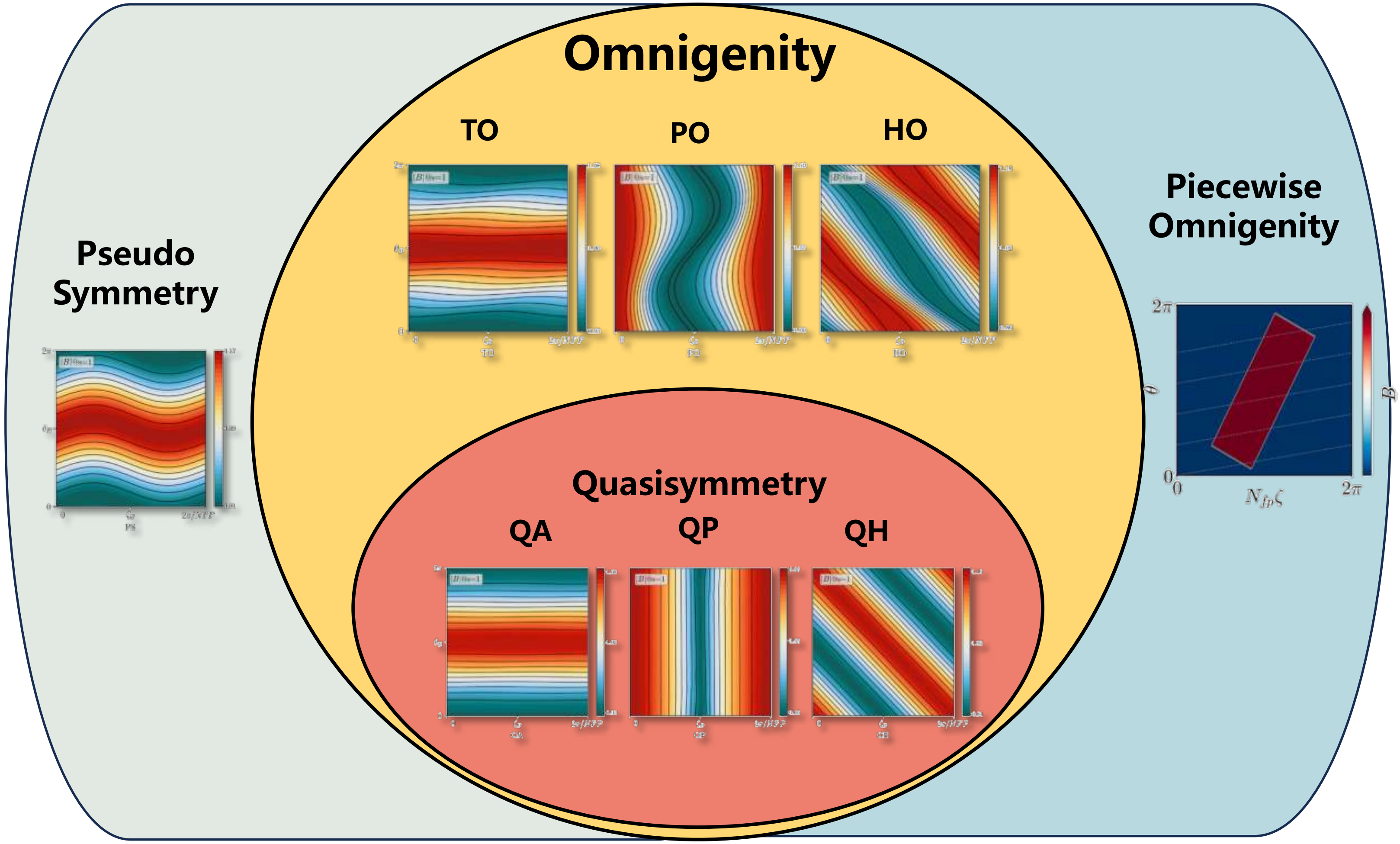
- B contours close toroidally, poloidally, or helically
- "Bounce distance" is independent of the field line label

$$\langle \vec{v}_d \cdot \nabla\psi \rangle = 0 \Longleftrightarrow \frac{\partial \mathcal{J}}{\partial \alpha} = 0$$

$$\mathcal{J} = m\int v_\parallel dl = \sqrt{2m(u - q\phi)} \oint \sqrt{1 - \frac{B}{B^*}} dl$$

*Parra NF 2015

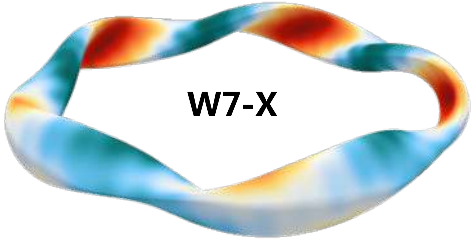
Various types of magnetic fields



$\frac{\partial J}{\partial \alpha} \neq 0$

B contours closing locally

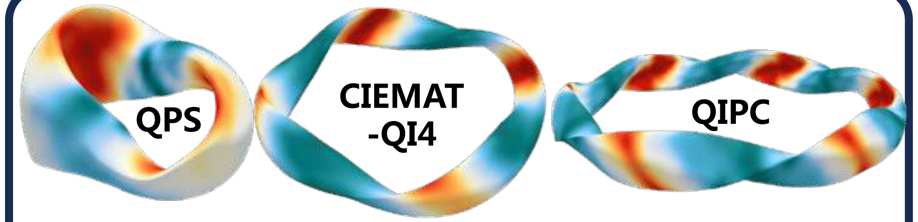
Optimization of omnigenity and quasi-symmetry



$\langle \vec{v}_d \cdot \nabla\psi \rangle = 0$

Optimizing particle losses

Comp. expensive



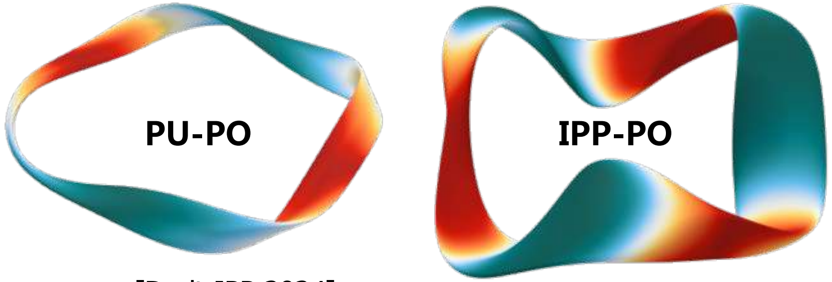
[Spong NF 2001] [Sánchez NF 2023] [Subbotin NF 2006]

$\mathcal{J}$ properties → $\partial\mathcal{J}/\partial\alpha = 0$

Direct or as proxies ……

Aligning $\mathcal{J}$ contours

Computationally expensive



[Dudt JPP 2024] [Goodman JPP 2023]

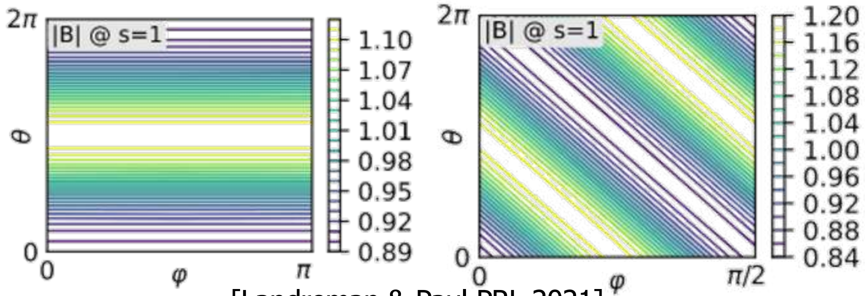
min $|\boldsymbol{B}|_{equilibrium} - |\boldsymbol{B}|_{omnigenity}$

Constructing an ideal field and minimizing the difference

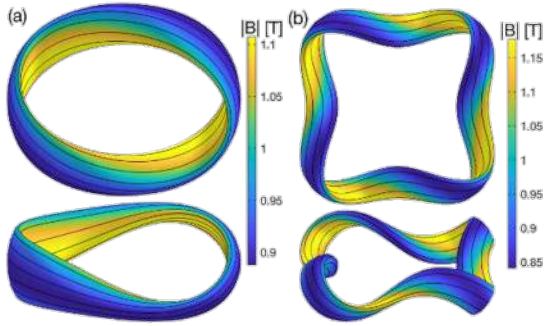
Might be over constrained

Reducing asymmetric modes

$$f_B = \sum_{\substack{n,m \\ n \neq \tilde{\alpha} m}} |B_{nm}|^2$$



[Landreman & Paul PRL 2021]



$$f_T = \nabla\psi \times \nabla B \cdot \nabla(\boldsymbol{B} \cdot \nabla B)$$

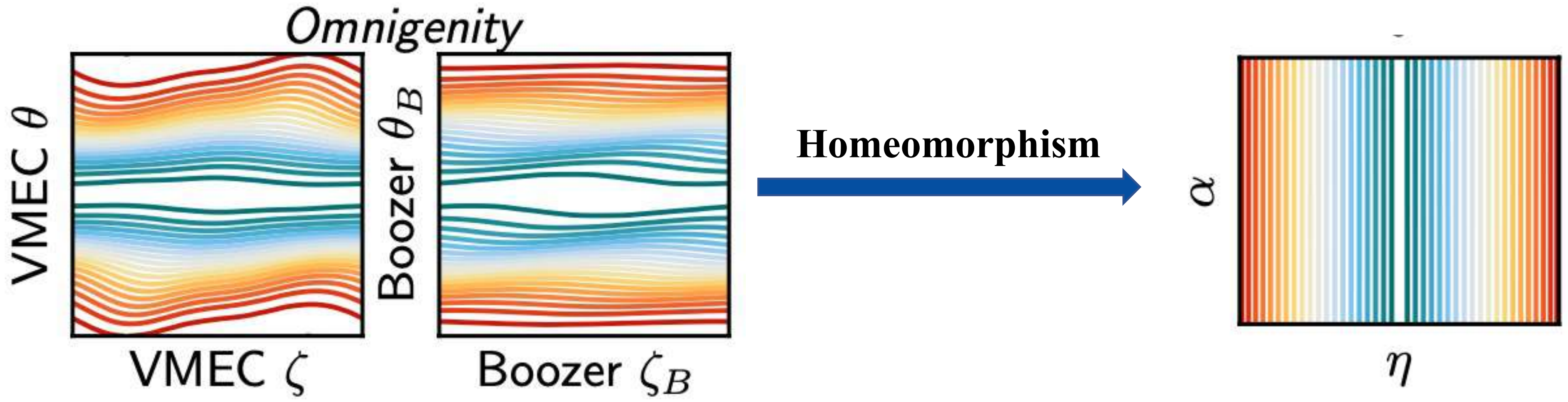
$$f_C = \boldsymbol{B} \cdot \nabla\psi \times \nabla B - C(\psi)\boldsymbol{B} \cdot \nabla B$$

Omnigenity OPtimization like quasi-Symmetry (OOPS)

H. Liu, G. Yu, C. Zhu, and G. Zhuang, Optimizing Omnigenity like Quasisymmetry for Stellarators, arXiv:2502.09350.

Homeomorphic coordinate mapping for omnigenity

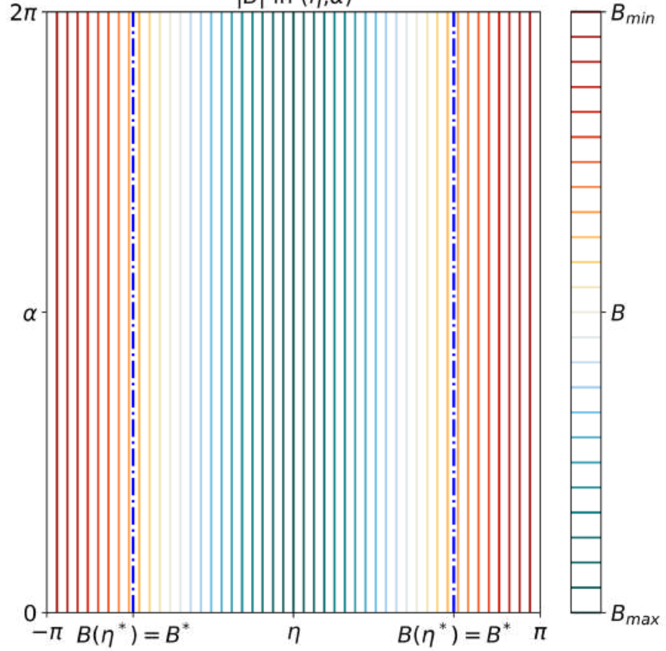
- Omnigenous fields have no locally closed contours.
- There exists homeomorphic coordinates where *B* contours are straight.



