Flexible and Fast Stellarator Optimization with DESC Suite

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DESC

Princeton Plasma Control
control.princeton.edu

Postdoc Opportunity!

- **Novel Stellarator Designs**
- **Stellarator/Divertor Topology**
- **Plasma SOL**
- **Plasma, Applied Math, CS welcome**
- **Work on DESC**

- **Email: ekolemen@princeton.edu (or talk to me)**

Tokamak Disruption (C-Mod)

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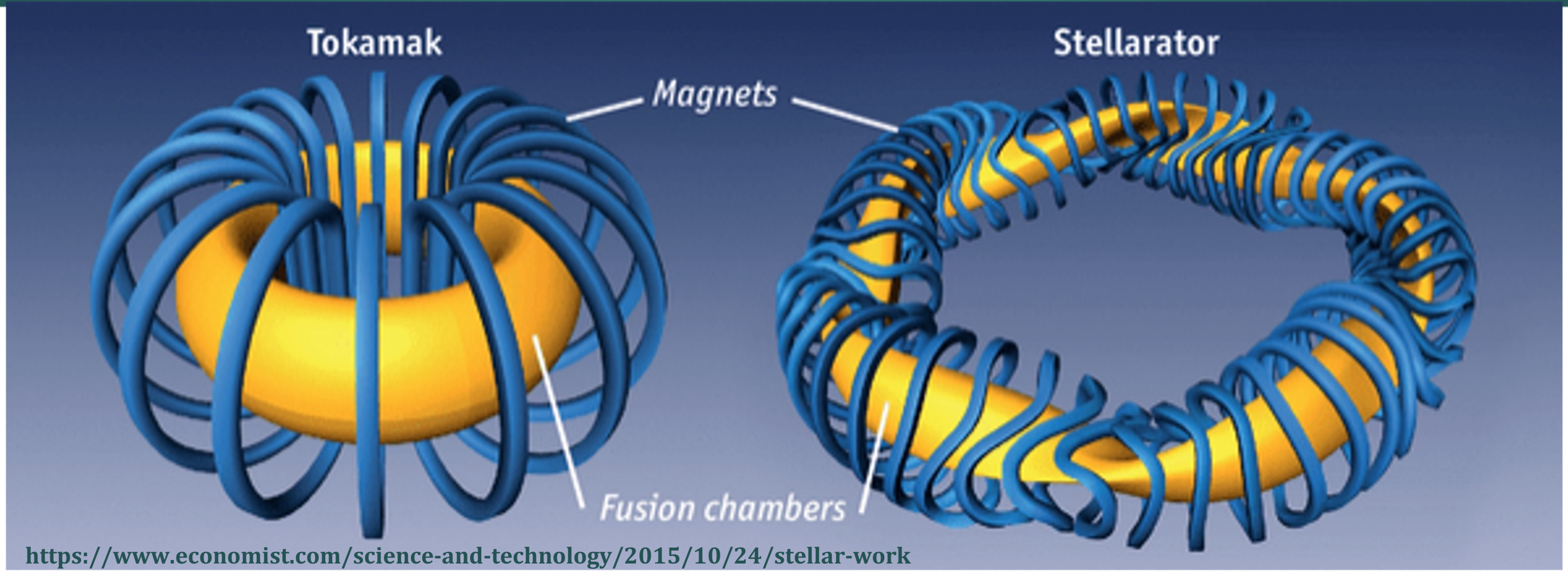
youtu.be/CUfR819hlDg

Magnetic Confinement Devices: Tokamak vs. Stellarator



Tokamaks

- **Axisymmetric**
  - Simpler geometry
  - Guaranteed particle confinement
- **Requires substantial plasma current**
  - Must be driven
  - NOT steady state!
- **Current leads to instabilities -> disruptions**

Stellarators

- **Inherently 3-D**
  - Complex geometry and coils
  - Confinement not guaranteed
    - certain fields exist which recover this (Quasisymmetry)
  - **Larger design space**
- **Does not need plasma current**
  - Steady state
  - No disruptions

What is the ideal way to optimize stellarators?

- **Constraints g(x):**
  - **MHD equilibrium**
  - ***Physicist insight: Analytical calculations (e.g. NEA)***
  - ***Engineer insight: e.g. A<5, …***
- **Objectives f(x):**
  - **Quasi-symmetry**
  - **Turbulence**
  - **…**
- ***Physicist/engineer insight: relative importance of f(x)***

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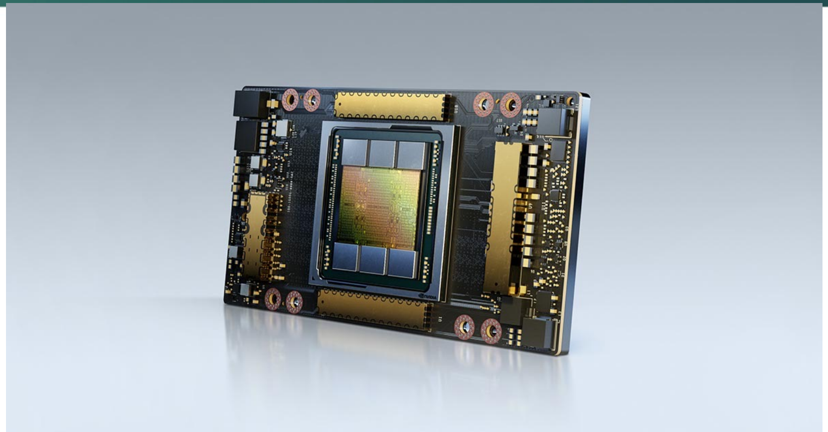
Then Call A Fast Code

$$\min_x f(x)$$

$$\text{subject to} \quad g_{eq}(x) = 0$$
$$g_{ineq}(x) \geq 0$$

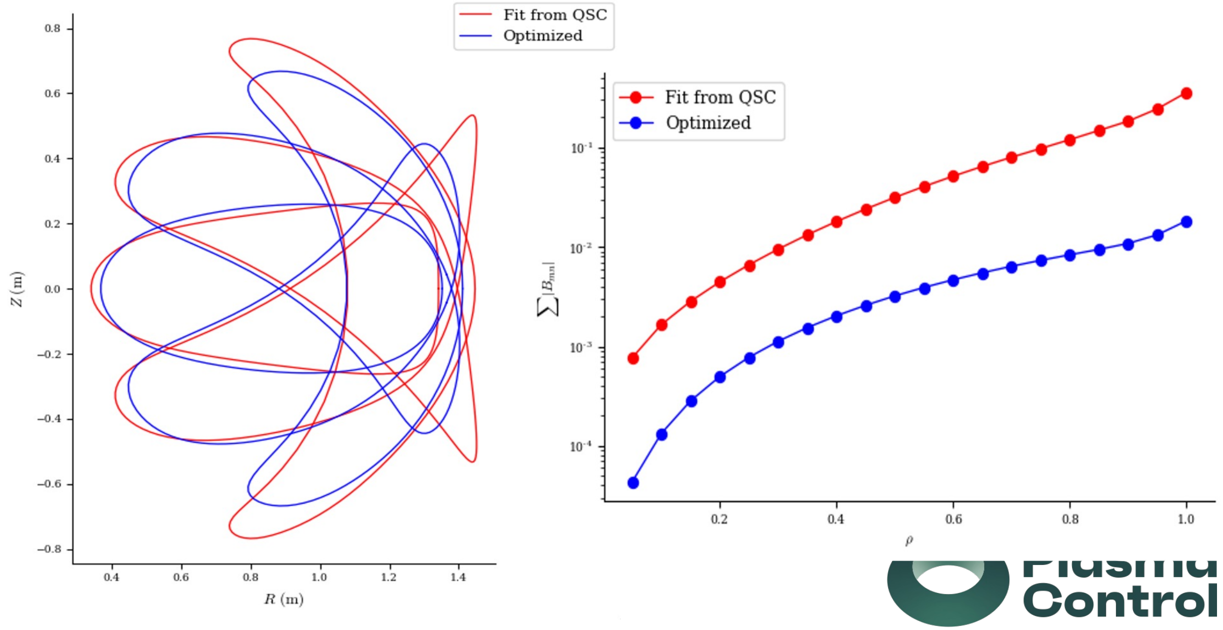
→



Fast= GPU + Jacobian

Constraints: $g_{physicist}$ =Fix NEA
+ g= MHD Eq.
Optimize remaining volume

→



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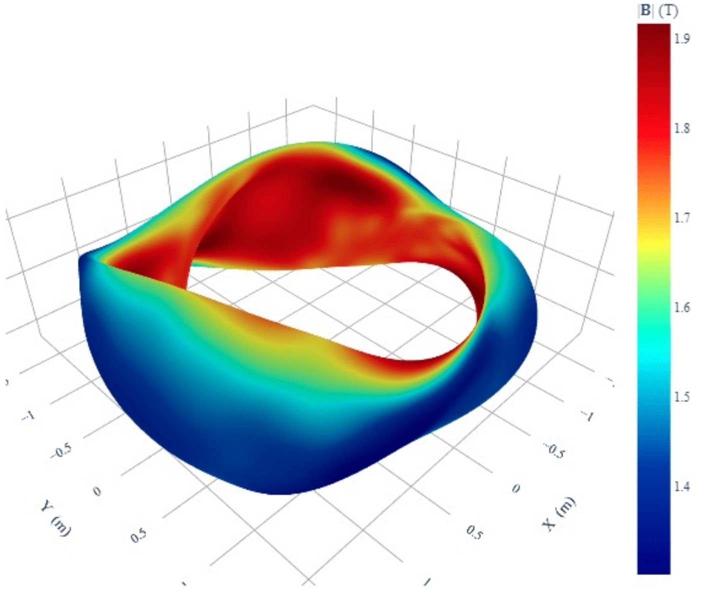
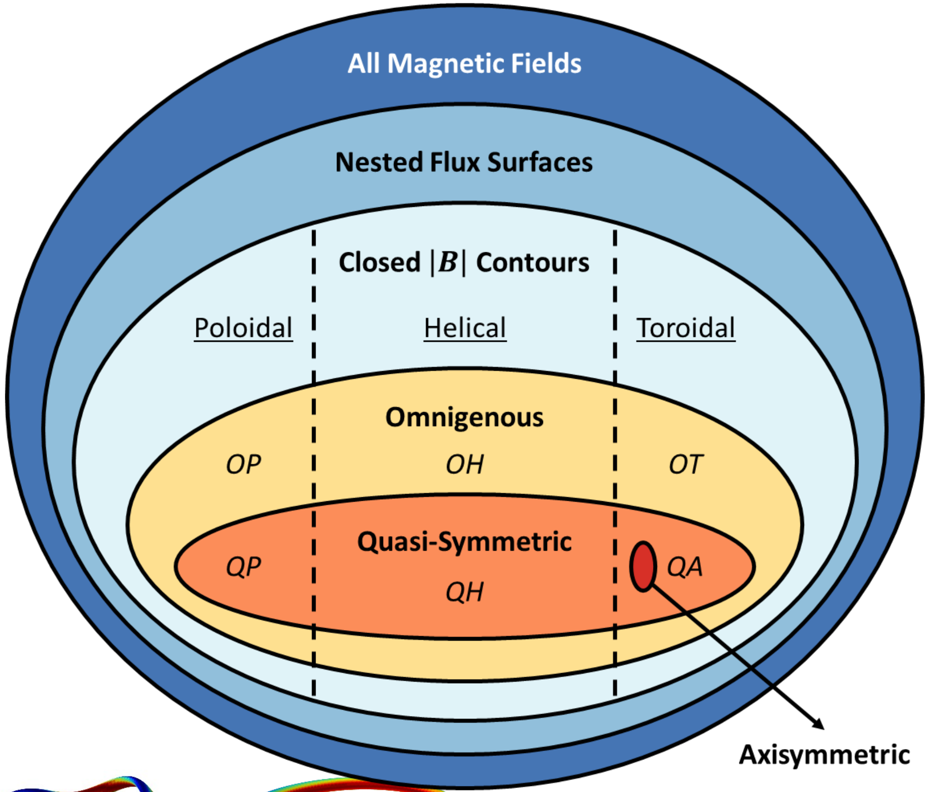
DESC is a new tool for stellarator optimization

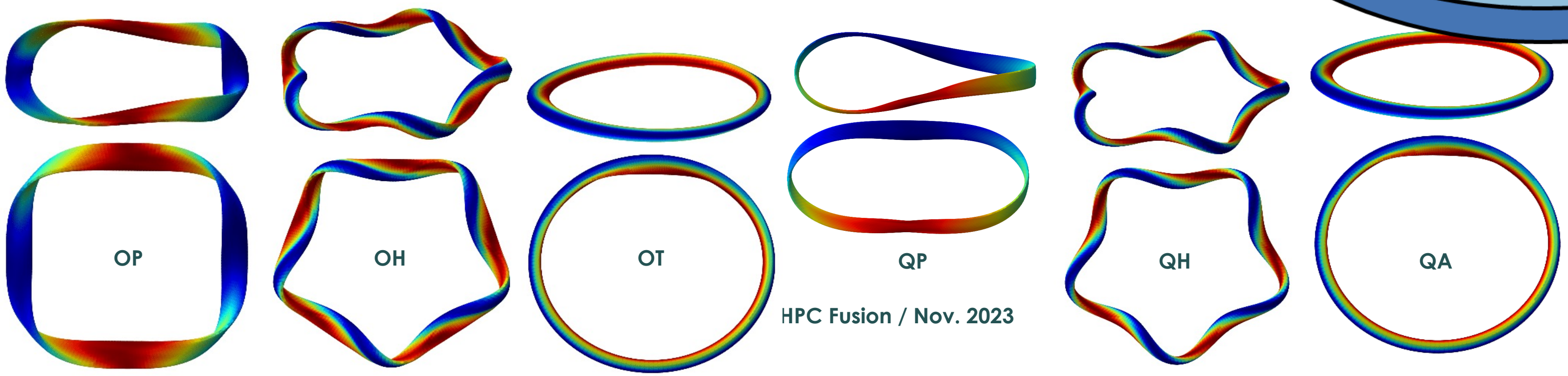
Accurate Equilibria



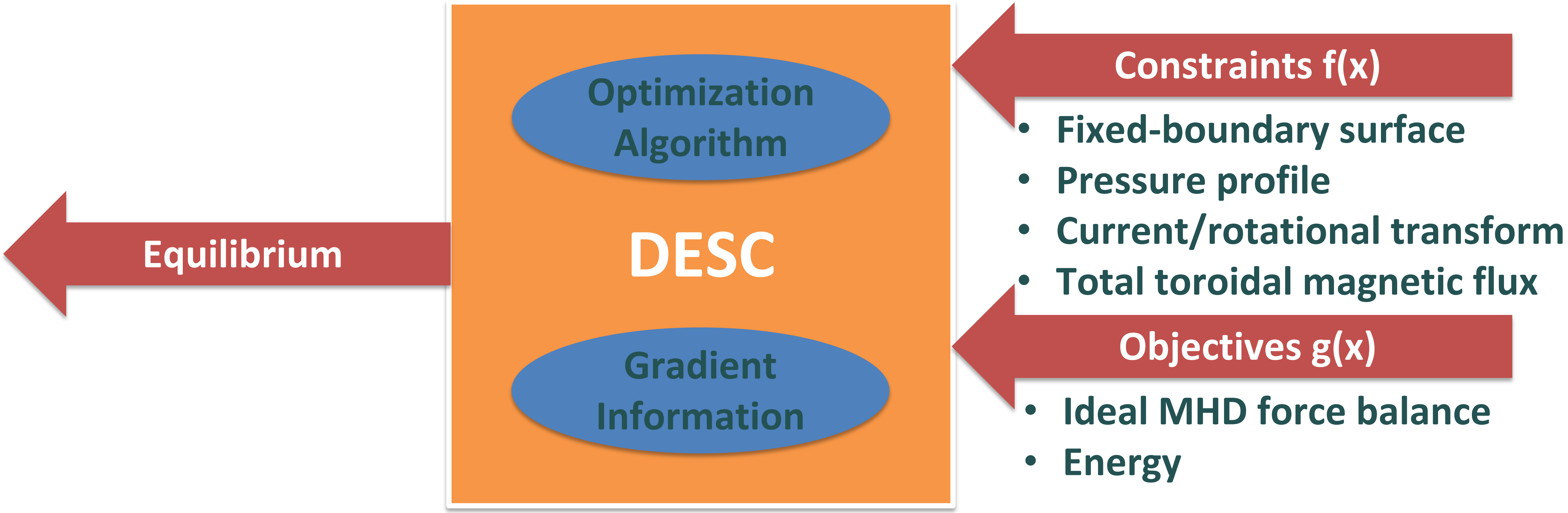
- **Stellarator equilibria are complicated**
- **Design space is much larger than tokamaks**

