



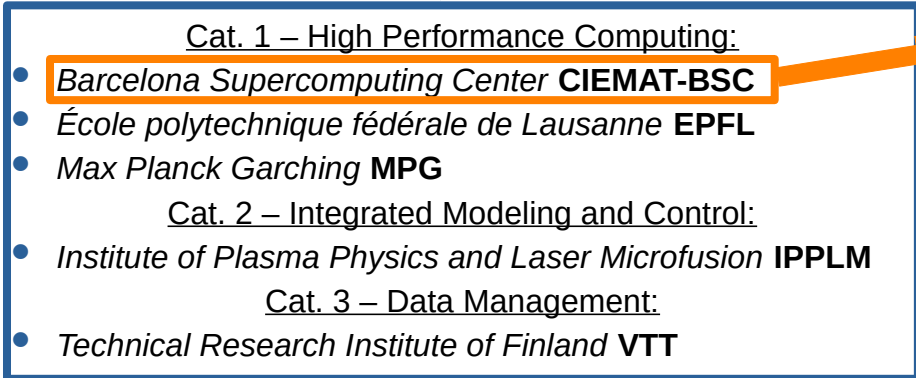
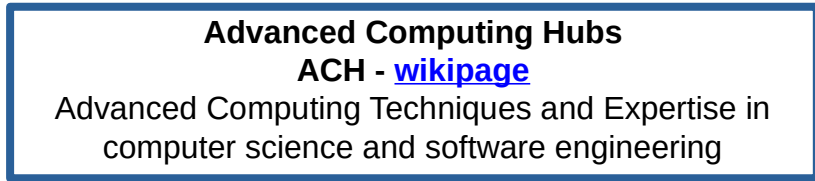
# Code optimizations in the BSC Advanced Computing Hub: Implementation of matrix compression for the coupling of JOREK to the 3D realistic conducting wall structures

F. Cipolletta, N. Schwarz, M. Hoelzl, S. Ventre, N. Isernia, G. Rubinacci, A. Soba

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1. Brief introduction to JOREK and the response matrices
2. Matrix Compression
  - a. Chosen method
  - b. How it is implemented
3. How the method performs within applications
  - a. Vertical Displacement Event (VDE)
  - b. Tearing Mode (TM) instability
  - c. Issues and Ideas
4. Next steps



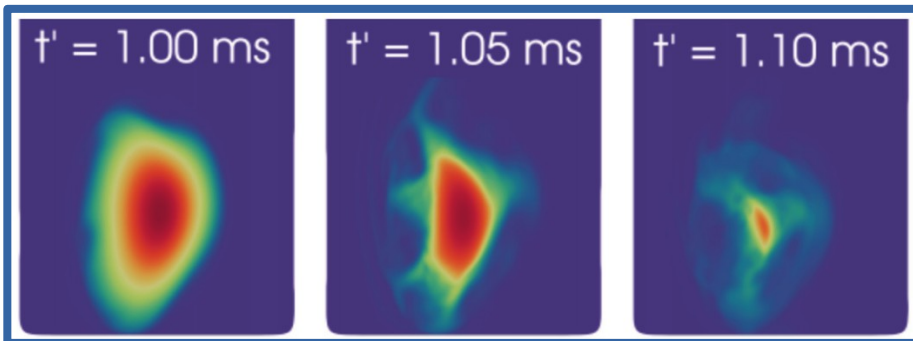
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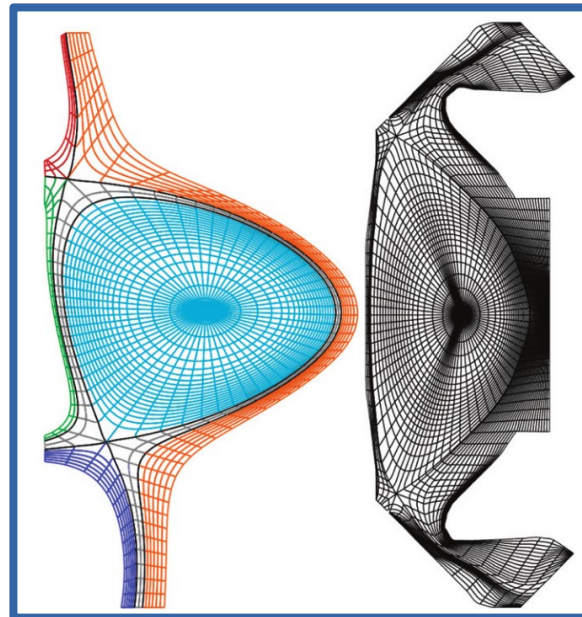
- Non-linear MHD + many extensions including kinetic & hybrid models
- Bezier finite element + toroidal Fourier expansion
- Fully implicit time-evolution
- Divertor tokamaks including X-point(s) – (1)
- Adopted for simulations of plasma instabilities – (2)
- Originally developed at CEA Cadarache:
  - [Czarny and Huysmans \(2008\)](#); [Huysmans and Czarny \(2007\)](#)
- In the present work Reduced MHD for simplicity
- Fortran 90/95
- MPI + OpenMP hybrid parallelization