- For quasi-symmetry (QS), Boozer coordinates satisfy such conditions.
- For non-QS omnigenity, one have to find a new coordinate system that meets the requirements and **preserves omnigenity**.

Constructing omnigenous fields in Boozer coordinates is easier

- Several mappings (Cary & Shasharina PoP 1997, Landreman & Catto PoP 2012, Dudt JPP 2024) have been proposed to construct omnigeneous fields.
  - ➢ **Omnigenity is preserved**
  - ➢ **Constant $\boldsymbol{\eta}$ is constant B**
- A similar mapping is applied here.



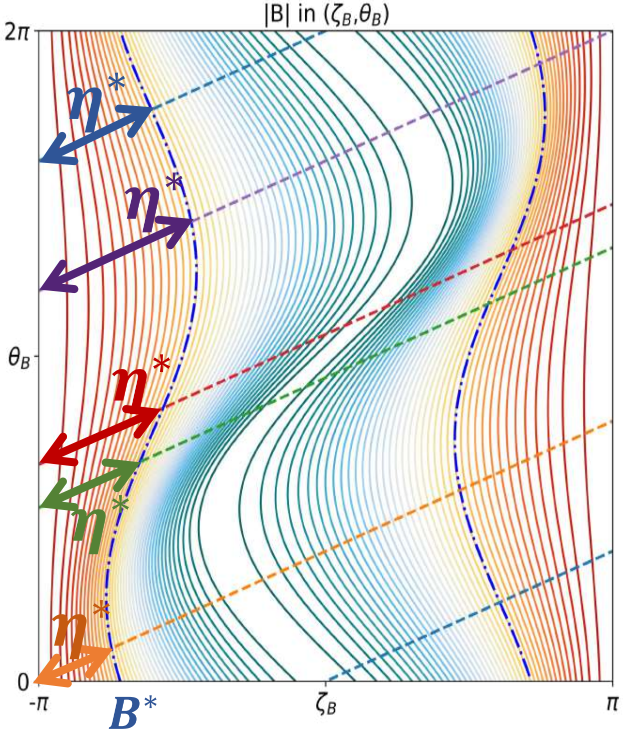
Preserve $\frac{\partial \tilde{J}(B^*, \alpha)}{\partial \alpha} = 0$

$$\tilde{\zeta}(\alpha, \eta) = \eta - D(\eta) \cdot S(\alpha, \eta)$$

$$\tilde{\theta} = \alpha$$

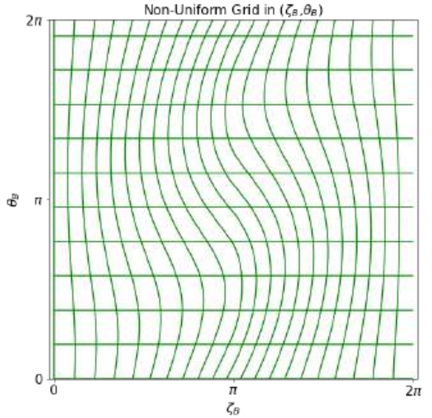
S controls the contour shape

D determines the bounce distance



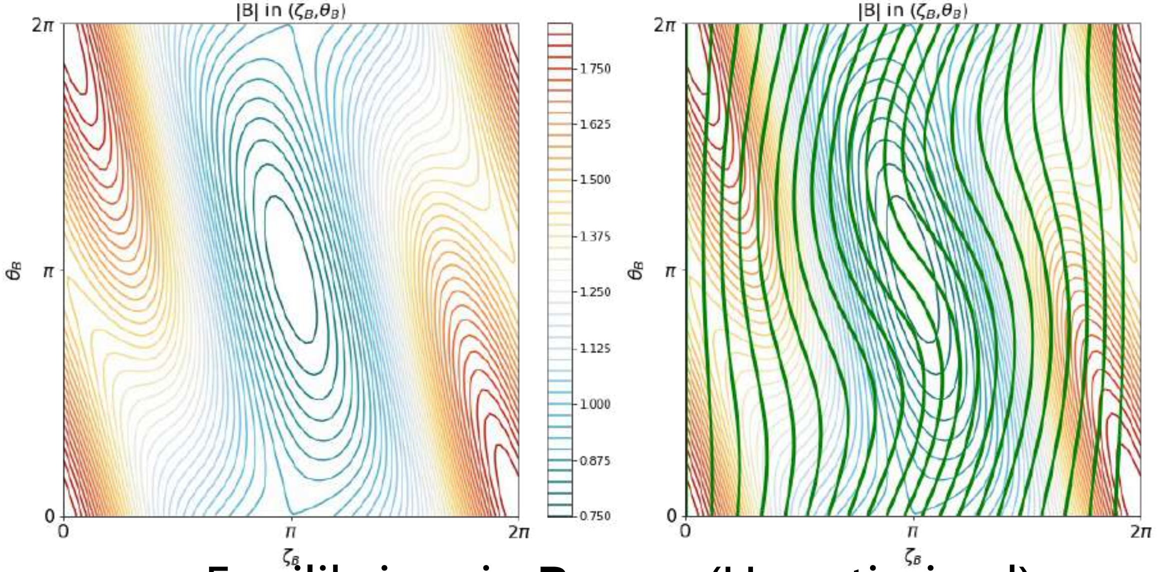
Optimizing omnigenity like quasisymmetry

Pick up an **arbitrary** omnigenity mapping

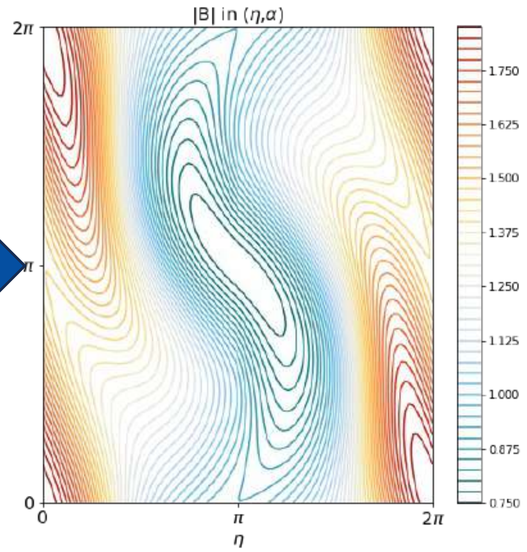


For quasisymmetry, we can use $\eta = \zeta_B, \alpha = \theta_B$

The omnigenity mapping can be varied during the iteration of optimization.
(For simplicity, we keep it fixed.)

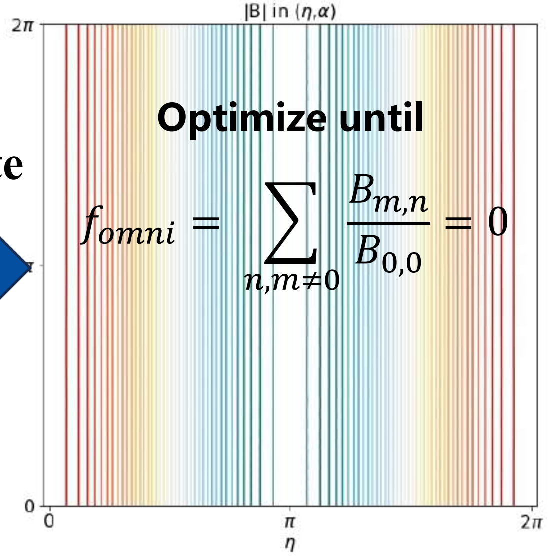


Equilibrium in **Boozer**(Unoptimized)