Fast Optimization





A flexible stellarator optimization suite



A flexible stellarator optimization suite

DESC

- Optimization Algorithm
- Gradient Information

← Optimized Stellarator

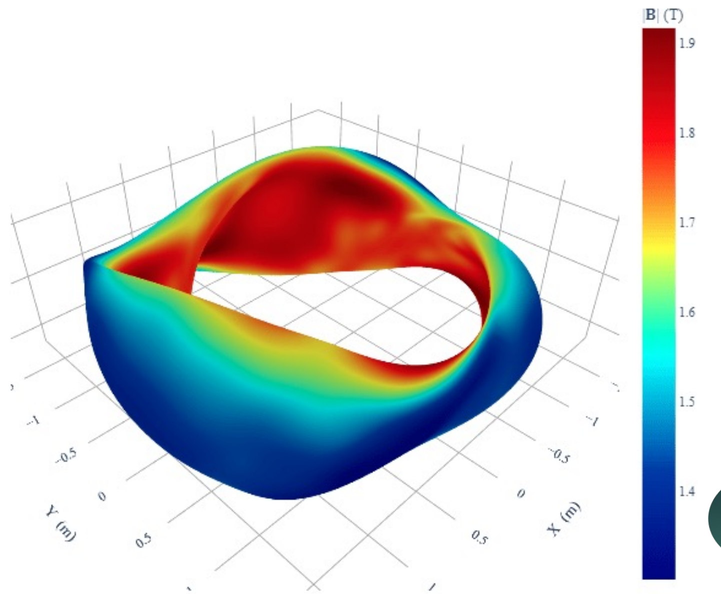
Constraints f(x) →
- Ideal MHD force balance
- Equilibrium profiles
- Some boundary modes

Objectives g(x) →
- Quasi-symmetry
- Mercier stability
- Aspect ratio
- etc.

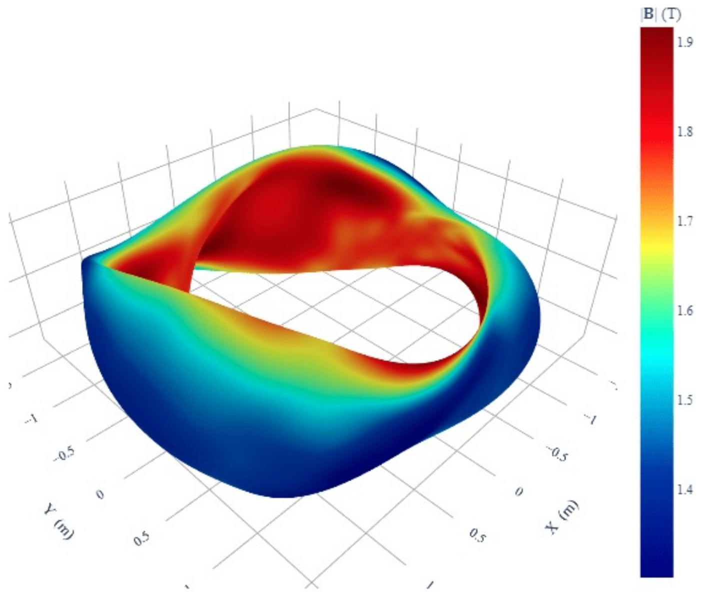
DESC Developed with the following design principles:

1. **Simple user interface**
   - Open-source Python code
   - Well documented
   - High test coverage
   - Easy to install
2. **Local error quantification**
   - Pseudo-spectral (collocation) methods
3. **Properly resolve the magnetic axis**
   - Global basis functions
   - Zernike polynomials
4. **Exact derivatives of all objectives**
   - Automatic differentiation (JAX)
5. **Hardware agnostic**
   - Run on CPUs, GPUs, and TPUs
6. **Extendable to new applications**
   - Modular & flexible code structure

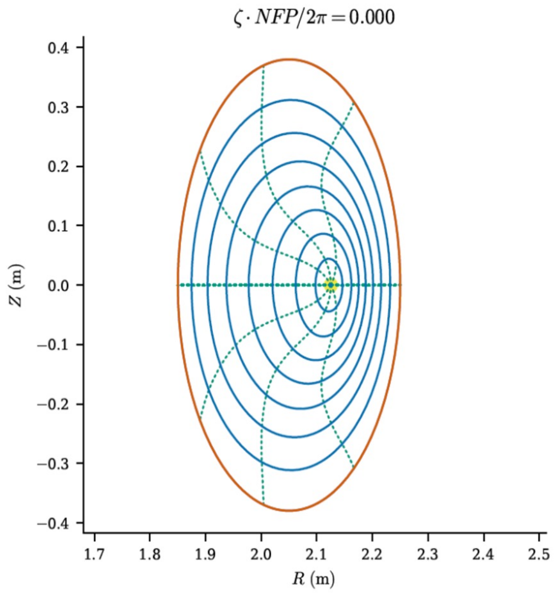
https://github.com/PlasmaControl/DESC

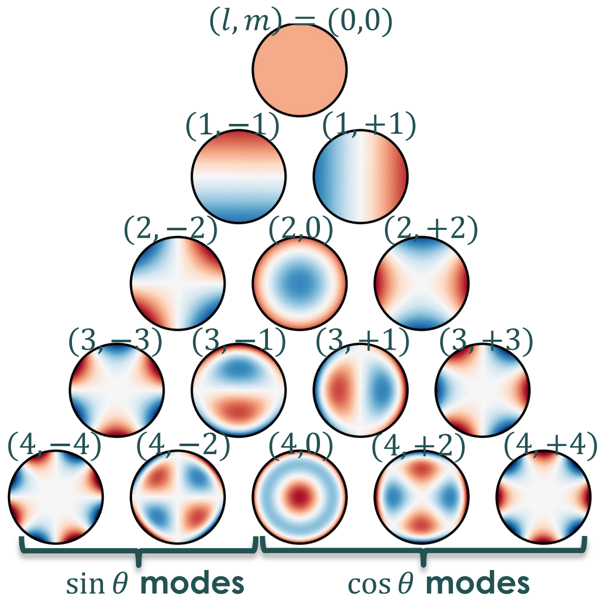


Zernike spectral basis better than concentric circle basis



Poincare section





spectral coefficients

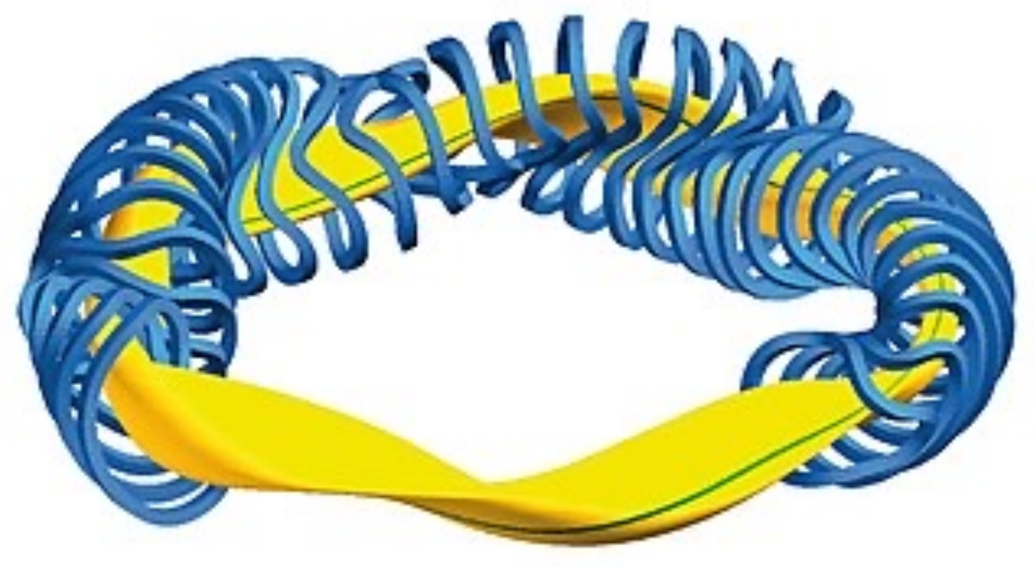
Zernike polynomials

$$X(\rho,\theta,\zeta) = \sum_{lmn} X_{lmn} \mathcal{Z}_l^m(\rho,\theta)\mathcal{F}^n(\zeta)$$

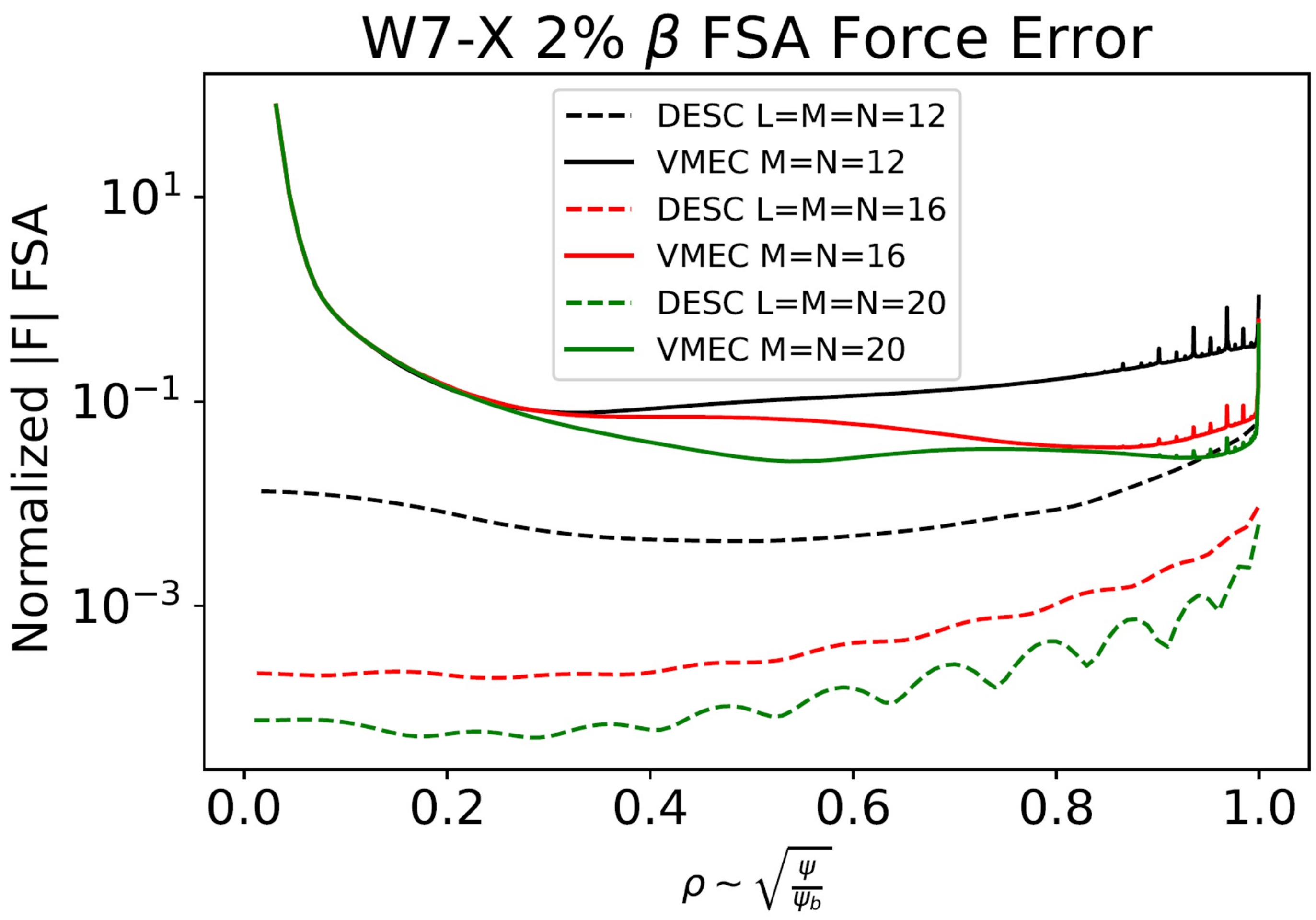
Fourier series

- **Satisfies analyticity conditions at the magnetic axis**
- **Exponential convergence (if solution exists and is smooth)**