More details in the presentation [M. Hoelzl \(2012\)](#)

(2) Snapshots of evolution of the pressure in the  $\phi = 0$  plane in a 3D Vertical Displacement Event (VDE)



(1) Typical grids used in JOREK

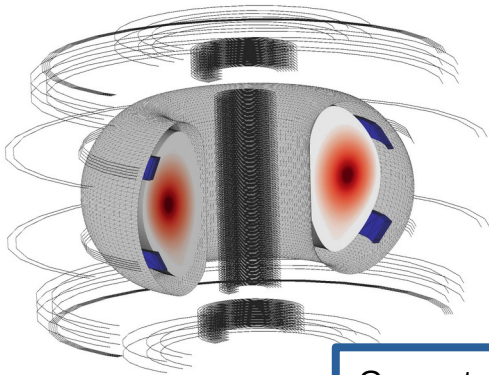


See [Hoelzl et al \(2021\)](#)

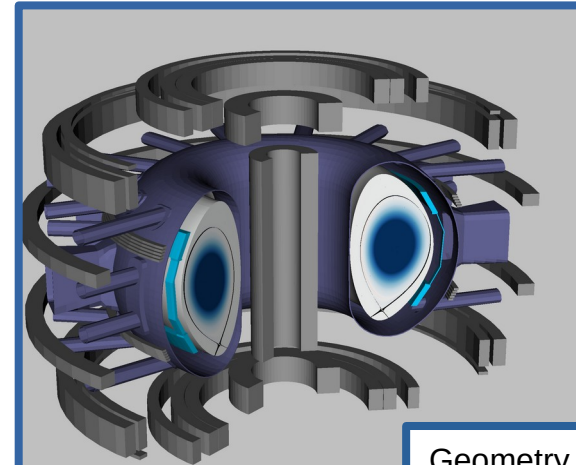


The free-boundary and resistive wall extension of JOEAK are obtained through coupling to external codes, which provides **response matrices**, in particular:

- **STARWALL** → 3D thin resistive walls response – see [Merkel, Strumberger \(2015\)](#)
- **CARIDDI** → 3D volumetric resistive walls – see [Isernia et al \(2022\)](#) + in preparation



Geometry used in **STARWALL**  
credits to **N. Schwarz**



Geometry used in **CARIDDI**  
credits to **N. Schwarz**



## Goal

Enable modeling capabilities of realistic and accurate 3D wall structures within MHD simulations of plasma instabilities inside a Tokamak

## Objective related to JOREK

**Reduce Memory** required by response matrices and **Improve Performance** → Apply factorization and compression techniques to matrices provided by STARWALL or CARIDDI

## Challenges

TASK-RELATED – connected to matrix dimensions (could become big for realistic geometries)

CODE-RELATED – complex collection of several Fortran files with ID computed externally

PROJECT-RELATED – many parallel developments → “code-orthogonality” is mandatory



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# Singular Value Decomposition (SVD)



Given a dense matrix, with one accurate SVD, we could write

$$A = (U \times \Sigma) \times V^T$$

A dense  
 $U, V$  orthogonal  
 $\Sigma$  diagonal

## Required Dimensions for the factorized representation:

$\text{rank}(\Sigma) = k$ , storing  $(U\Sigma)$  and  $V^T \Rightarrow (2nk)$  elements instead of  $n^2$

A rectangular  $(m,n) \Rightarrow (mk+nk)$  elements instead of  $(mn)$  (size scales linearly with k)

## Powerful Features in view of applications:

1. The SVD can be always performed
2. An SVD with singular values in descending order always exists
3. The SVD is an optimal approximation with respect to the residual computed via Frobenius norm

## Implementation:

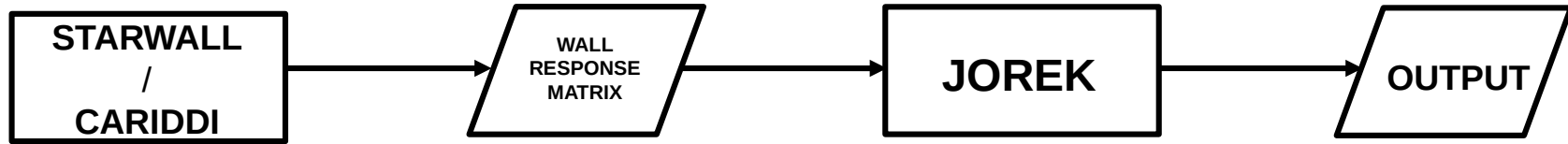
The Scalable Linear Algebra PACKage (ScaLAPACK) parallelized library offers the routine `pdgesvd` to compute the SVD of a given matrix (<https://netlib.org/scalapack/>)



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## Starting point for this work