Equilibrium in $(\boldsymbol{\eta}, \boldsymbol{\alpha})$

iterate



Optimize until

$$f_{omni} = \sum_{n,m\neq 0} \frac{B_{m,n}}{B_{0,0}} = 0$$

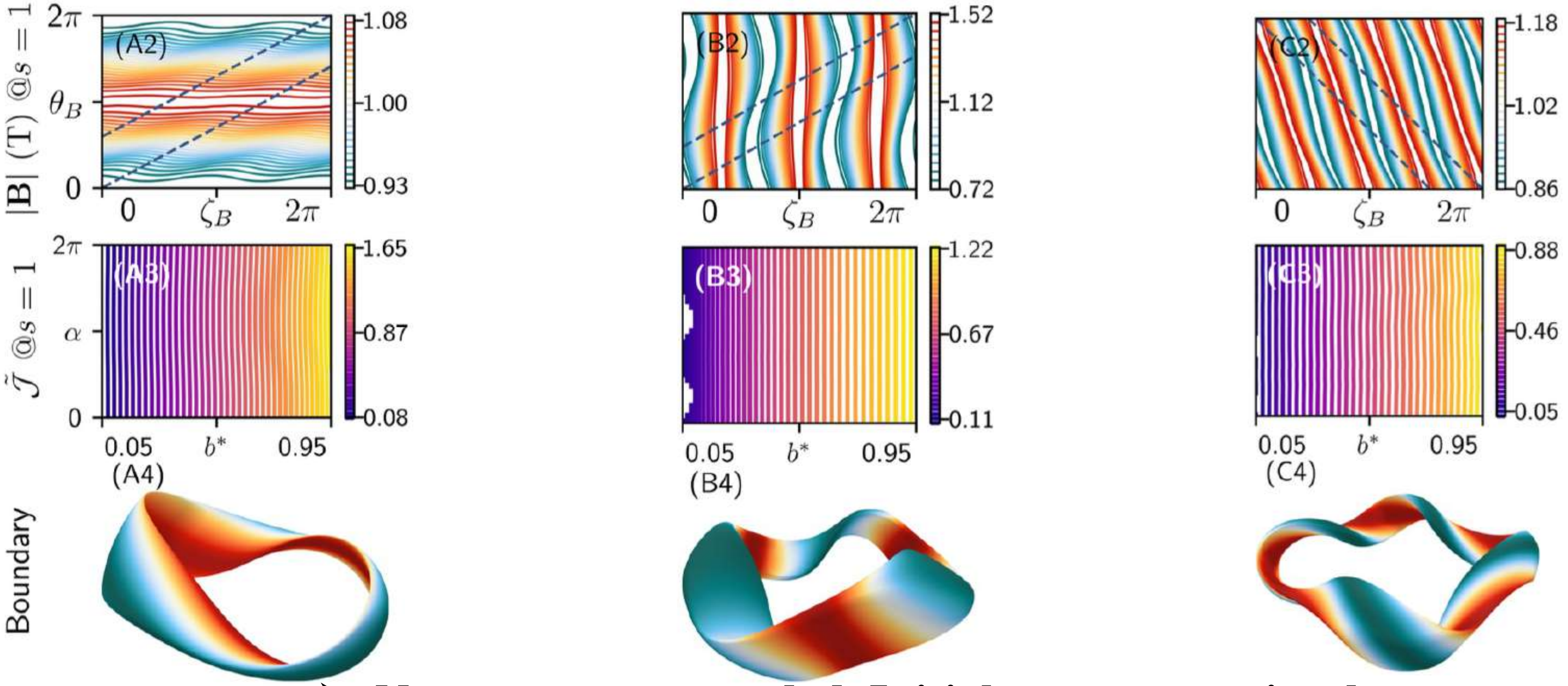
Target

Precise, compact omnigenous configurations have been obtained

Toroidal Omnigenity (TO) Nfp=2 Ap=6 $\iota_{edge} = 0.711$

Poloidal Omnigenity (PO) Nfp=3 Ap=6.5 $\iota_{edge} = 0.765$

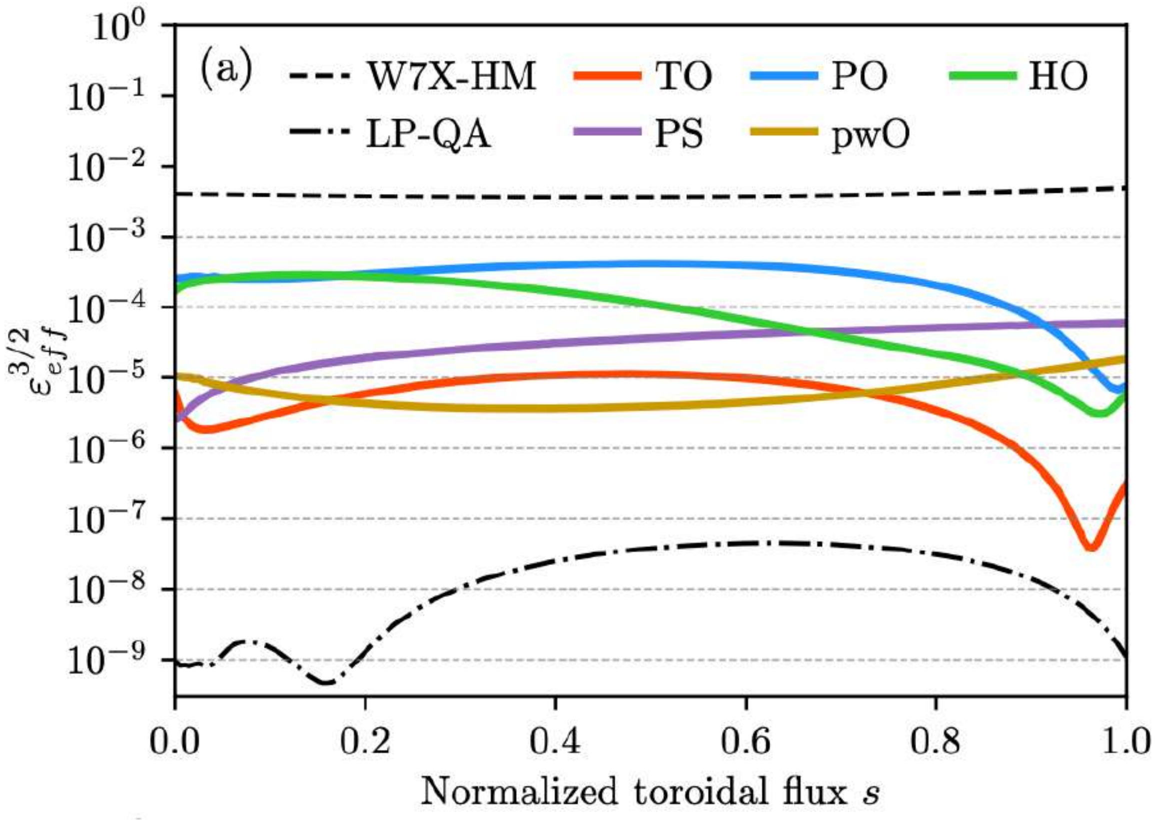
Helical Omnigenity (HO) Nfp=4 Ap=8 $\iota_{edge} = 1.326$



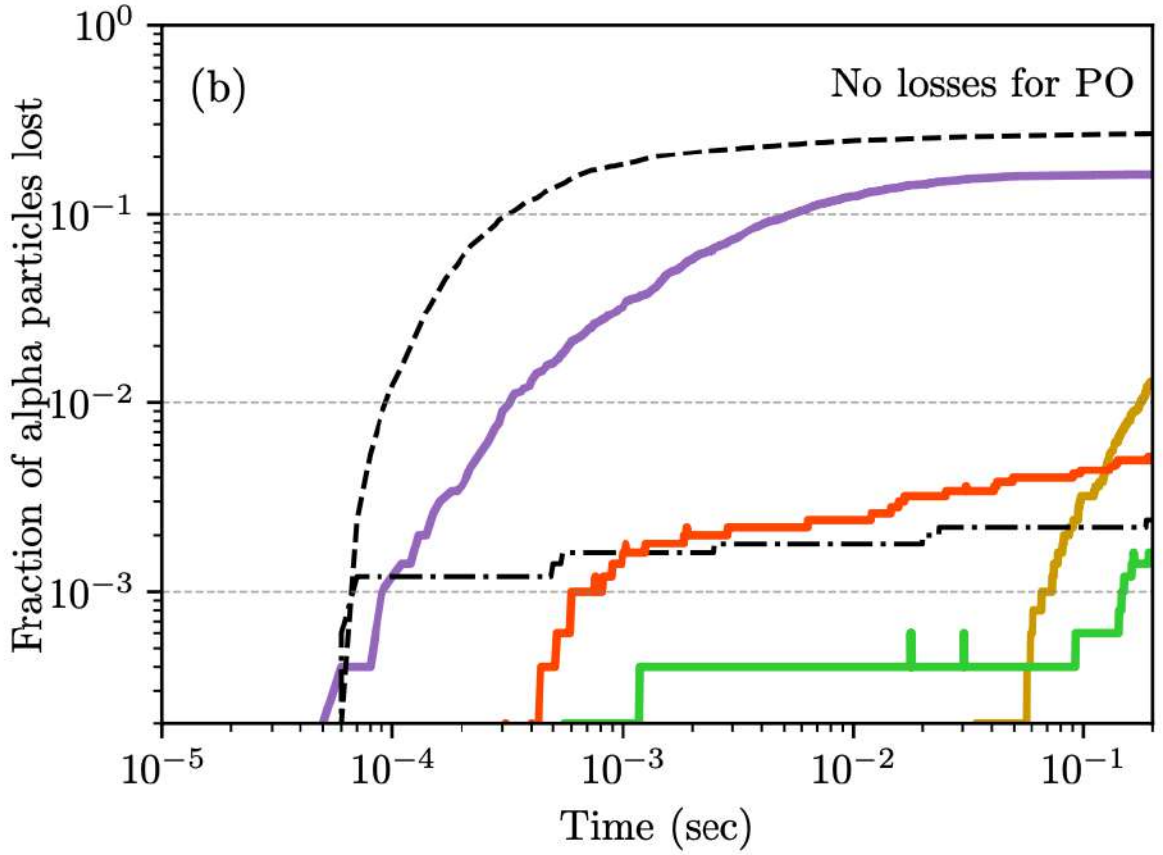
- ➢ **No warm starts needed. Initial guesses are circular torus.**
- ➢ **Simplest mapping is used. (One Fourier coefficient)**

Good confinement properties are achieved

Effective Ripple $\epsilon_{eff}^{3/2}$



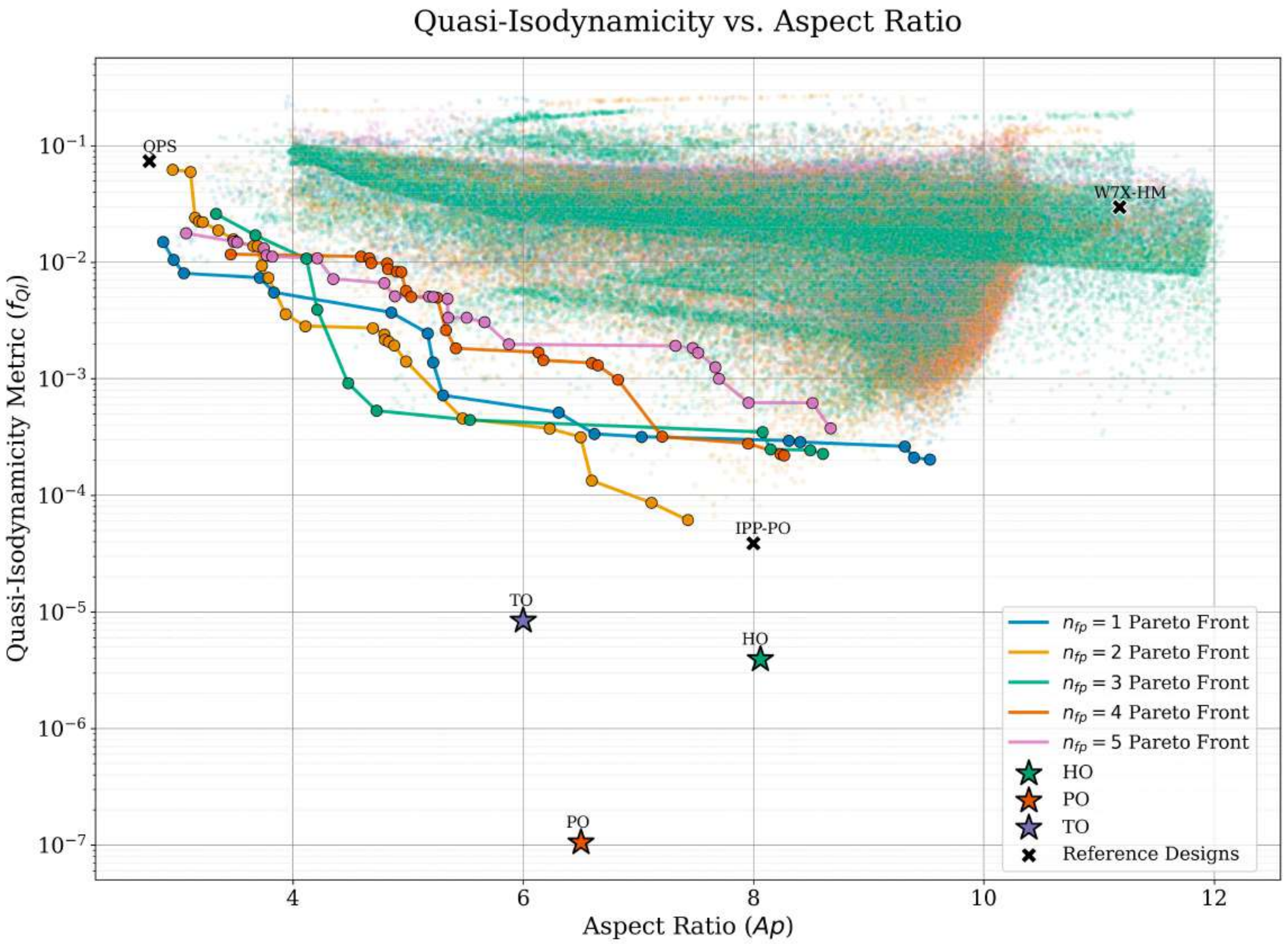
Loss Fraction of Collisionless 3.5 MeV α-Particles