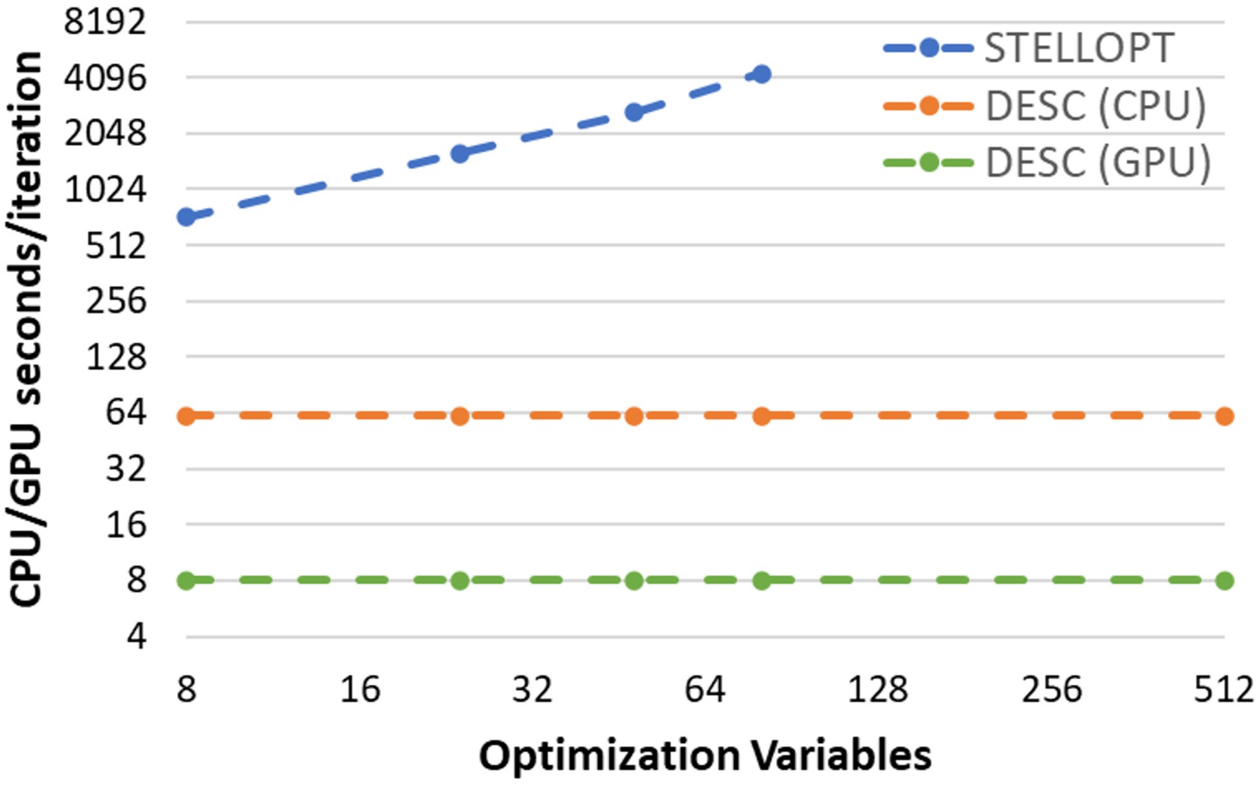
DESC spectral methods yield more accurate equilibrium solutions



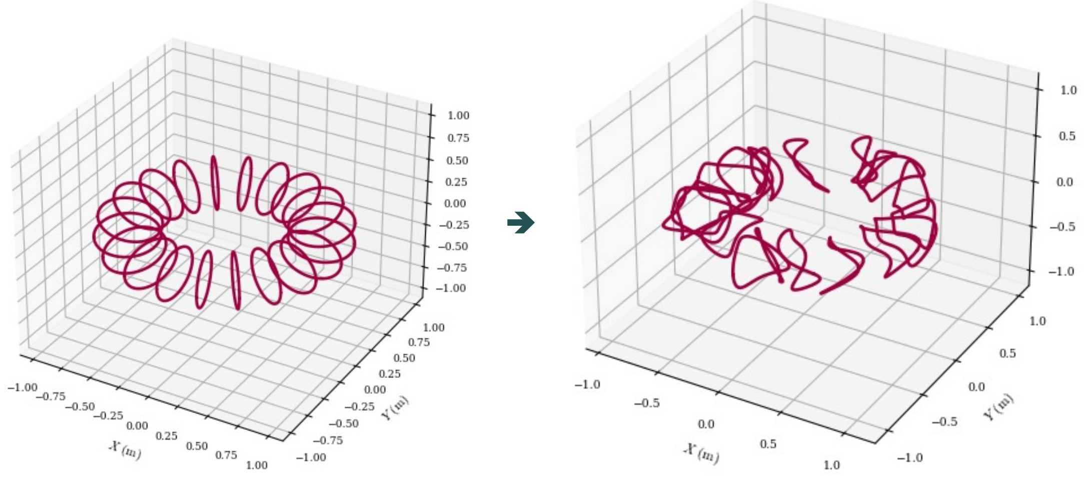
W7-X



DESC Allow Much Faster Stellarator Optimization



Example Coil Optimization



W7-X $\beta = 2\%; L = 24, M = N = 12$

Hardware	Run Time
Intel Cascade Lake CPU	48 min
NVIDIA A100 GPU	20 min

https://github.com/PlasmaControl/DESC

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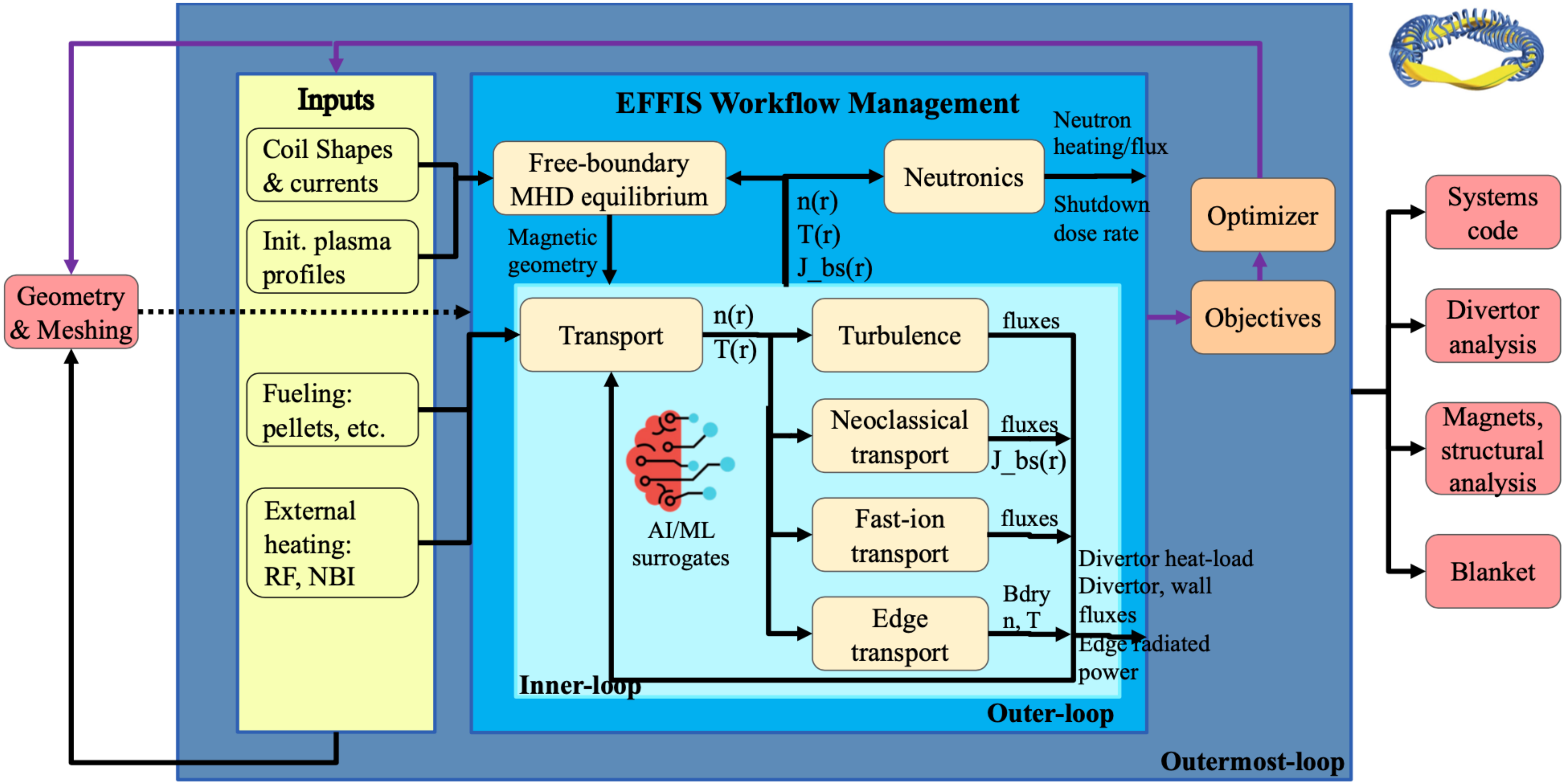
Next up: End-To-End Stellarator Optimization with AutoDiff



Scidac work M. Churchill PPPL

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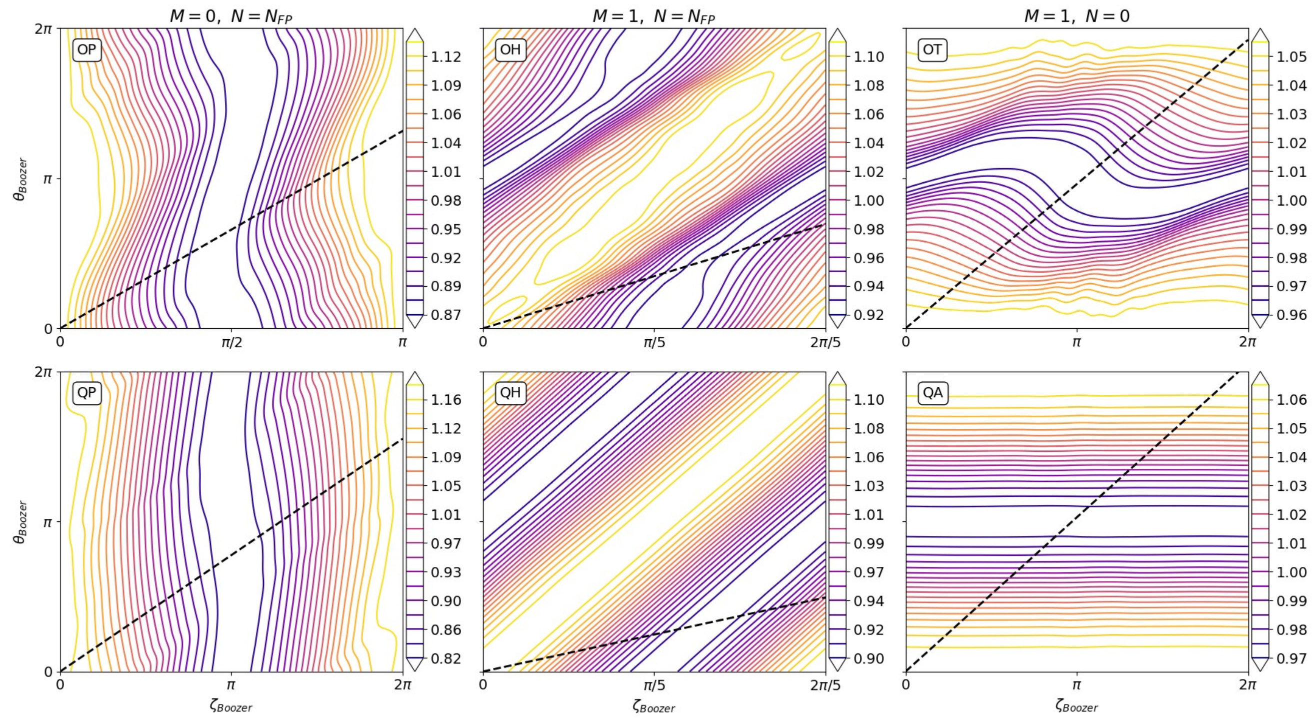
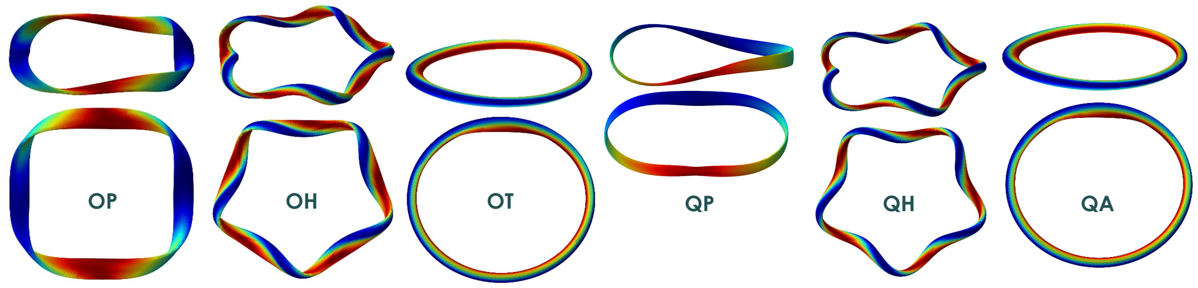
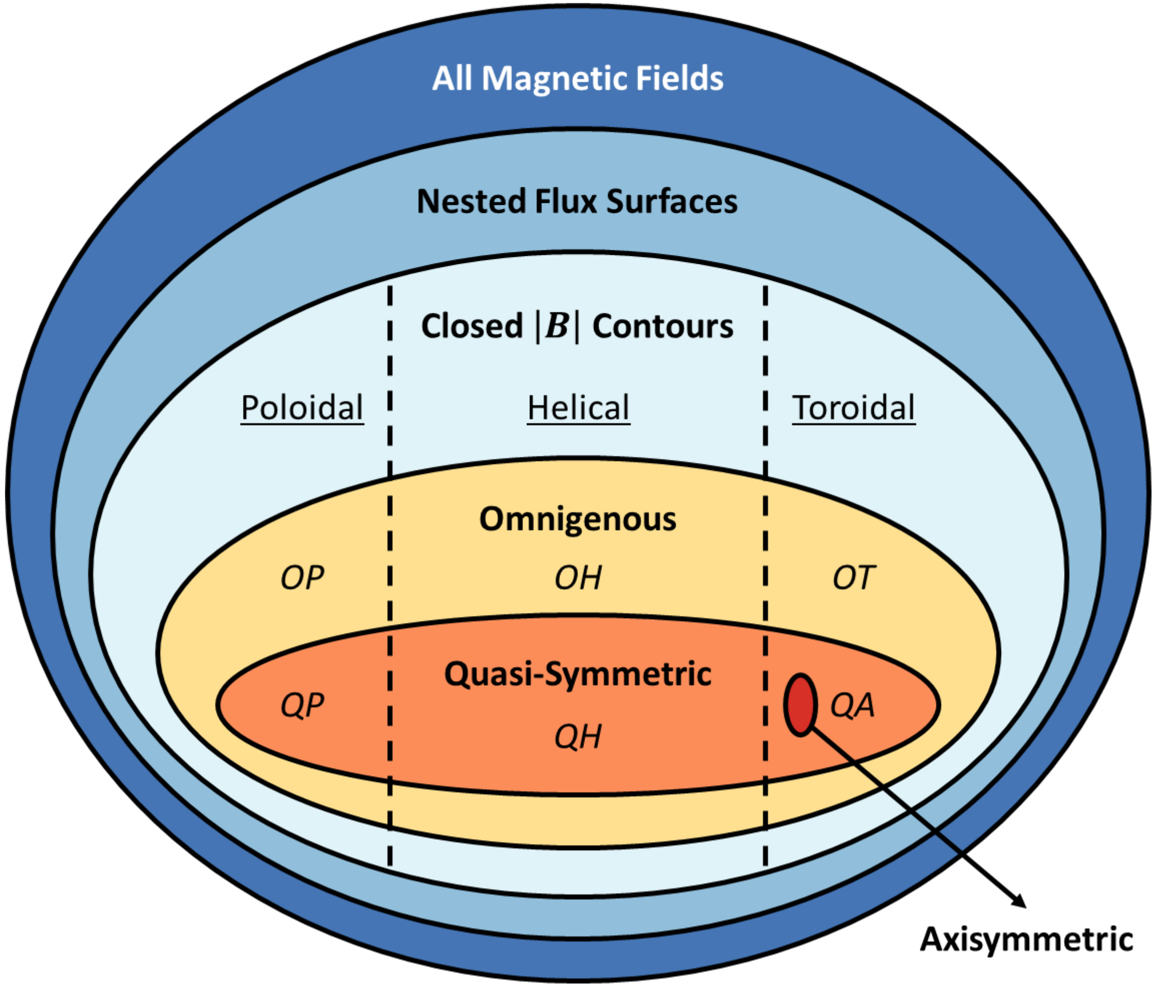
Main Result: General Omnigenity Easily Obtained with DESC







Current optimization methods: Sum of Squares

Combine equality + inequality constraints

$$\min_x f(x) + w_1[g(x)]^2$$

Choose small weight for inequality constraints to enforce "approximately"

Choose large weight for equality constraints to penalize a lot

Limitations:

- **Hard to guess a-priori what weights should be**
- **Even small weights for "inequality" constraints can overly penalize things we don't care about**

Better methods: Augmented Lagrangian

- Combination of traditional Lagrangian + quadratic penalty

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda^T \mathbf{g}(x) + \mu g^2(x)$$

- Doesn't introduce any non-smooth terms
- "Exact" method - doesn't need μ → infinity
- Solve sequence of subproblems for increasing μ, λ
- Provides estimate of true Lagrange multipliers - useful information about trade-offs

- Open source packages available (LANCELOT, NLopt, etc). Also python/JAX version implemented in DESC

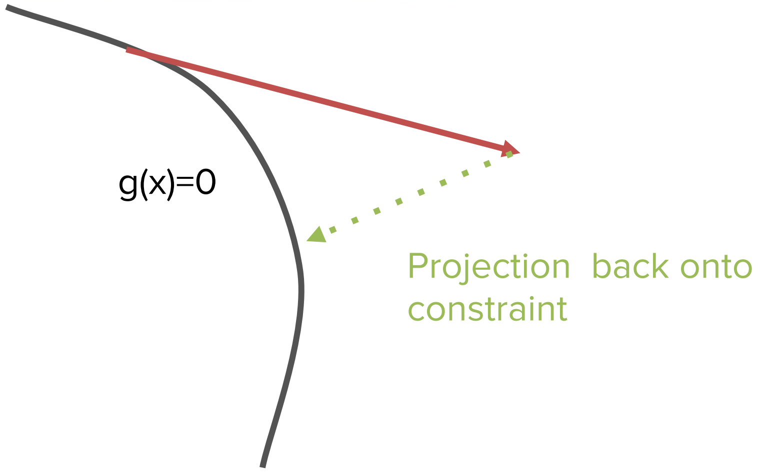
DESC Allow Combined Constraints + Optimization

```
while optimization stopping criteria are not met:

    perturb equilibrium solution to improve objective

    while equilibrium stopping criteria are not met:

        solve equilibrium force balance
```



Find x_{k+1} = min $L_A(x,\lambda_k,\mu_k)$ → **Calculate constraint violation $c(x_k)$ and check tolerances** → **Depending on $c(x_k)$, update λ_k or μ_k** → (back to Find x_{k+1})

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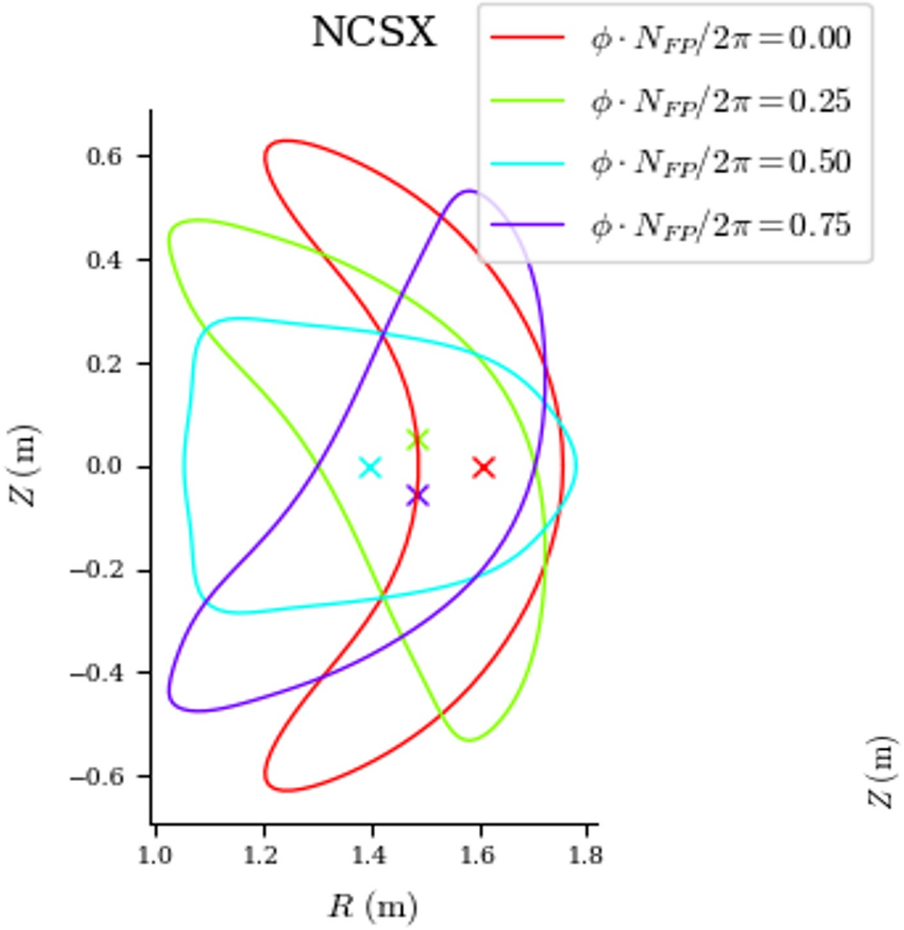
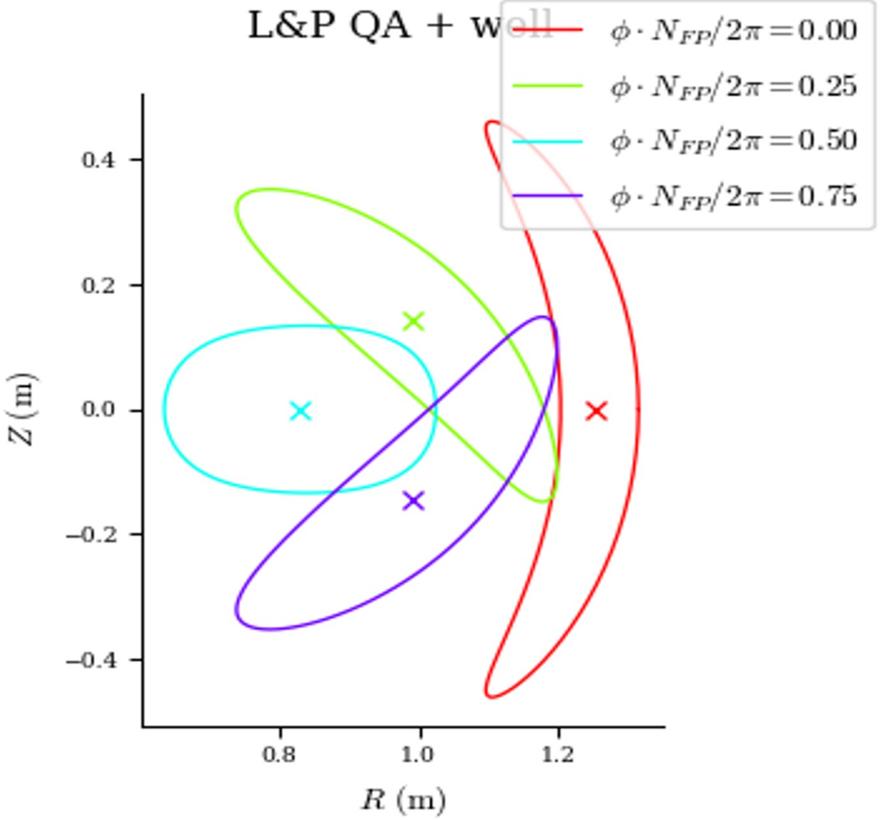
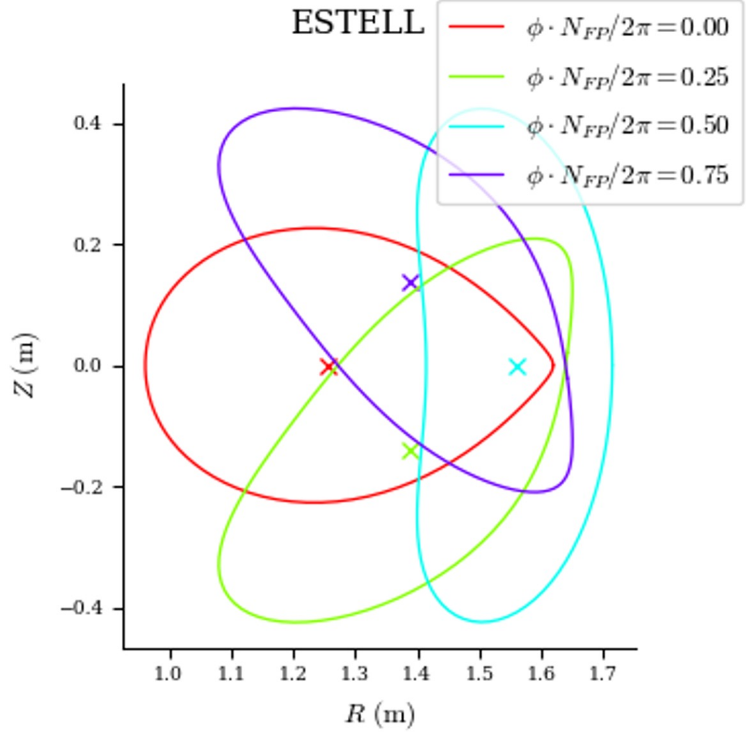
Application of Multi-Objective Optimization in DESC



NCSX + L&P precise QA: Magnetic well, but large concave regions



ESTELL: simpler geometry, but magnetic hill



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Application of Multi-Objective Optimization in DESC

Minimize f_{QS} ("two term" metric) subject to

Force balance: $$\mathbf{J} \times \mathbf{B} - \nabla p = 0$$

Iota: $$0.41 \leq \iota \leq 0.43$$

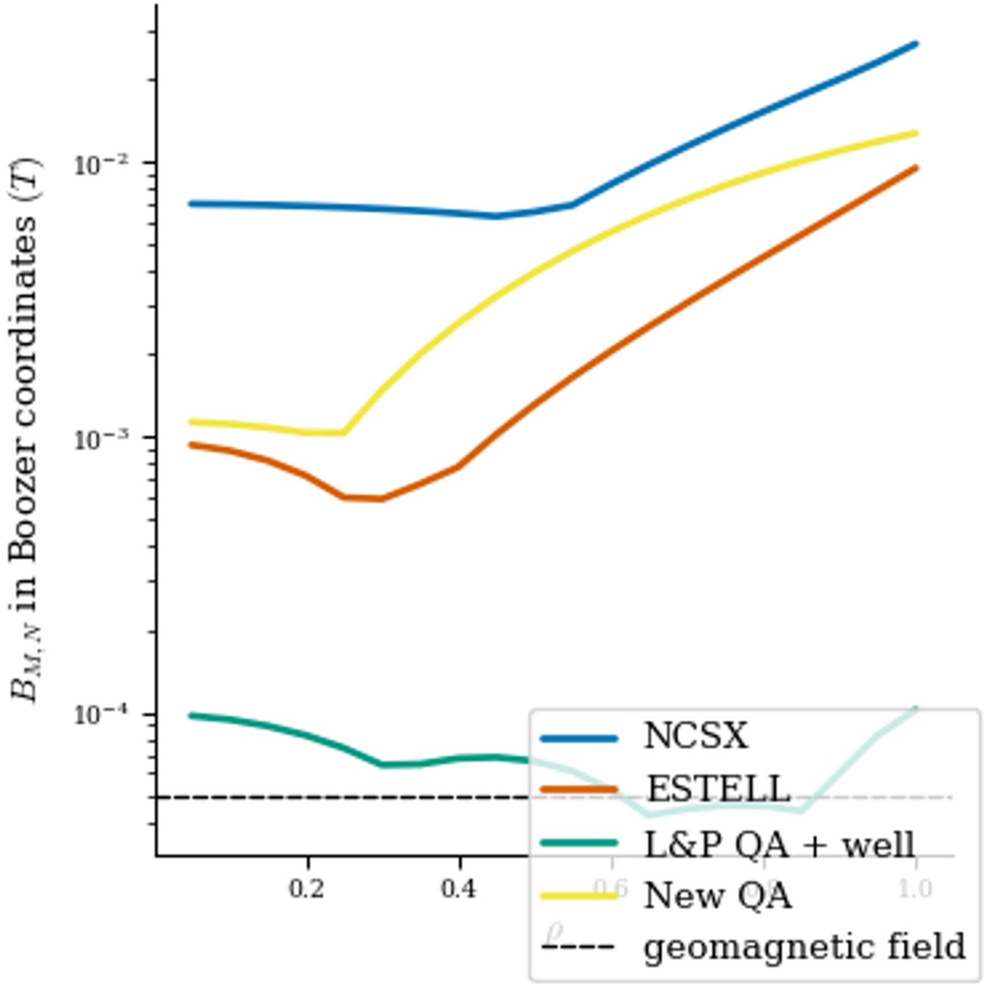
Magnetic Well: $$\frac{\partial_\rho V (2\mu_0 \partial_\rho p + \partial_\rho \langle B^2 \rangle)}{V \langle B^2 \rangle} > 0$$