Beyond the Pareto fronts of the "Constellaration" database

- Proxima Fusion recently released the "Constellaration" database [Cadena arXiv 2025].
- 158, 000 QI configurations were optimized using DESC and VMEC++.
- Pareto fronts of f_{QI} and Ap for each period are identified.

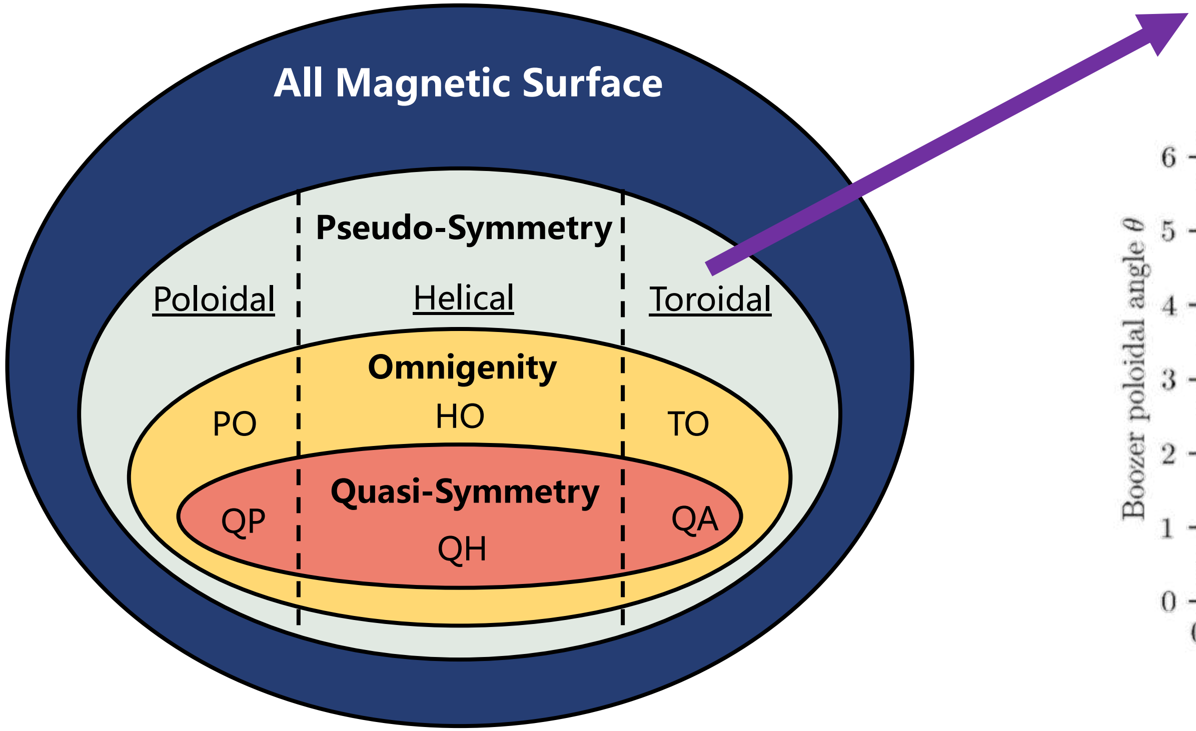
$$f_{QI} = \frac{1}{4\pi^2} \iint (B(\theta,\phi) - B^{QI}(\theta,\phi))^2 d\theta d\phi$$

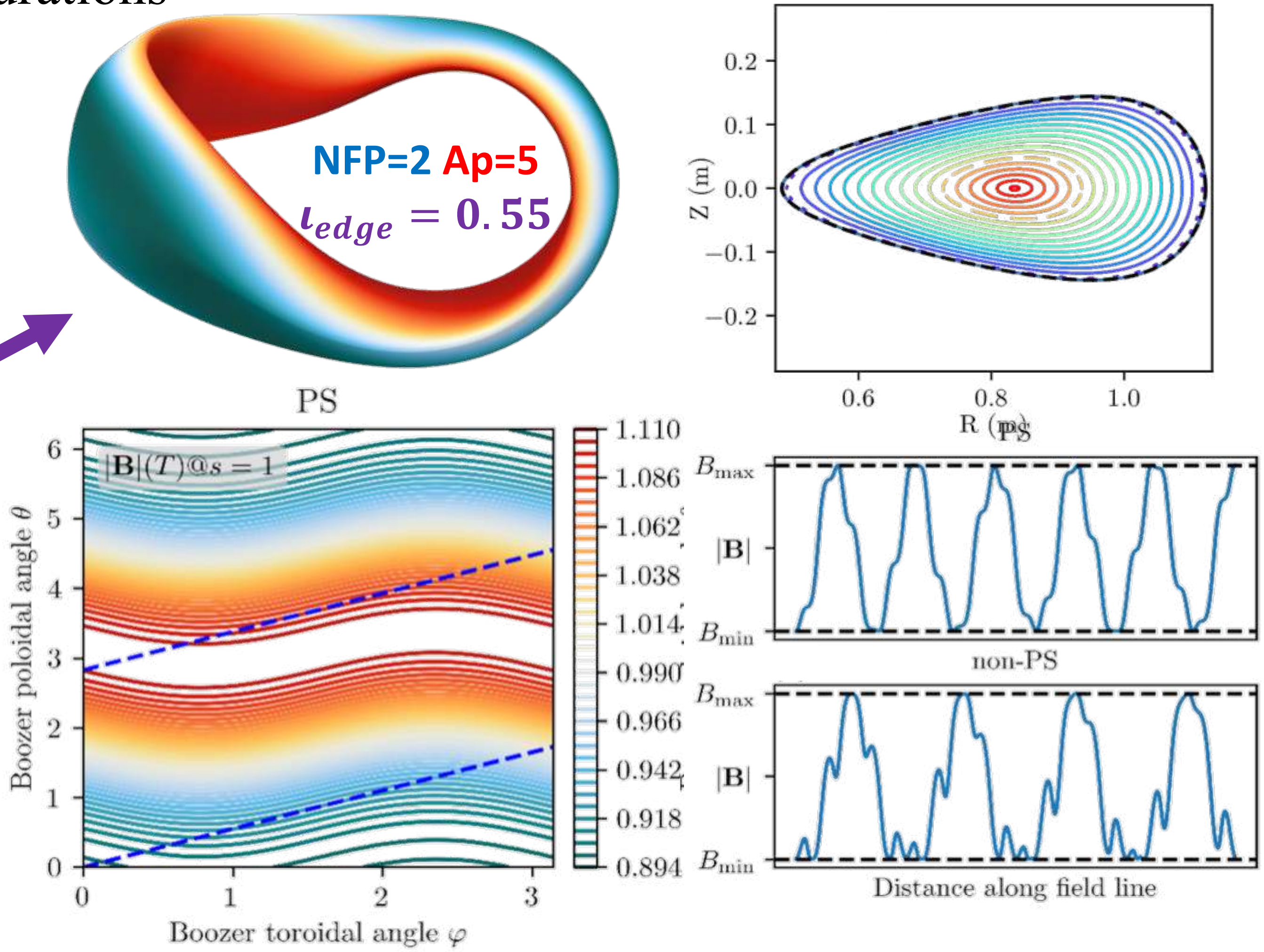


Pseudo-symmetry can also be easily optimized

Pseudo-symmetry (PS): Magnetic configurations with no locally trapped particles

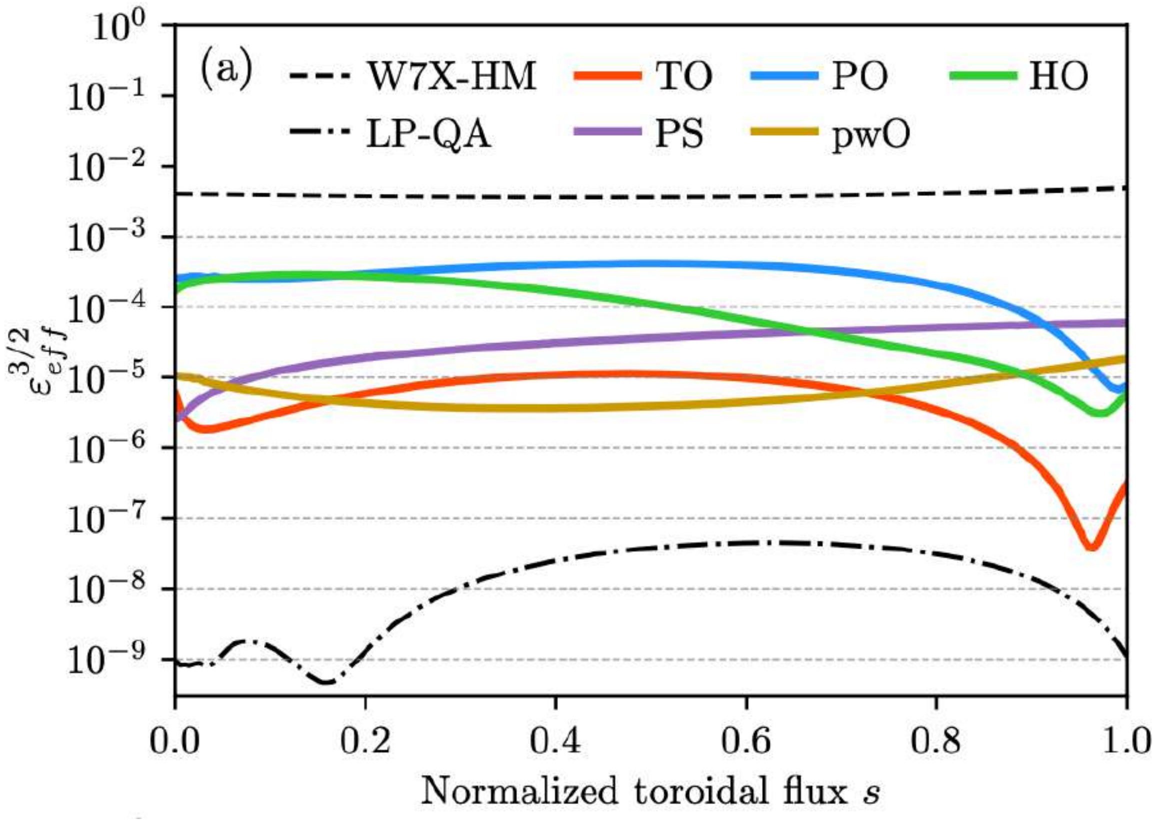
- ➢ Relax constraint of constant bounce distance
- ➢ B has no locally closed contours
- ➢ Field lines do not cross B contours twice