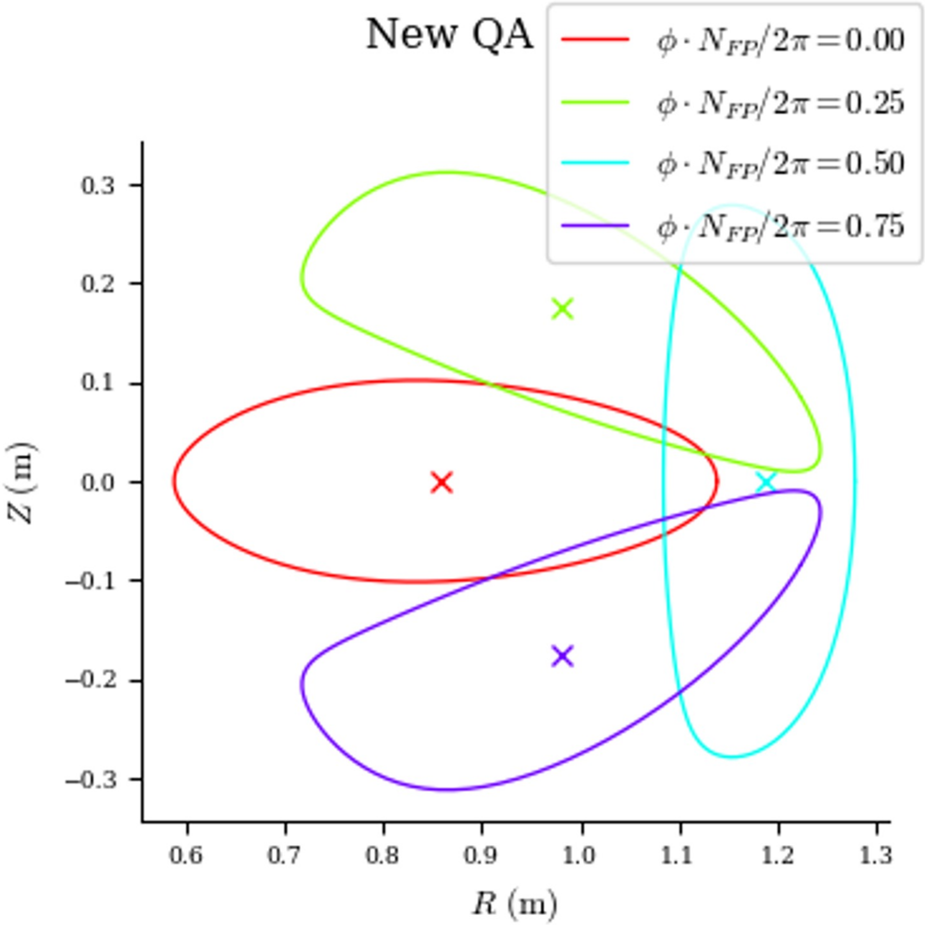
Mean Curvature: $$H = (\kappa_1 + \kappa_2)/2 < 0$$

κ_1, κ_2 eigenvalues of second fundamental form

Solve with least squares augmented Lagrangian method in DESC

QS better than NCSX + stability without concavity





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Free Boundary DESC

Free boundary constraints:

Flux surface:

$$\mathbf{B} \cdot \mathbf{n} = 0$$

Pressure balance:

$$B_{\rm out}^2 - B_{\rm in}^2 - 2\mu_0 p = 0$$

Tangential jump:

$$\mathbf{n} \times [\mathbf{B}_{\rm out} - \mathbf{B}_{\rm in}] - \mu_0 \mathbf{K} = 0$$

B_{in} = B from fixed boundary DESC

$B_{out} = B_{coil} + B_{VC} + B_K$

- **Parameterize unknown sheet current K, along with R, Z**
- **Use high order singular integration scheme (Malhotra et al 2019) to compute virtual casing + sheet current field $B_{VC} + B_K$**
- **Minimize all 3 equations simultaneously**

Free Boundary DESC

Free boundary constraints:

Pressure balance:

$$B_{\mathrm{out}}^2 - B_{\mathrm{in}}^2 - 2\mu_0 p = 0$$

B_{in} = B from fixed boundary DESC

Virtual Casing: -nxB_{out} = K_{out}

Biot-Savart(K_{out})=B_{coil}

- **Alternatively:**
- **Use high order singular integration scheme (Malhotra et al 2019) to compute virtual casing for B_{out}**
- **Just check for pressure balance.**

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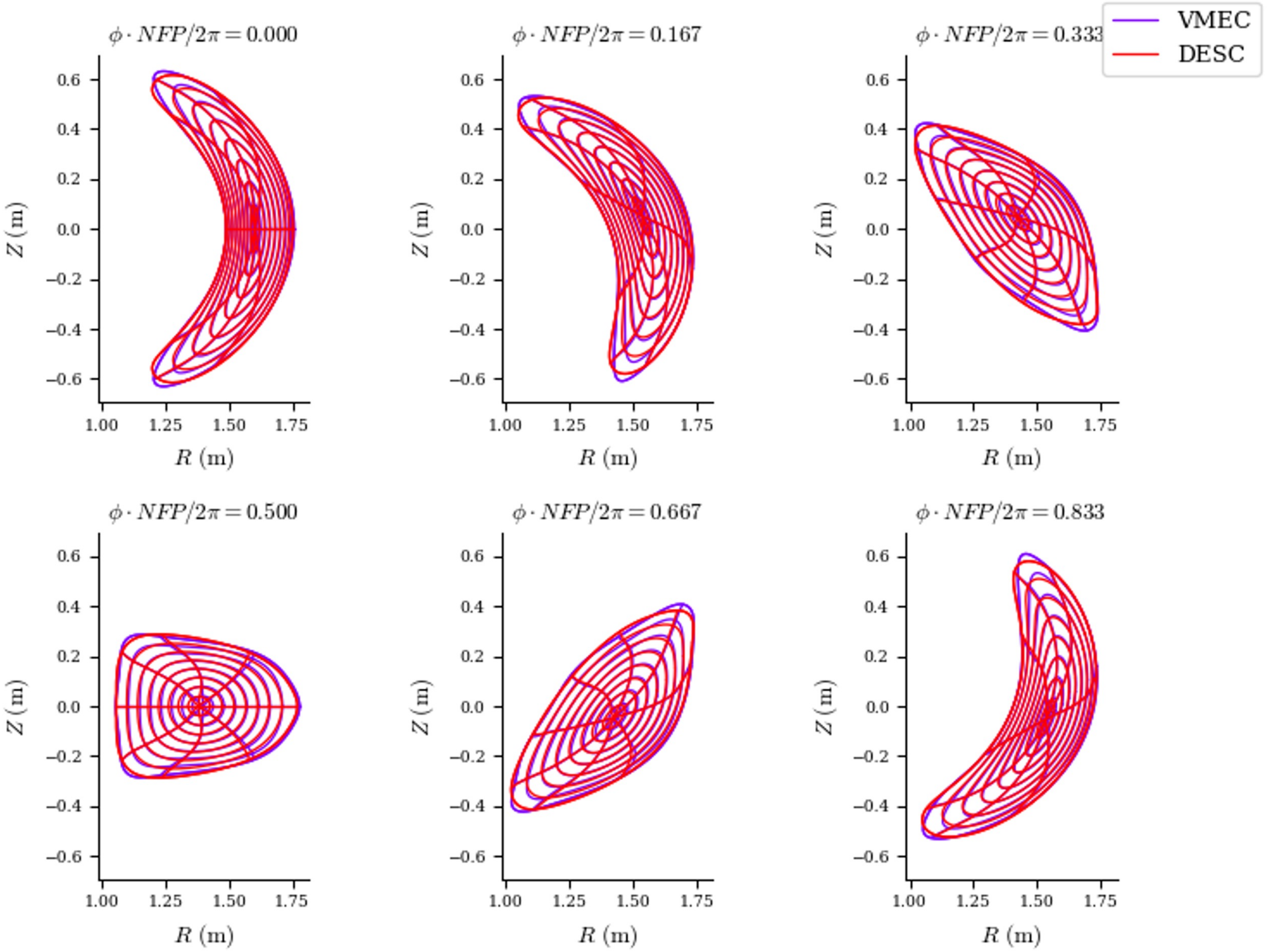
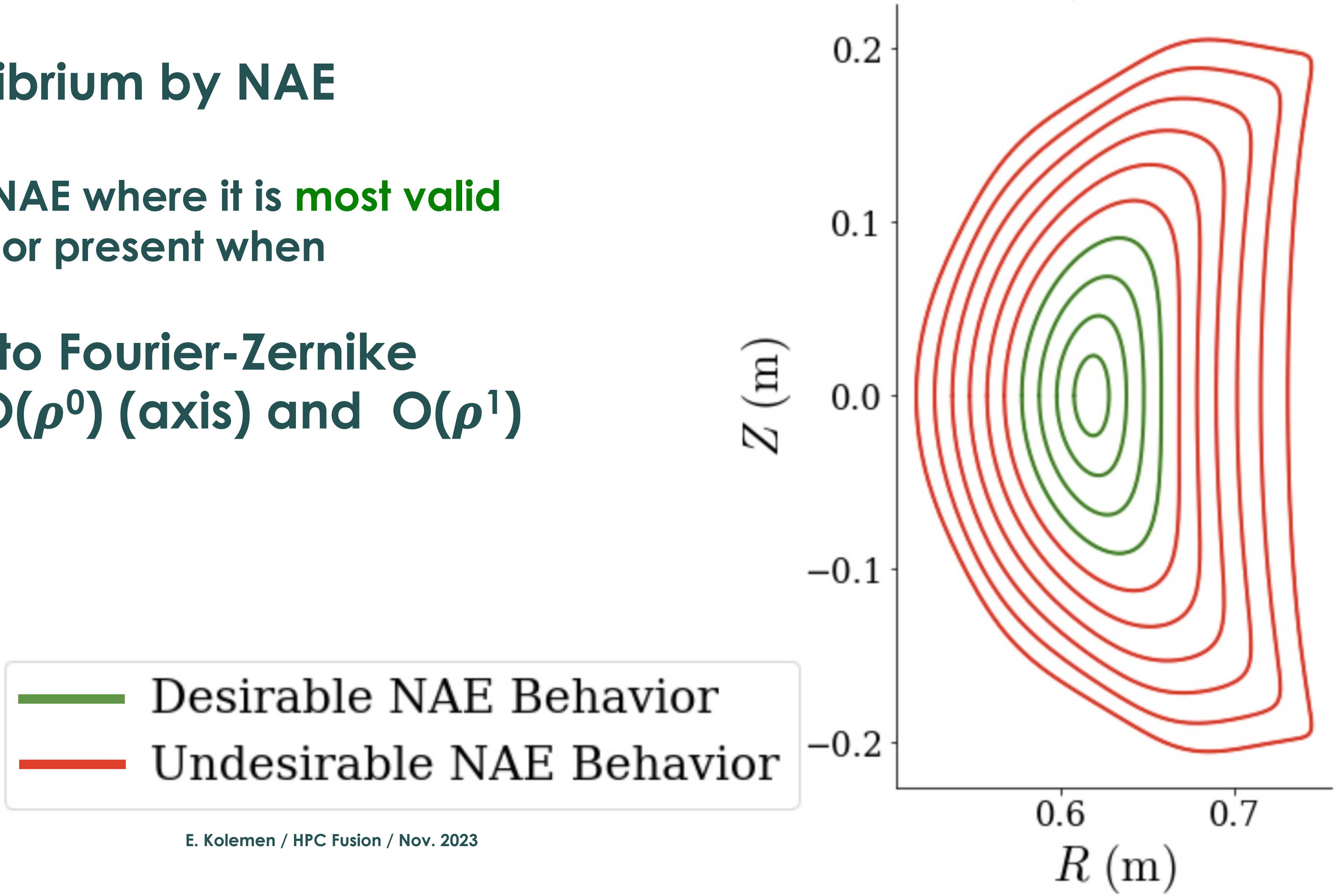
Free Surface DESC vs VMEC



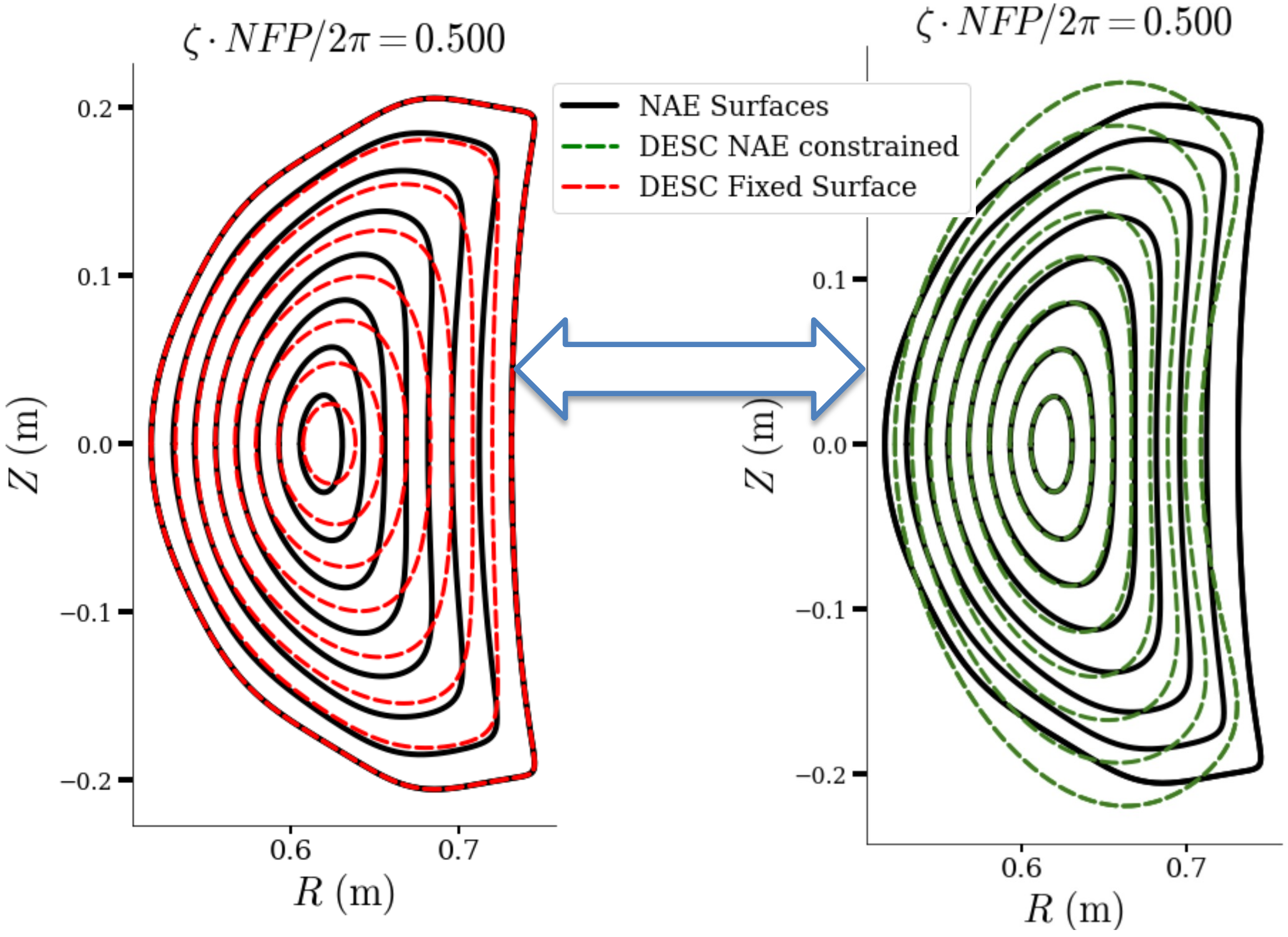
Near-Axis Expansion (NAE) Constraints in DESC