Good confinement properties are achieved

Effective Ripple $\epsilon_{eff}^{3/2}$



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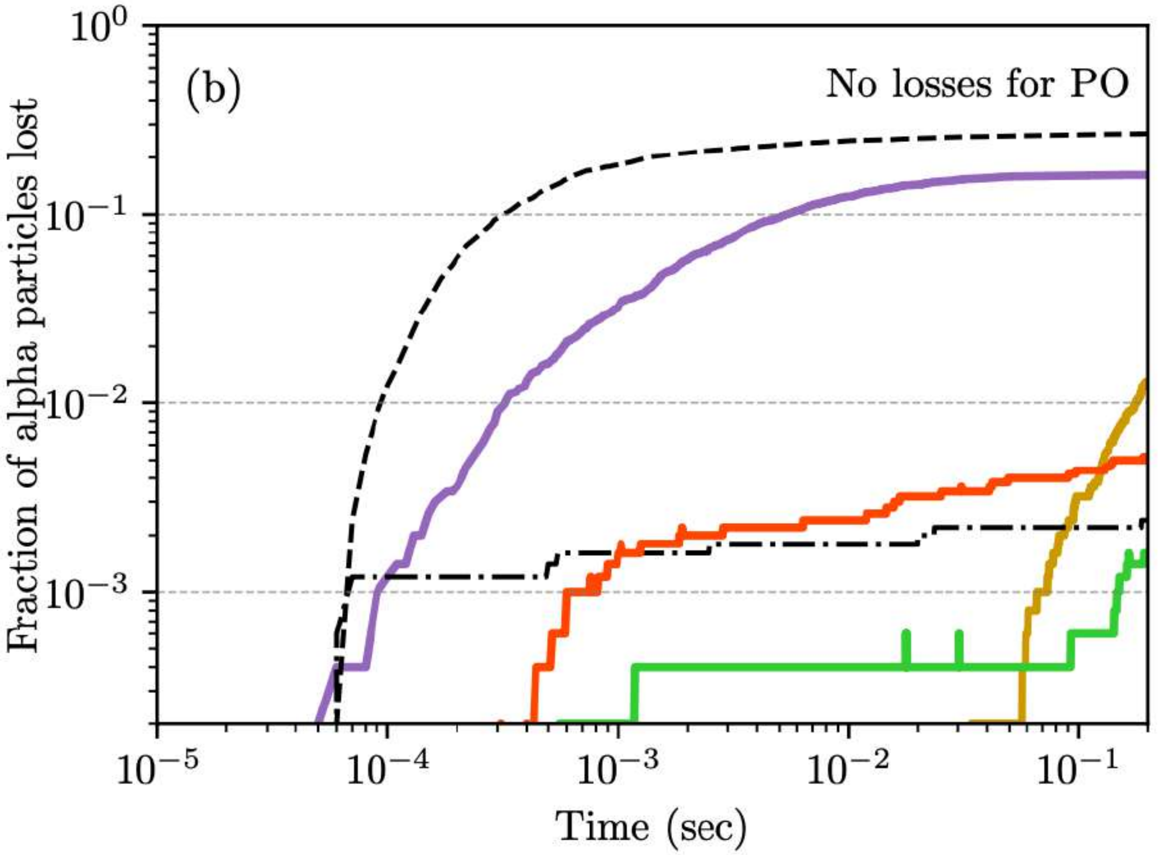
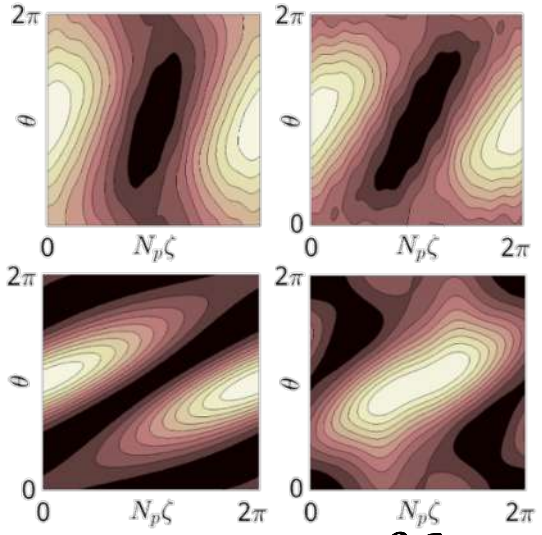


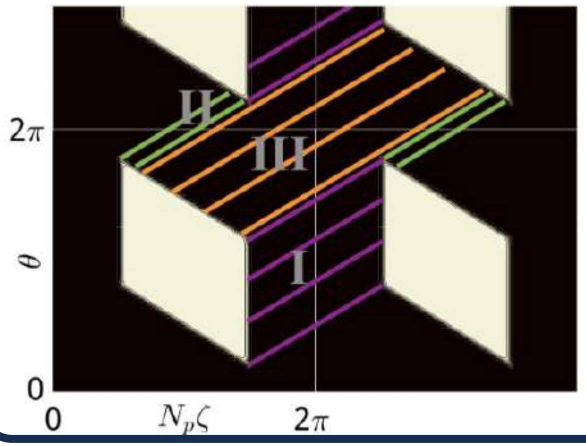
Configurations Combining Omnigenity and Piecewise Omnigenity

- Liu, *et al*., Optimization of stellarator configurations combining omnigenity and piecewise omnigenity (to be submitted)
- Velasco, et al., Nearly-quasi-isodynamic piecewise omnigenous fields for negligible neoclassical radial transport and bootstrap current (to be submitted)

Piecewise omnigenity is a generalized concept



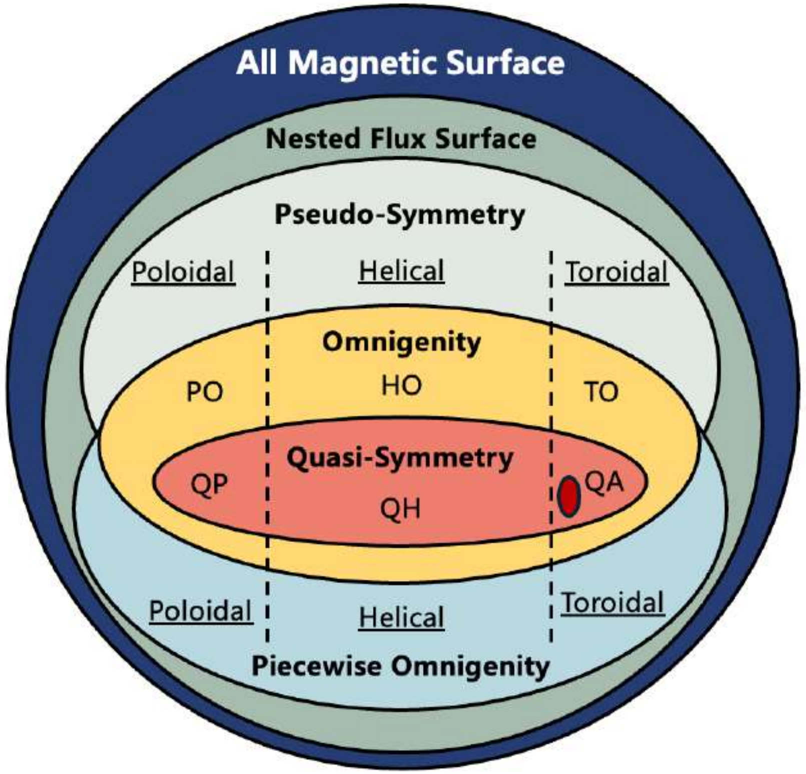
$$\mathcal{J}_1 + \mathcal{J}_2 = \mathcal{J}_3 \rightarrow \frac{\partial \mathcal{J}}{\partial \alpha} = 0$$



W7-X, LHD inward-shifted and conf. A [Bindel 2023 PPCF] configurations have relatively low ϵ_{eff} and alpha-particle losses (LHD), even with significant deviations from perfect omnigenity.

Omnigenity conditions
(1) B-contours are closed
(2) Bounce distance between neighboring lines is constant

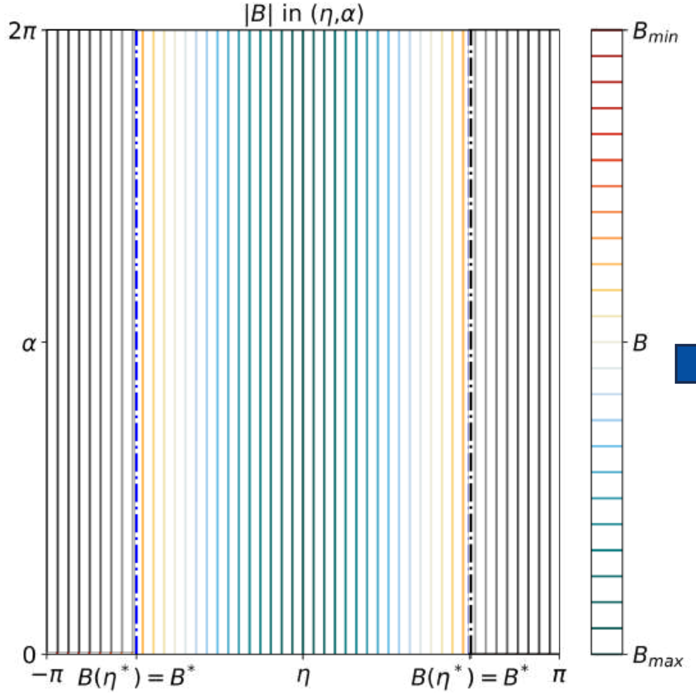
Piecewise omnigenity (pwO): omnigenity is satisfied piecewisely