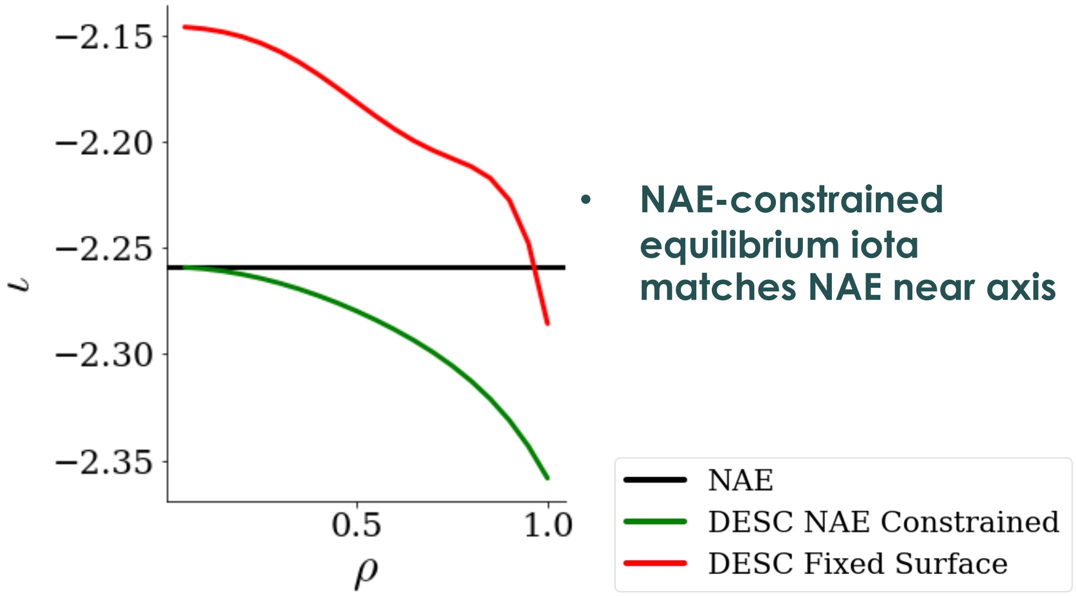
- Constrain global equilibrium by NAE behavior as $\rho \rightarrow 0$
  - Use information from NAE where it is most valid
  - Avoid singular behavior present when evaluating at large r
- Map NAE coefficients to Fourier-Zernike modes of DESC to fix $O(\rho^0)$ (axis) and $O(\rho^1)$ behavior

NAE equilibrium evaluated at r =0.1

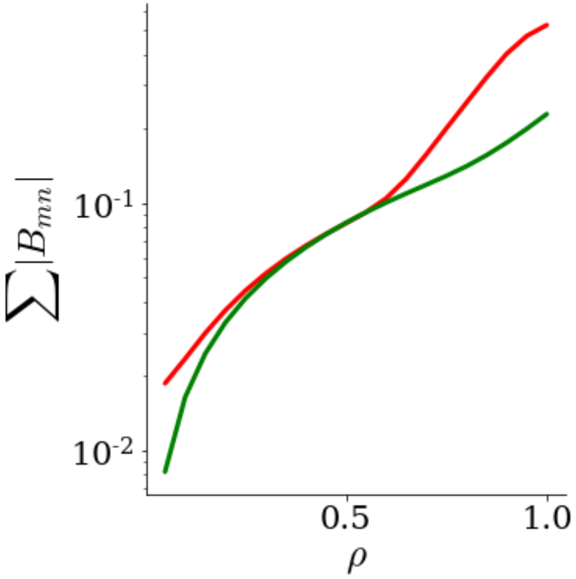


Near-Axis-Expansion Constrained Equilibria in DESC





- NAE-constrained equilibrium iota matches NAE near axis



- NAE-constrained equilibrium maintains better QS near axis

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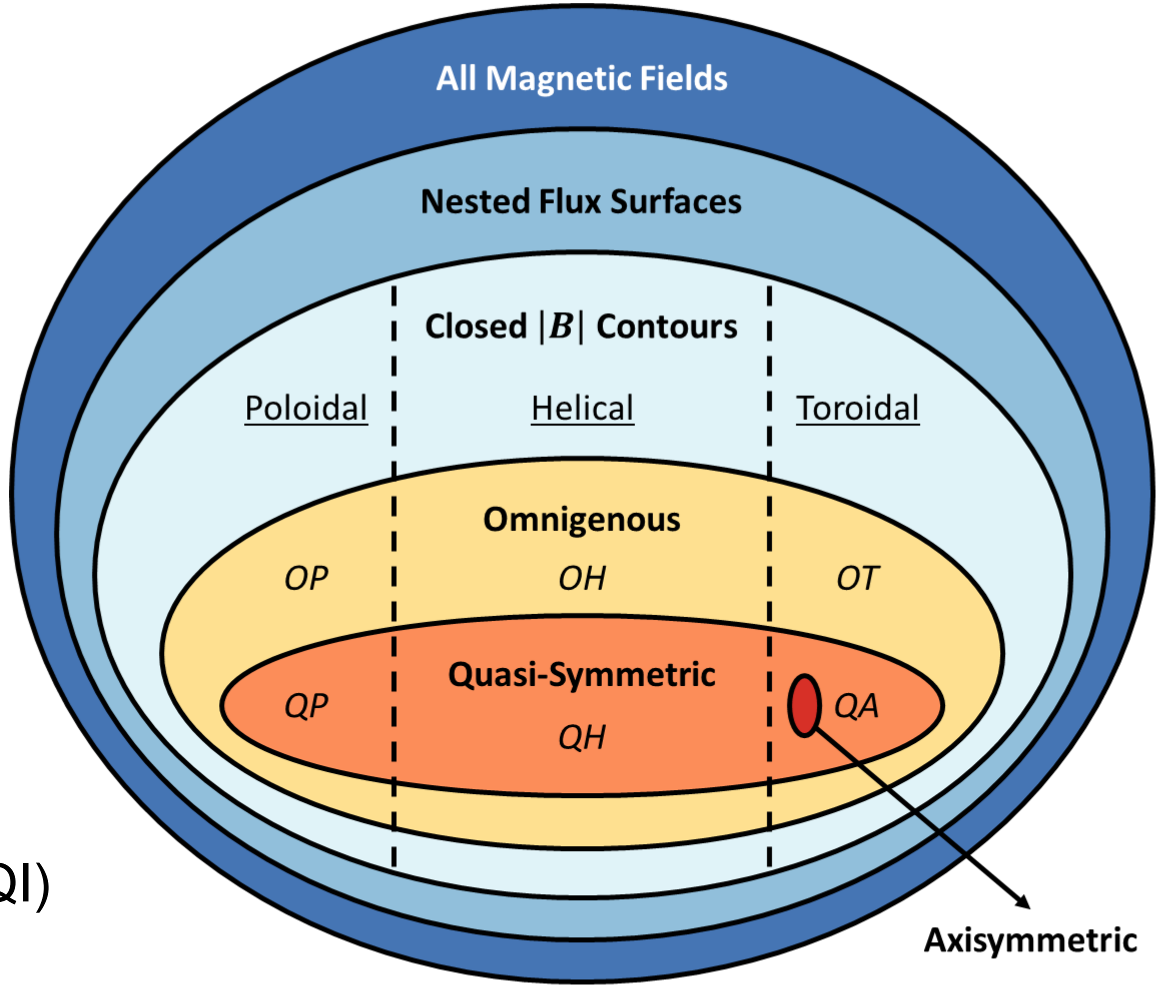
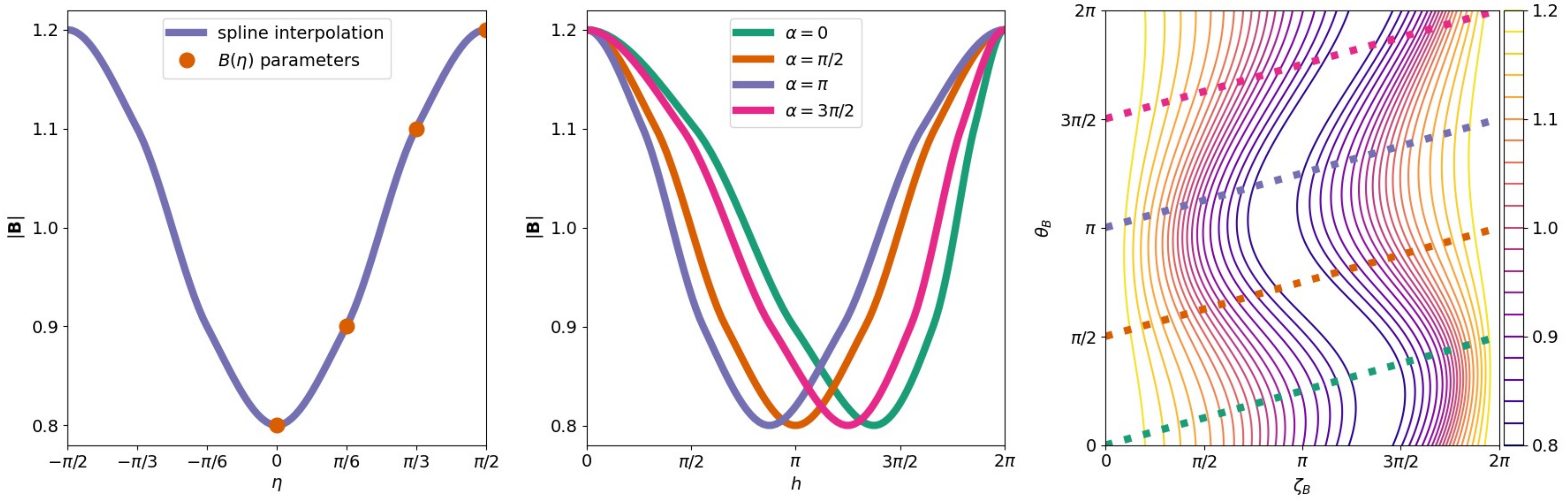
General Omnigenity: confined particles without Quasi-Symmetry

- **Omnigenity:** the class of magnetic fields in which the bounce-averaged radial drifts of trapped particles vanish
  - $B = |\boldsymbol{B}|$ contours are closed curves
  - B_{max} contour is straight in Boozer coords
  - "Bounce distances" between consecutive B points are independent of the field line α
- Previous omnigenous equilibria were limited to:
  - Quasi-Axisymmetry (QA)
  - Quasi-Helical symmetry (QH)
  - Omnigenity with Poloidal contours (OP aka QI)
- General omnigenity = larger design space!



General omnigenity optimization is implemented in DESC



1. **Define a target magnetic well "shape" (in computational coordinate η)**
2. **Define a target "shift" on each field line (preserves constant bounce distances)**
3. **Optimize to minimize the errors: $B_{equilibrium} - B_{target}$**

Omnigenity is parametrized by a coordinate mapping

- Map between computational and Boozer coordinates:

helicity = (M, N)

$$h(\rho,\alpha,\eta) = h(\theta_B,\zeta_B)$$

$$2\eta + \pi + \sum_{l=0}^{L_\rho}\sum_{m=0}^{M_\eta}\sum_{n=-N_\alpha}^{N_\alpha} x_{lmn} T_l(2\rho-1)\mathcal{F}_m(\eta)\mathcal{F}_{nN_{FP}}(\alpha) = \begin{cases} N\zeta_B & \text{for } M = 0 \\ -\theta_B + \dfrac{N}{M}\zeta_B & \text{for } M \neq 0 \end{cases}$$

ρ = flux surface label
α = field line label
η = coord along field line

free parameters (x_{lmn}) — Chebyshev polynomials (T_l) — Fourier series ($\mathcal{F}_m$, $\mathcal{F}_{nN_{FP}}$)

- Constant bounce distances are preserved:

$$\delta \propto \Delta h = h(\rho,+\eta,\alpha) - h(\rho,-\eta,\alpha)$$

$$= 4\eta + \sum_{l=0}^{L_\rho}\sum_{m=0}^{M_\eta}\sum_{n=-N_\alpha}^{N_\alpha} x_{lmn}\left[T_l(2\rho-1)\mathcal{F}_{nN_{FP}}(\alpha)[\mathcal{F}_m(+\eta) - \mathcal{F}_m(-\eta)]\right]$$

$$= 4\eta$$

$$\therefore \frac{\partial\delta}{\partial\alpha} = 0$$

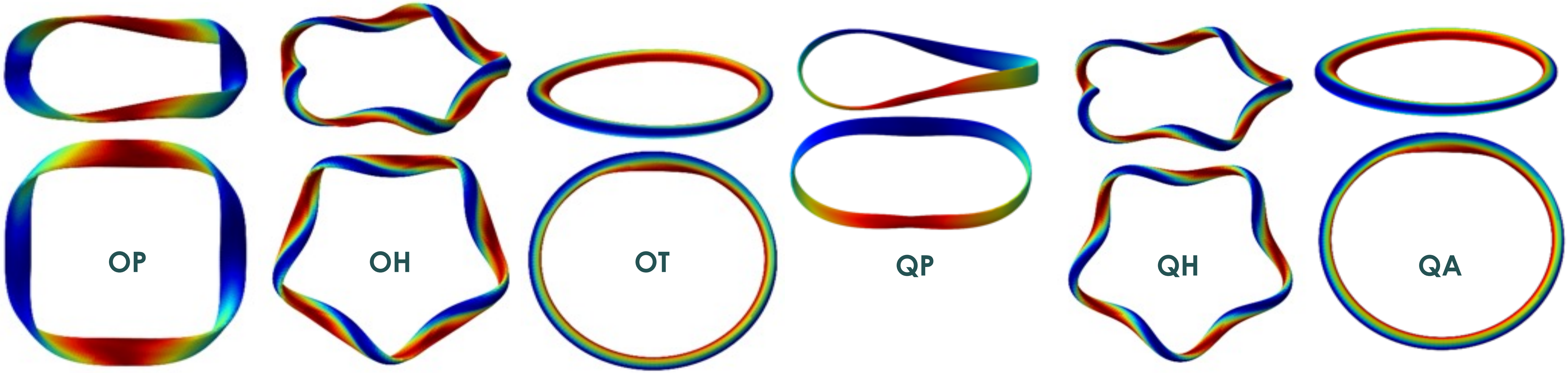
$= 0$ because $\sum_{m\geq 0}\mathcal{F}_m(\eta)$ is an even function of η