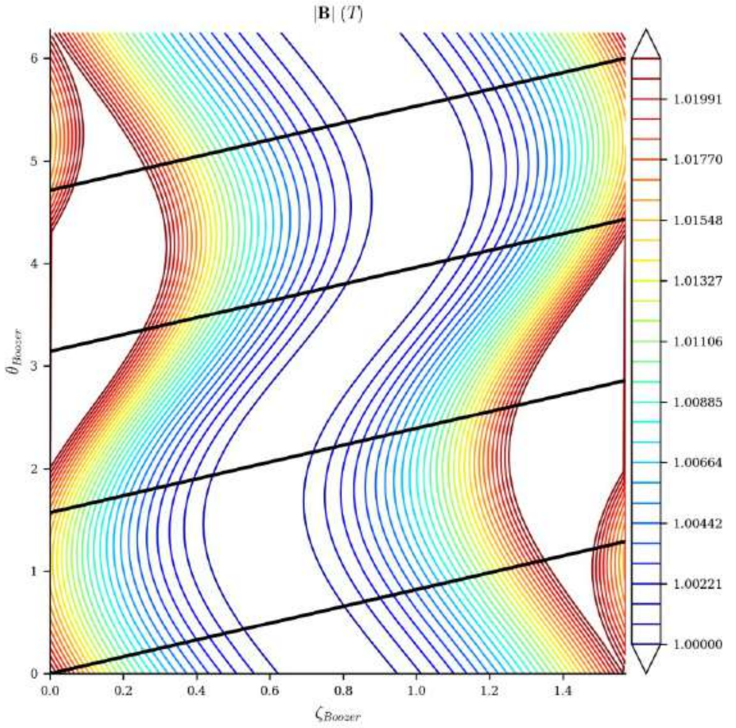


Velasco PRL 2024

Optimizing pwO using OOPS

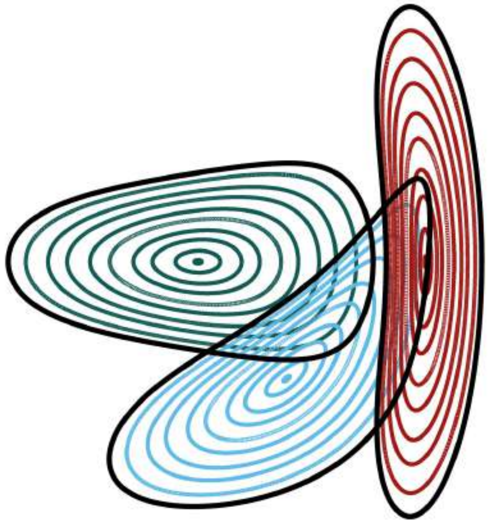
- pwO configurations have locally closed contours, however:
- OOPS can still work if we **bound *η*** and focus out of the Bmax region.
- *B* contours have to be **strongly shaped** such that B_{max} regions can close and form “parallelograms”.



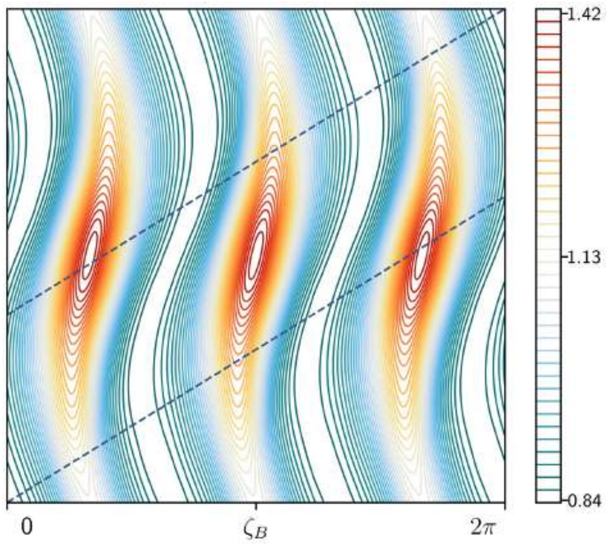


A three-period pwO + QI (OpwO)

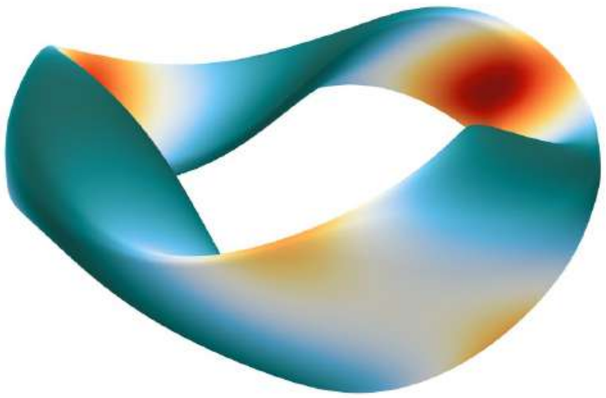
- Start from a PO, choose a strongly shaped mapping with $\eta \in [-0.225\pi, 0.225\pi]$



Flux surfaces and Poincare plots



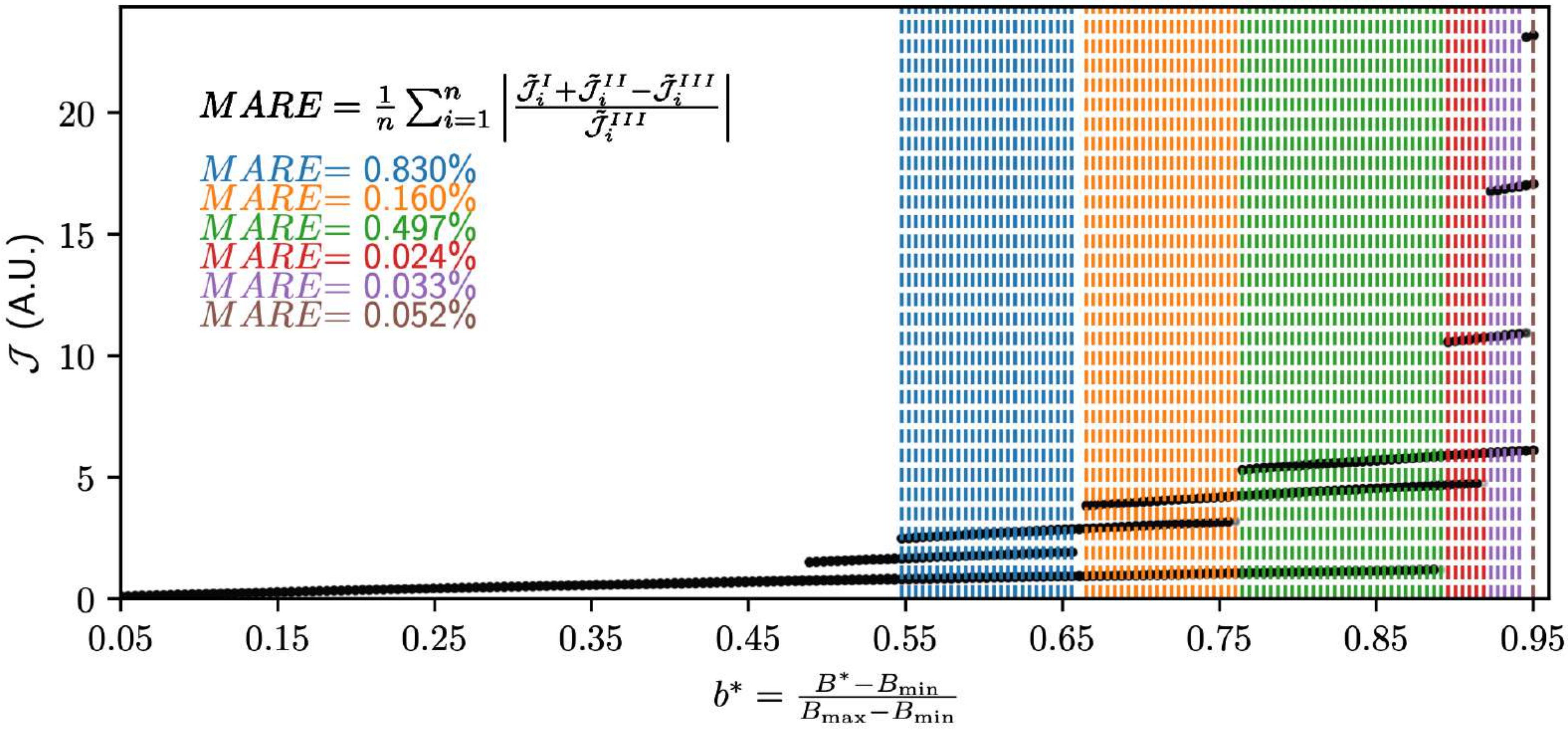
B contours on the boundary



Plasma boundary

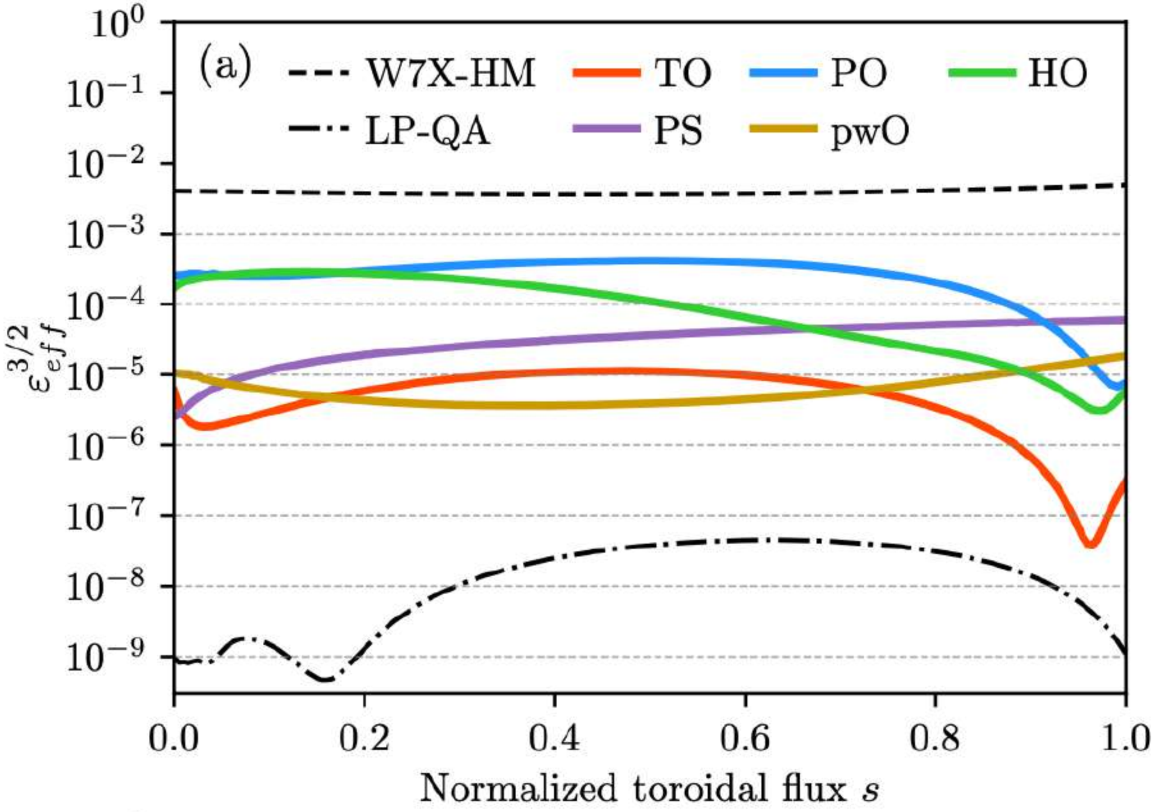
Nfp=3 Ap=6 $\iota_{edge} = 0.62$

$\mathcal{J}$ distributions confirm the feature of pwO + QI

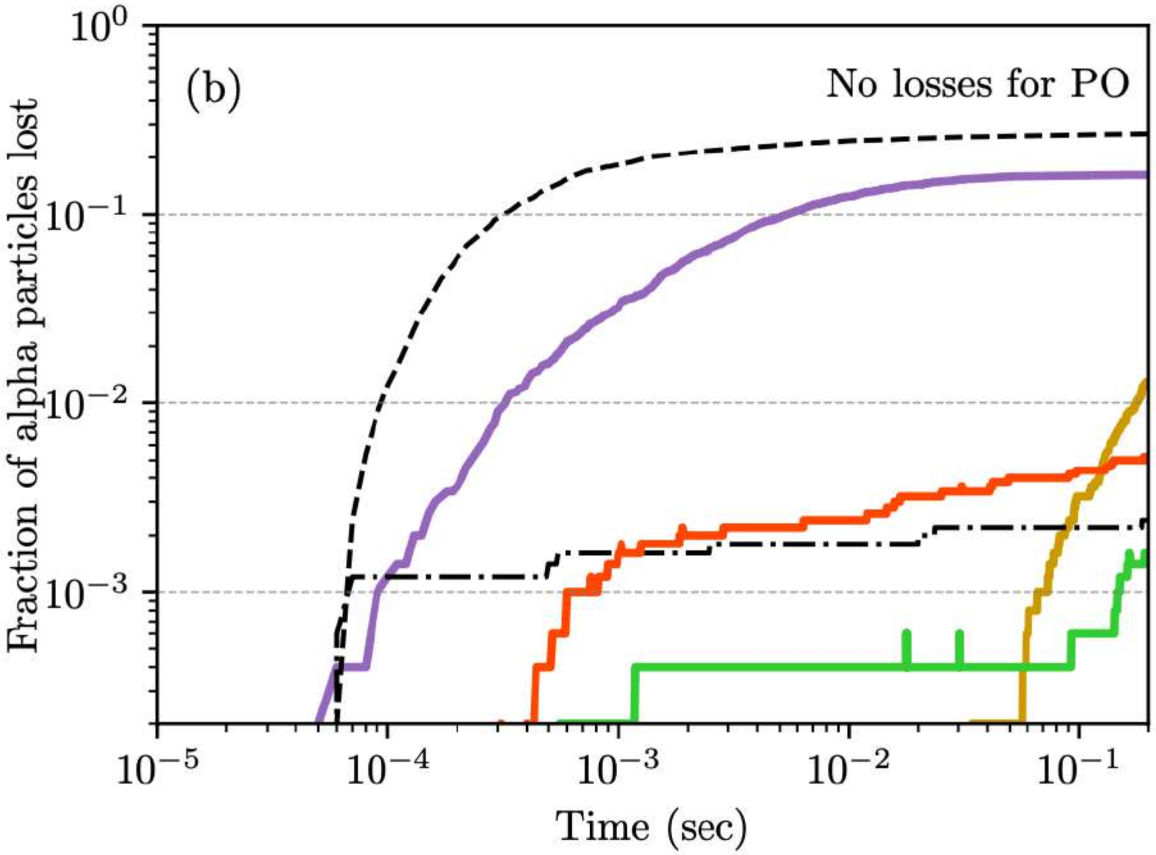


Good confinement properties are achieved

Effective Ripple $\epsilon_{eff}^{3/2}$

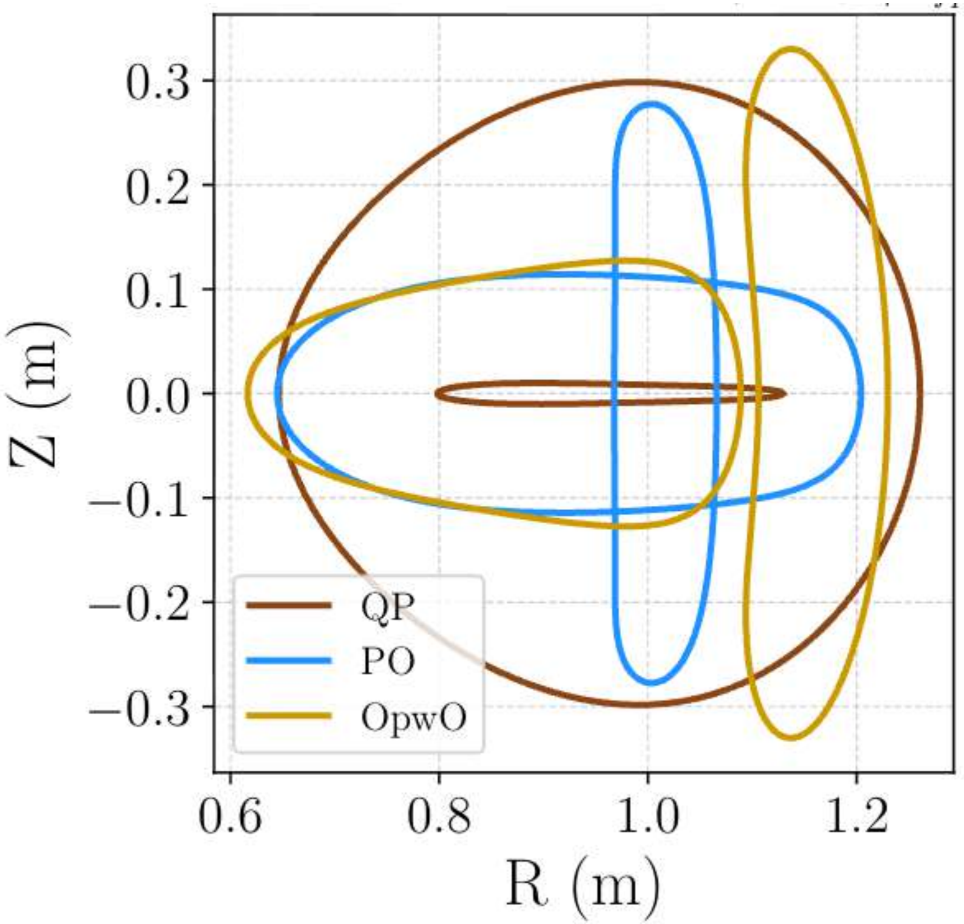


Loss Fraction of Collisionless 3.5 MeV α-Particles

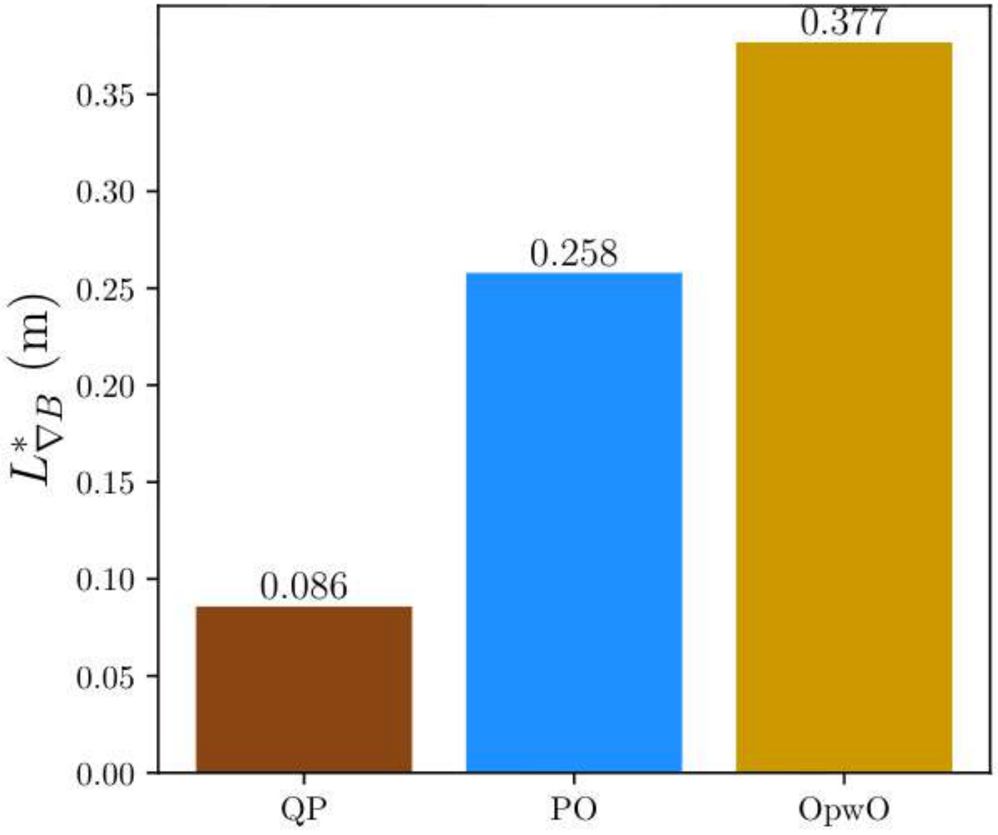


Relaxing constraints leads to better engineering feasibility

Nfp = 3, Ap=6~6.5, iota~0.7

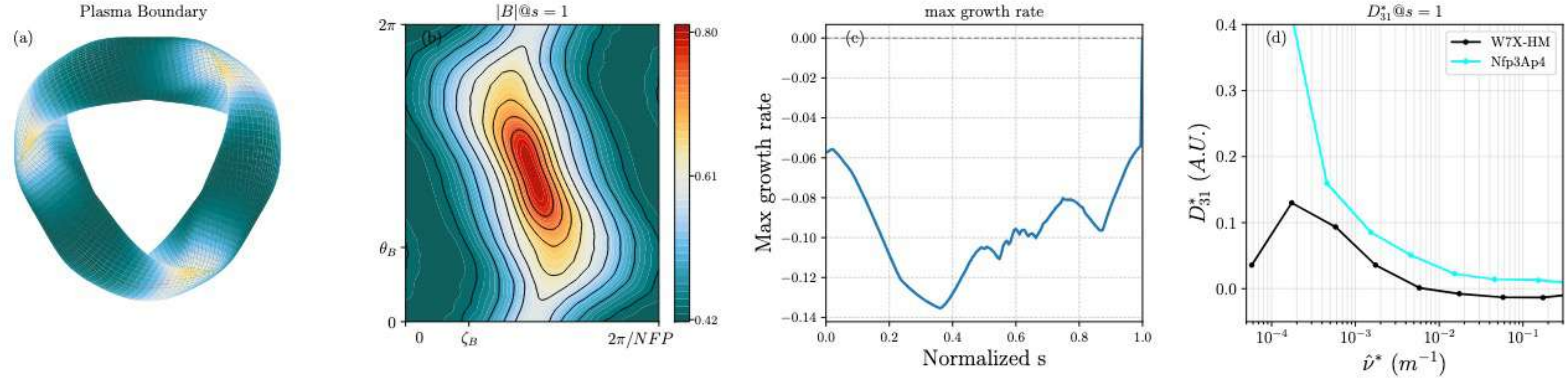


Boundaries get less elongated



The coil complexity proxy, $L^*_{\nabla B}$ [Kappel PPCF 2024], gets larger.