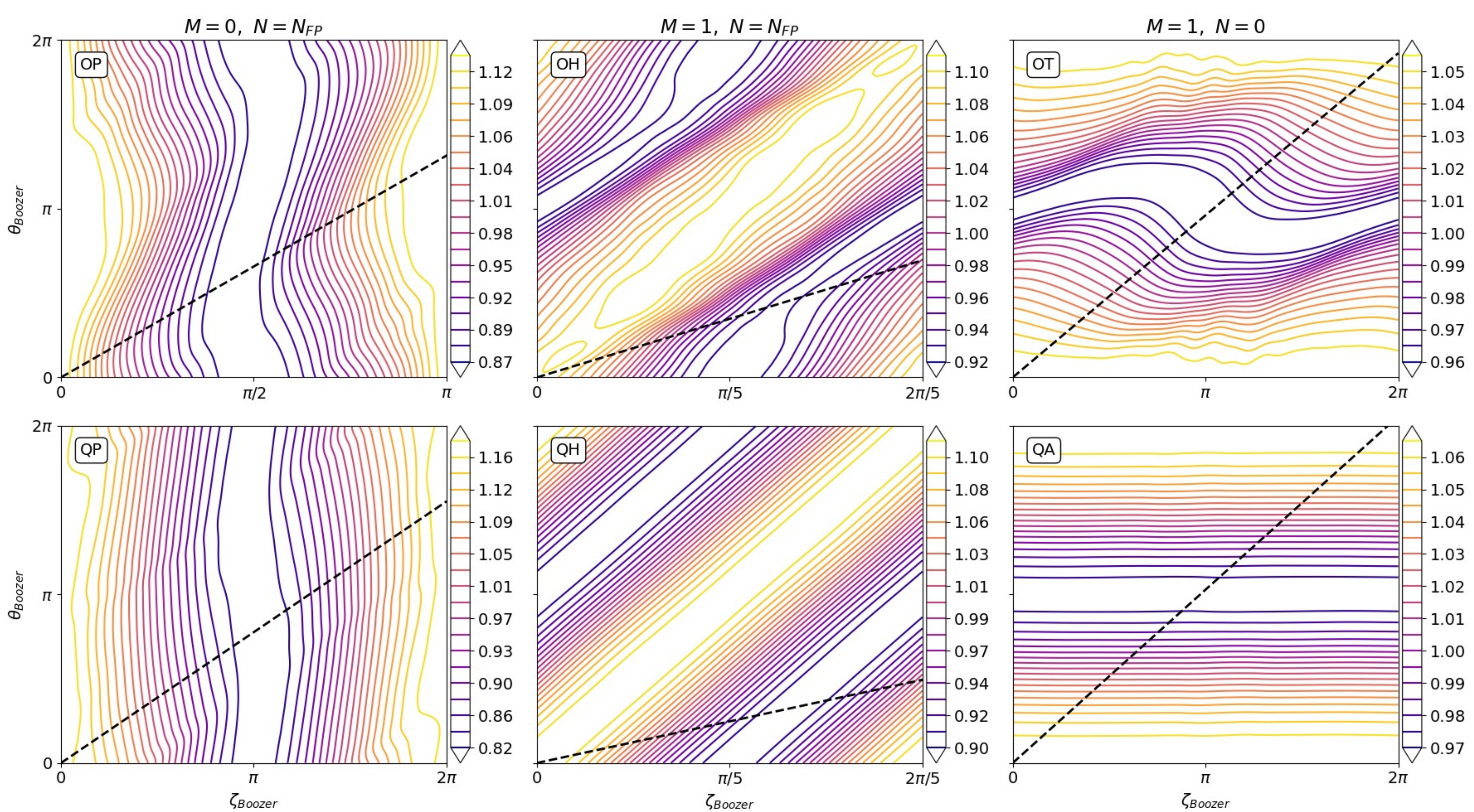
Examples of the different types of omnigenity

General Omnigenity with poloidally/helically/toroidally-closed B contours (OP, OH, OT)

Quasi-Symmetry is a subspace of omnigenity (QP, QH, QA)

- All solutions used near-axis expansions as an initial guess (from pyQIC and pyQSC)
- Optimized for omnigenity at five surfaces (no other objectives)
- Aspect ratios ≈ 20

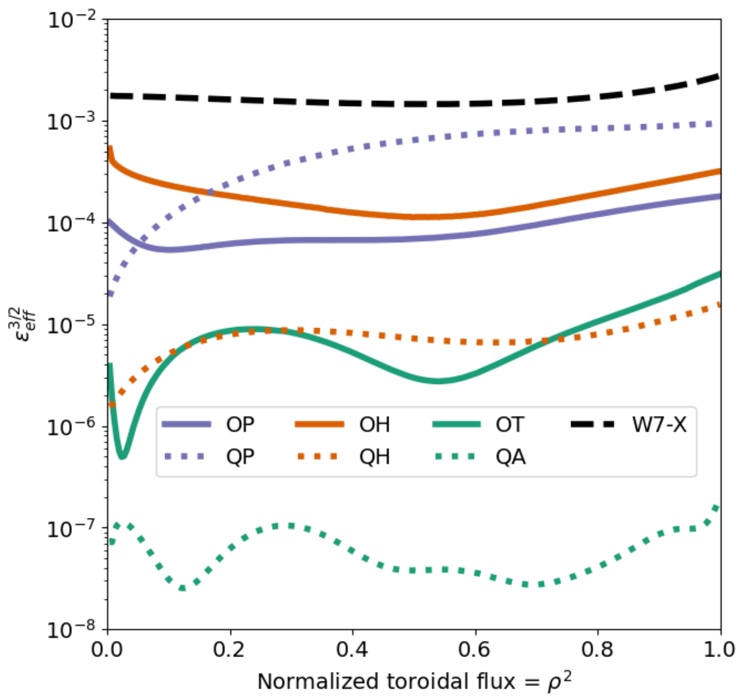


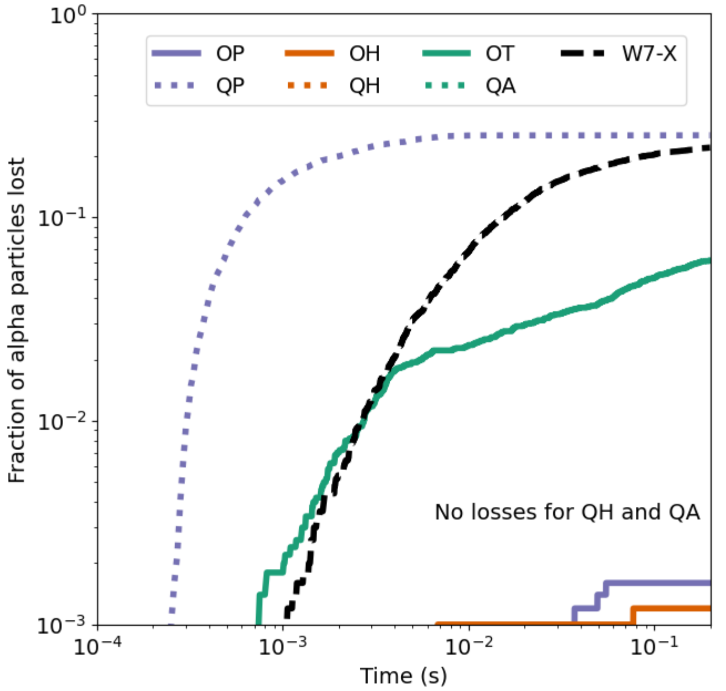


$M=0,\ N=N_{FP}$
$M=1,\ N=N_{FP}$
$M=1,\ N=0$
OP
OH
OT
QP
QH
QA
θ_{Boozer}
ζ_{Boozer}

Solutions show good particle confinement!

- **Top:** Neoclassical collisional transport magnitude
  - Computed by NEO
- **Bottom:** Collisionless losses of fusion-born alpha particles
  - Computed by SIMPLE
  - Particles initialized at $\rho = 0.5$
  - Configurations scaled to a and B of ARIES-CS
- Reference case is W7-X at $\beta = 4\%$
- Precise QP is difficult (impossible?) to achieve
- Higher alpha particle losses for OT case might be due to wide banana orbits?





GPU Allows Direct Optimization of Particles (Instead of proxies such as omnigenity)

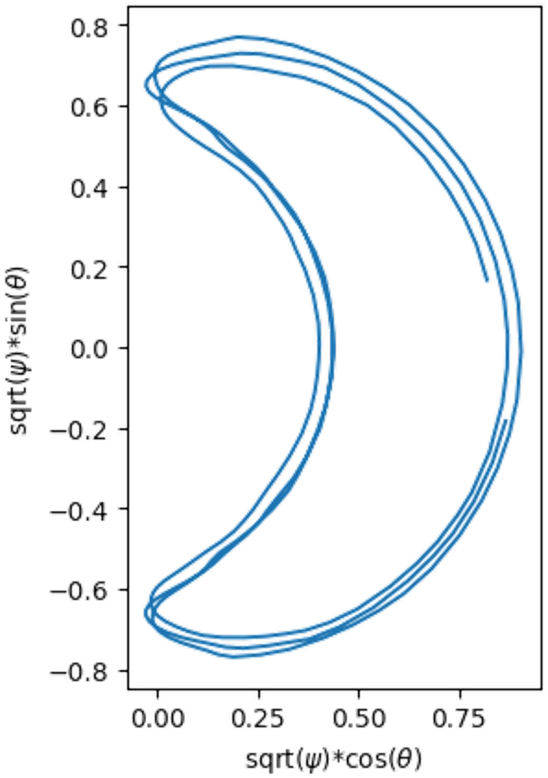
- **GPU has advantage in doing the same compute many times**
- ***Integrate lots of particle trajectories in DESC (guiding center equations of motion)***
- **JAX autodiff: Jacobian of trace particle trajectories wrt equilibrium**

$$\dot{\psi} = \frac{m}{qB^3}\left(\frac{v_\perp^2}{2} + v_\parallel^2\right) \boldsymbol{B} \times \nabla B \cdot \nabla \psi$$

$$\dot{\theta} = \frac{v_\parallel}{B}\boldsymbol{B} \cdot \nabla \theta \frac{m}{qB^3}\left(\frac{v_\perp^2}{2} + v_\parallel^2\right) \boldsymbol{B} \times \nabla B \cdot \nabla \theta$$

$$\dot{\zeta} = \frac{v_\parallel}{B}\boldsymbol{B} \cdot \nabla \zeta$$

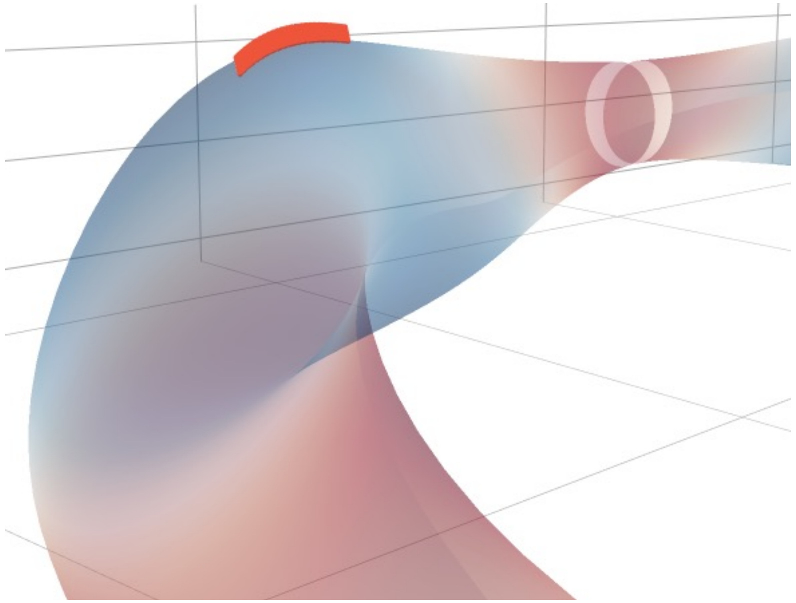
$$\dot{v_\parallel} = -\frac{v_\perp^2}{2B}\left(\frac{\boldsymbol{B}}{B} + \frac{m v_\perp^2}{2qB^3}\frac{1}{v_\parallel}\boldsymbol{B} \times \nabla B\right) \cdot \nabla B$$



Equilibrium optimization using a Particle Tracer

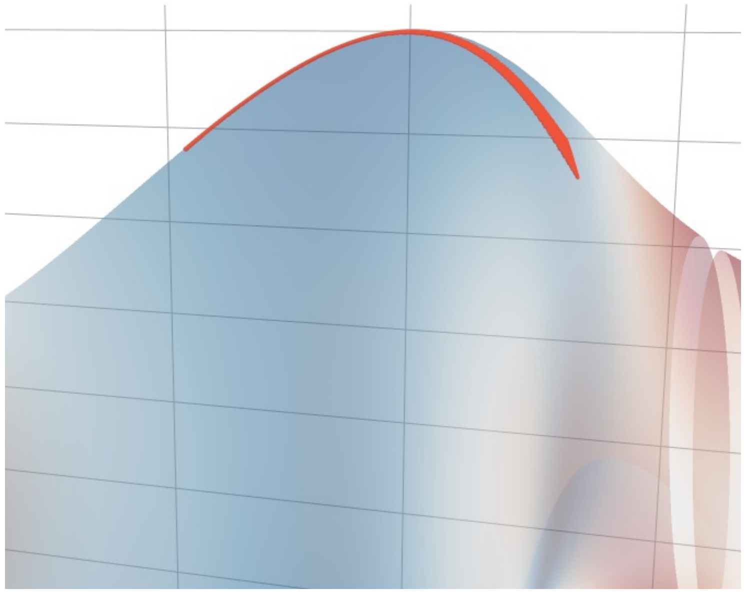
- Particle Tracer:
  - **Integrate Guiding Center EoM directly**
  - **Optimize the equilibrium from particle's trajectories using JAX autodiff**