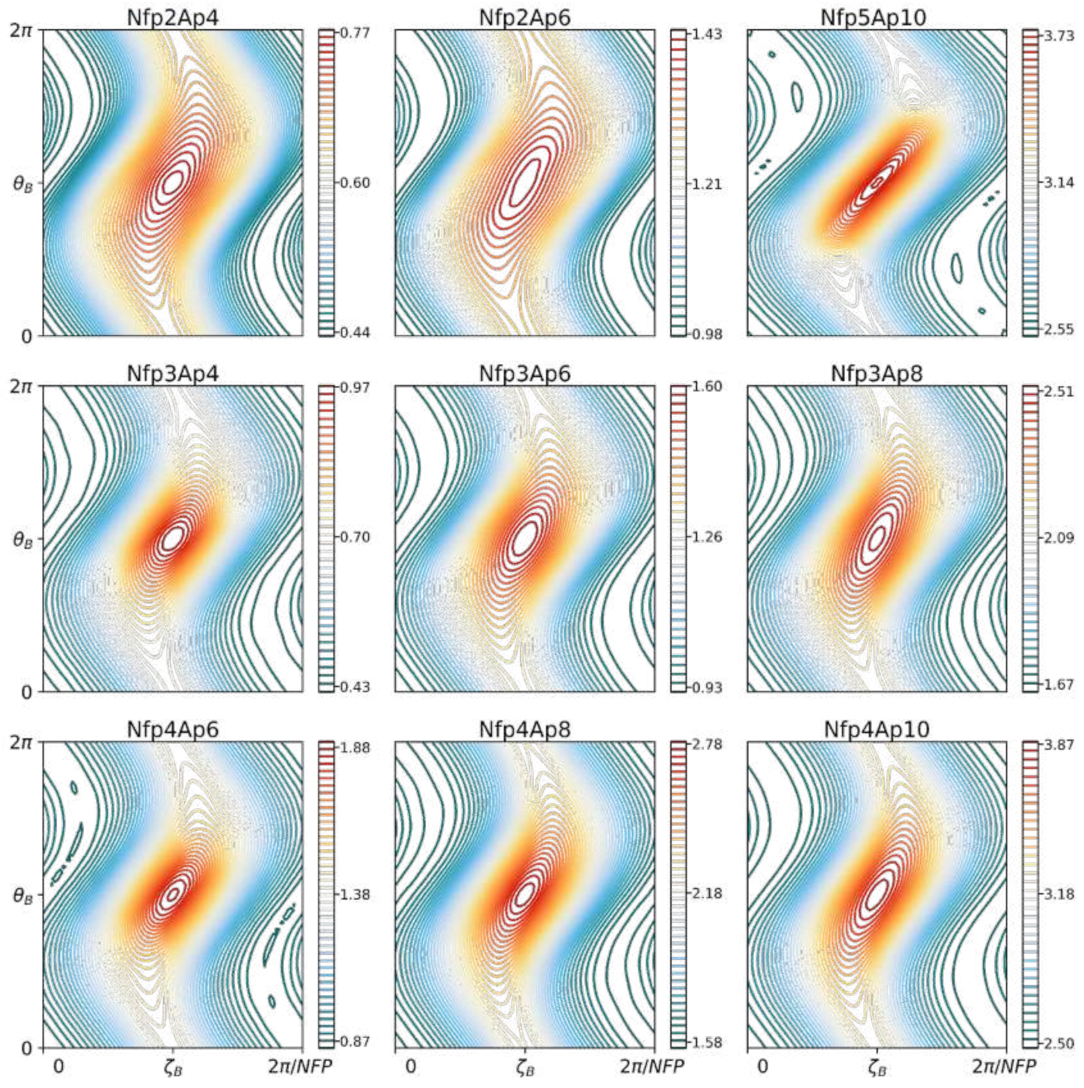
OpwO reactor design at Ap=4

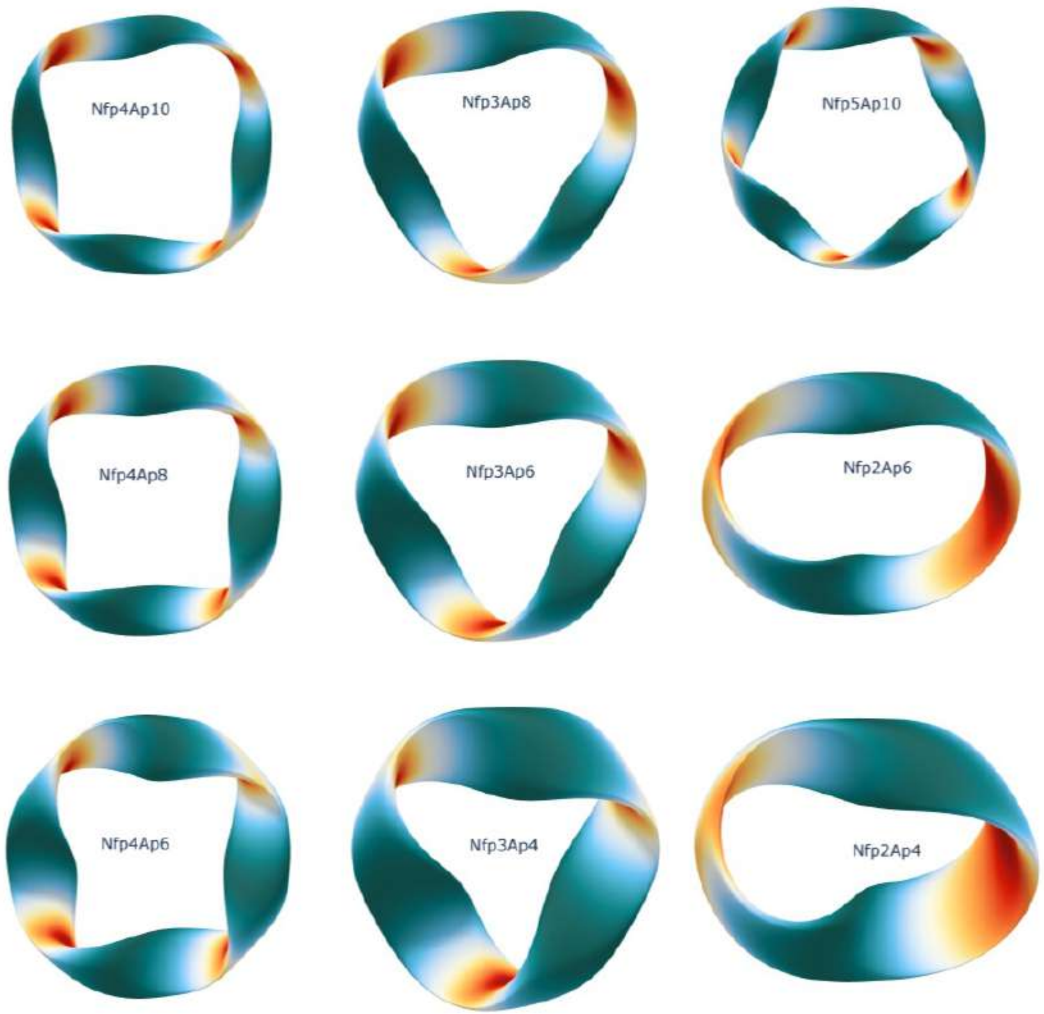


Nfp=3, **Ap=4**, $\beta = 3.0\%$, ballooning stable, modest bootstrap current

low neoclassical transport, good alpha-particle confinement

More pwO + QI configurations

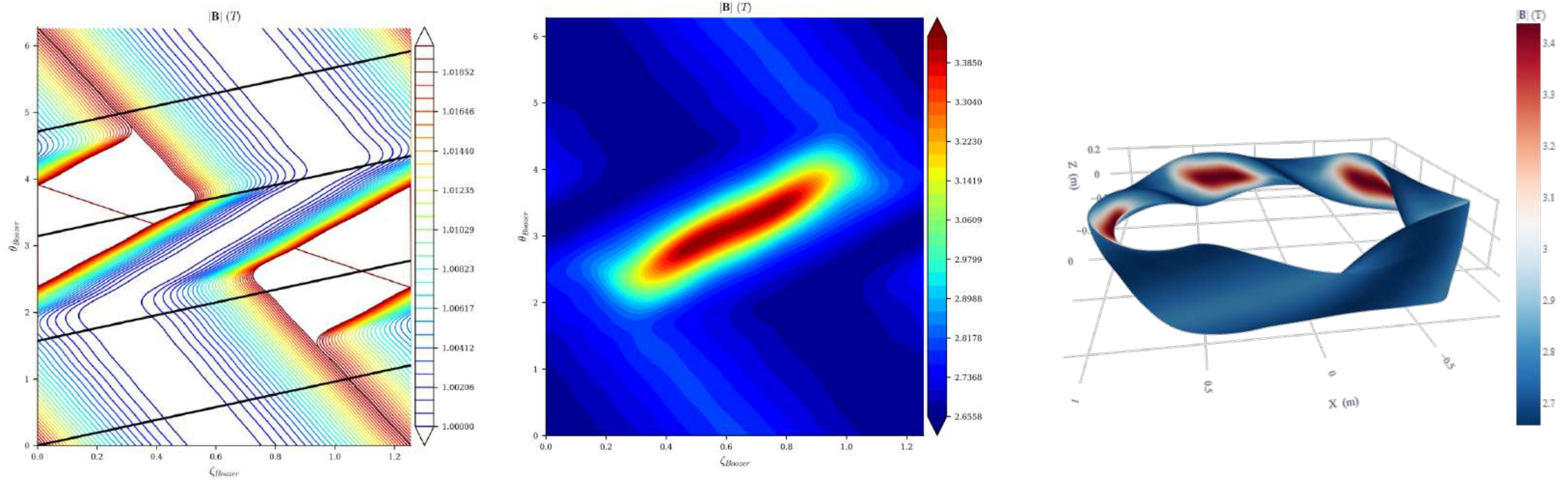




Tuning the mapping function to obtain configurations closer to a prototypical pwO

Try to fit the $B_{\max}$ contours in pwO paper [Velasco PRL 2024]

$$B_{pwO} = B_{\min} + (B_{\max} - B_{\min}) \times \lim_{p\to\infty} \exp\left[-\left(\frac{\zeta_B - \zeta_c + t_1(\theta_B - \theta_c)}{w_1} \right)^{2p} - \left(\frac{\theta_B - \theta_c + t_2(\zeta_B - \zeta_c)}{w_2} \right)^{2p} \right]$$



A five-period prototypical pwO (pwO5)

** Prototypical pwOs can also be obtained by PiecewiseOmnigenity objective in DESC [Fernández-Pacheco arXiv 2026; Velasco EPS 2025]*

Summary

- A new method, OOPS, has been proposed to optimize omnigenity. Precise, compact omnigenous configurations have been obtained.
- OOPS has the flexibility to optimize quasisymmetry, pseudo-symmetry, and piecewise omnigenity.
- Various configurations that combine omnigenity and piecewise omnigenity have been obtained.
- With relaxed constraints, pwO + QI has the potential to achieve more compact configurations and/or to use simpler coils while keeping good confinement.

Thank you for your attention