Starting Equilibrium

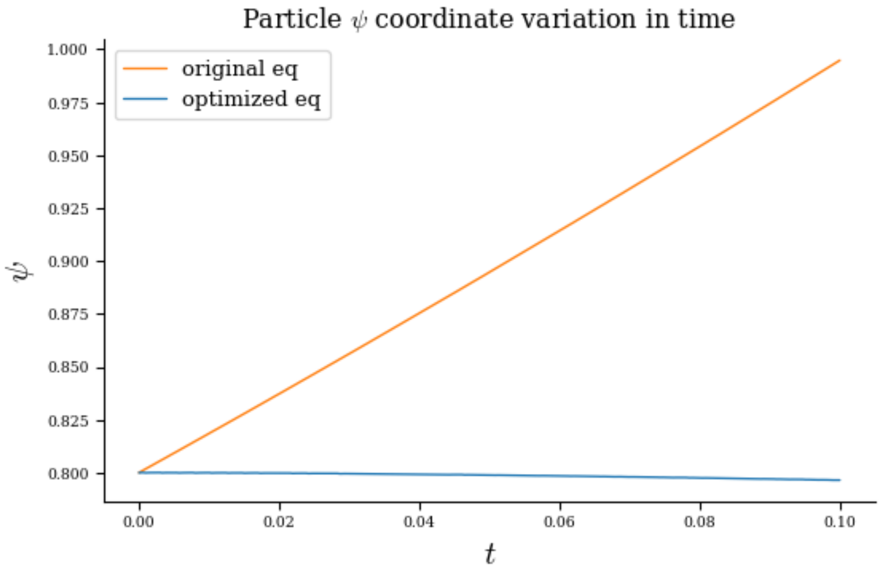


Particle drifting from flux surface ψ

Optimized Equilibrium



Particle confined in flux surface ψ



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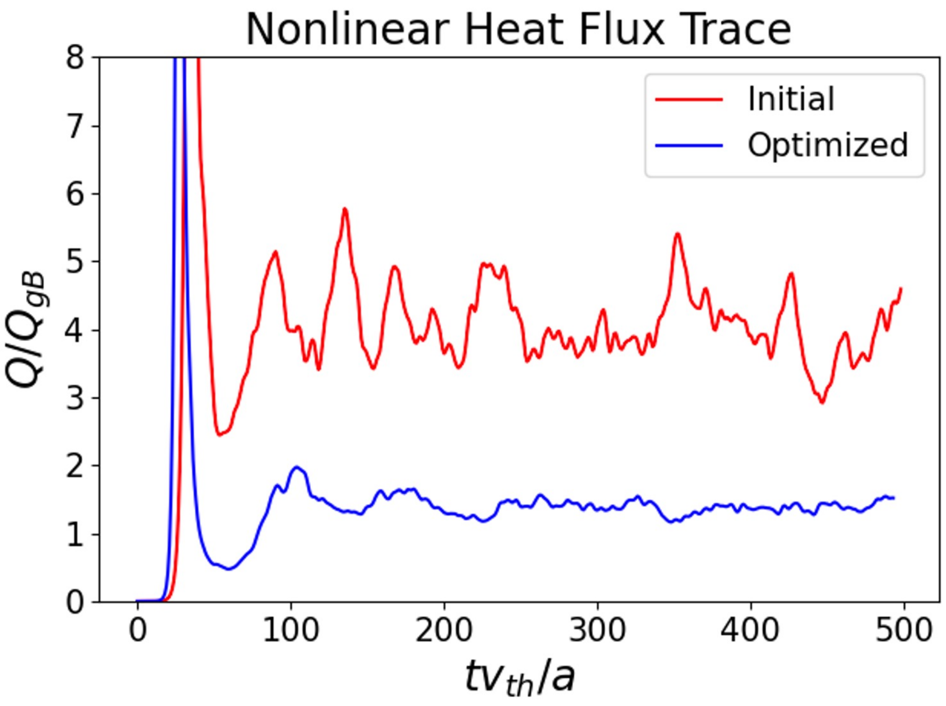
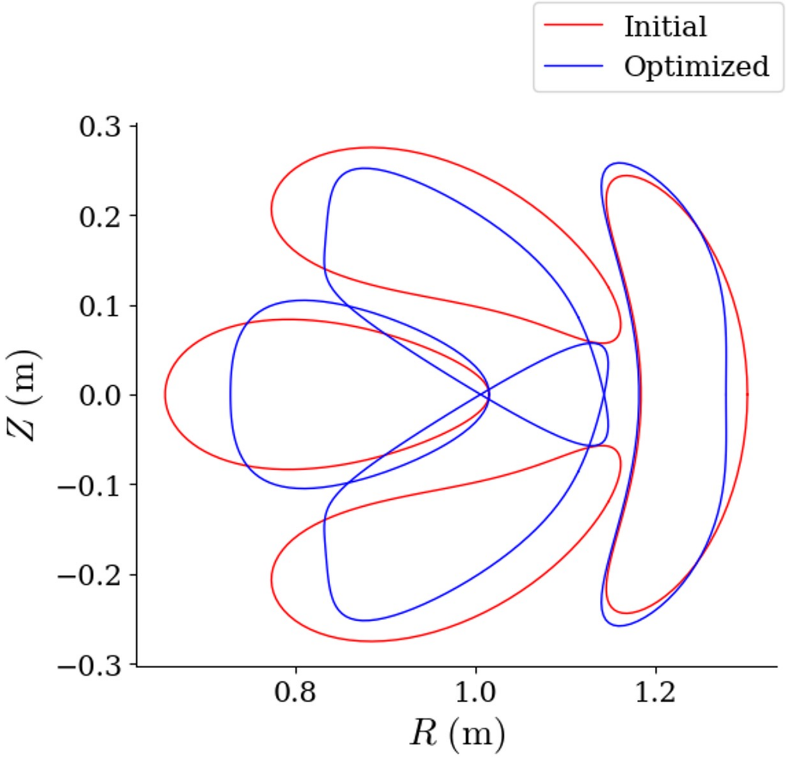
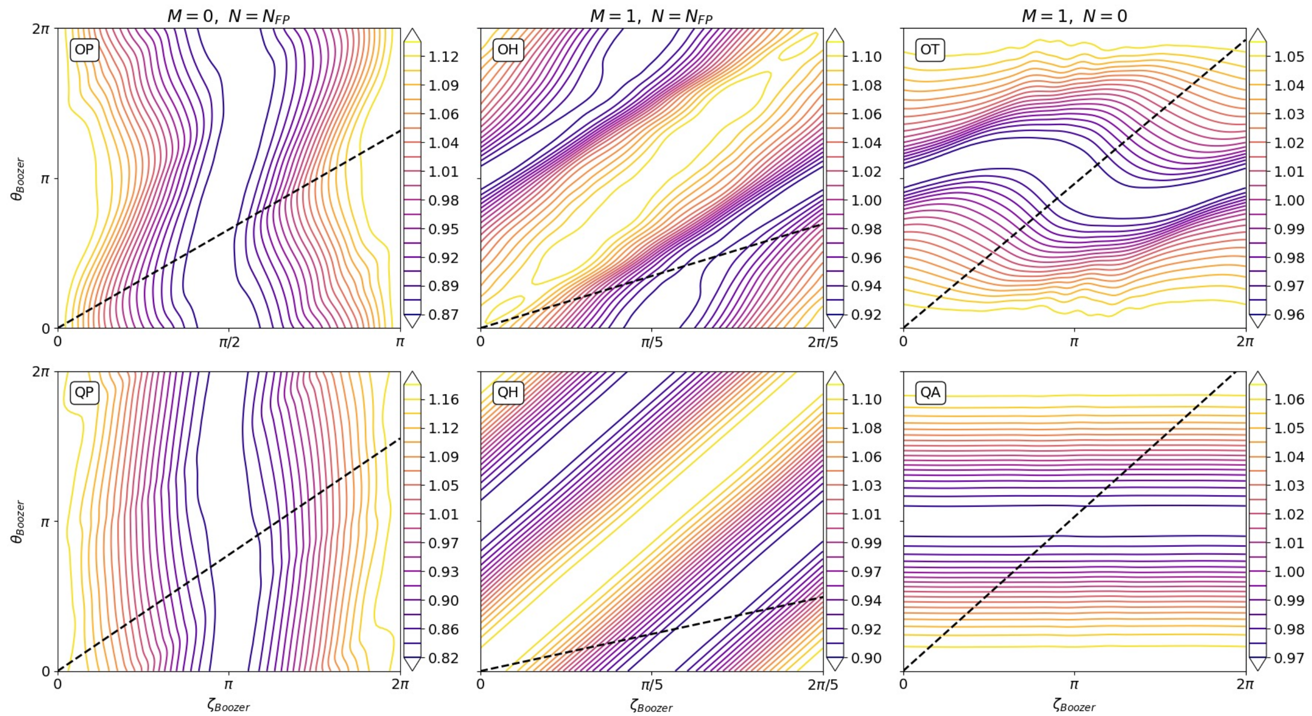
Turbulence + QS Optimization (Using GX+ DESC)

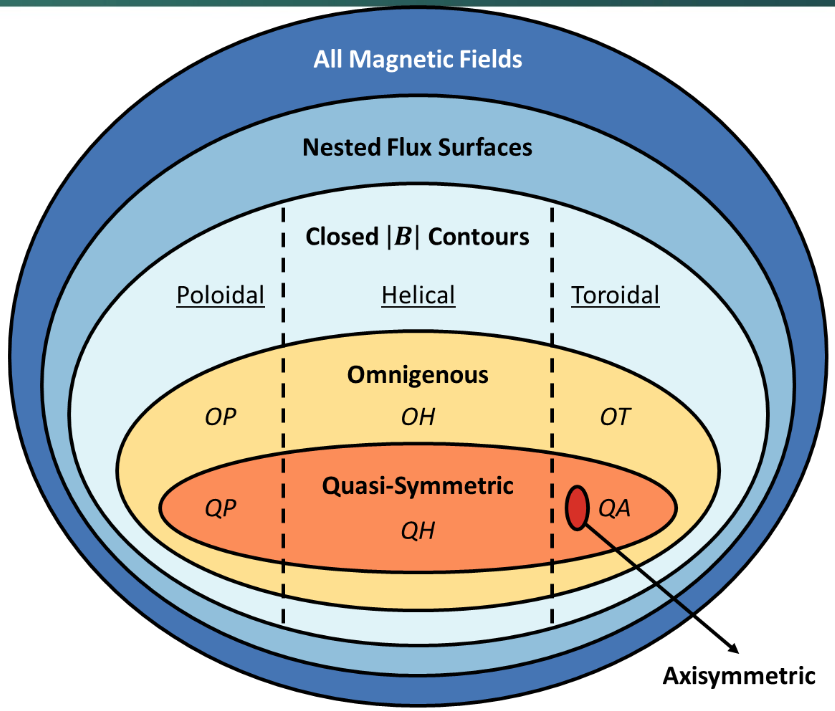
- GX + DESC coupling enables direct optimization of nonlinear heat fluxes with good quasi-symmetry.
- SPSA algorithm allows for cheaper gradient approximations for noisy objectives.
- Optimizer reduces nonlinear heat flux by a factor of 3, while maintaining good quasi-symmetry.

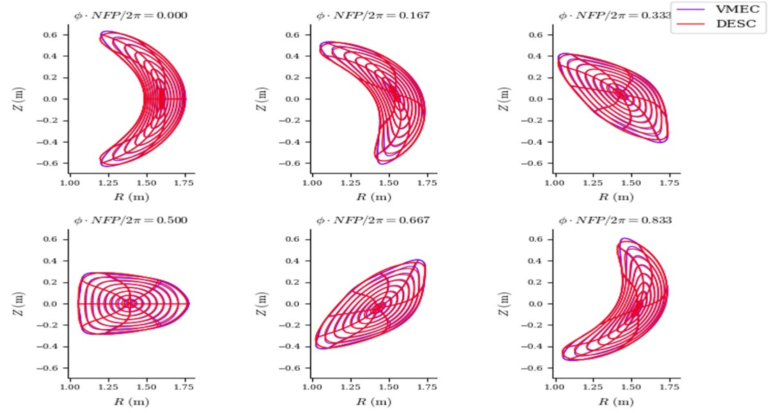


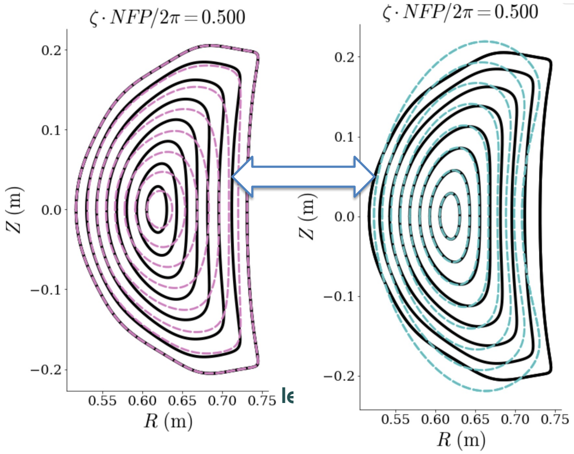


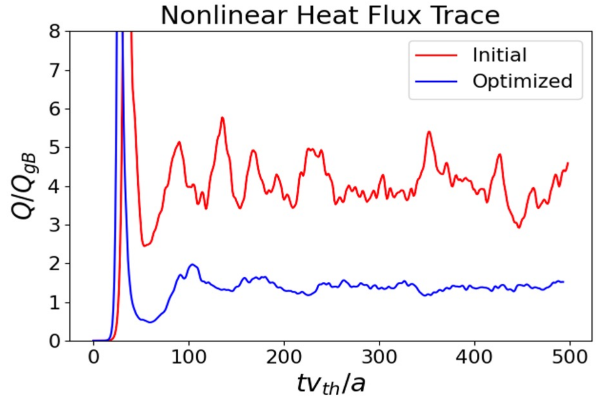
Conclusions: DESC multi-objective optimization applied to Turbulence Optimization, NEA, free surface stellarator & General Omnigenity











Postdoc Opportunity!

- **Novel Stellarator Designs**
- **Stellarator/Divertor Topology**
- **Plasma SOL**
- **Plasma, Applied Math, CS welcome**
- **Work on DESC**

- **Email: ekolemen@princeton.edu (or talk to me)**