## What we do





- At compilation, the user selects:
  - Which matrices to compress
  - What fraction of singular values of the SVD to retain
- Reads STARWALL/CARIDDI response file
- ScaLAPACK performs SVD and returns decreasing order Singular Values (SVs)
- **Compressing** = *Retaining the biggest SVs*
- An output file is printed with all the required information (factorization in  $(U\Sigma)$  and  $V^T$ )
- Possibility to re-aggregate the SVD decomposition (VALIDATION)
- Runs are typically fast (minutes), since they rely on the optimized ScaLAPACK library
- The compression needs to be run only one time per JOREK simulation (separate module)



## ADAPTATION of JOREK

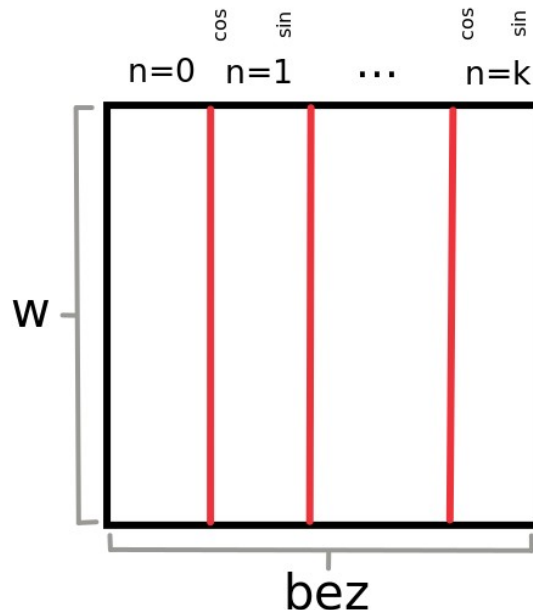
- Need to choose one or two matrices as a starting point
- Need to adapt JOREK to treat the compressed and factorized matrices
- Already implemented hybrid MPI/OpenMP parallelization for additional operation

The chosen matrices have dimensions  $(w, bez)$  or  $(bez, w)$

$W =$  wall dof  $\rightarrow$  kept constant for every order  
 $k$  of toroidal harmonics

$bez =$  Bezier dof times Fourier harmonics  $\rightarrow$  changes with different  $k$

**NOTE:** the rank of the matrix is always  $\leq \min(w, bez)$





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# Memory compression for VDE test case

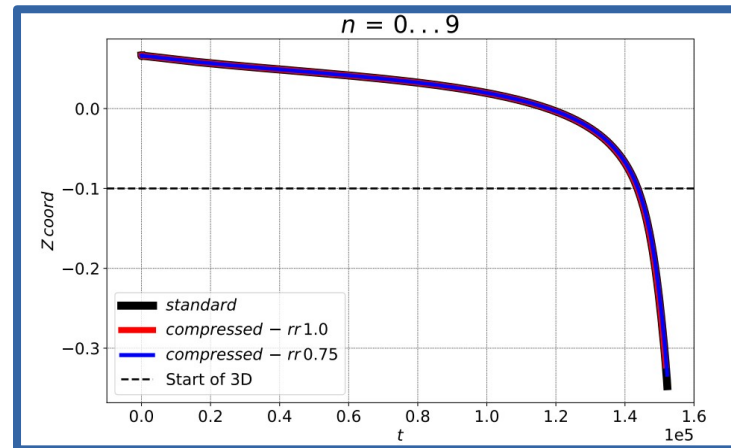
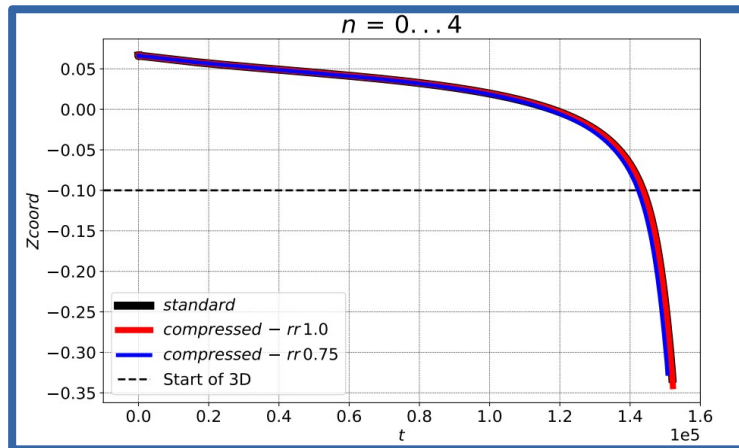
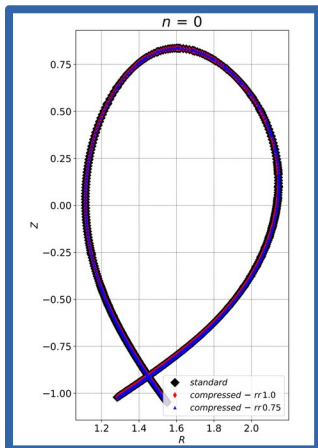


$r_r$  = rate of retained singular values  $\rightarrow$  smaller = more compression

Convergence + Accurate Results  
Inaccurate Results

- Selected rates of  $r_r$ :
  - 1.0 (factorized but not compressed)
  - 0.75 ( $\frac{3}{4}$  of the singular values are retained)
  - 0.5 ( $\frac{1}{2}$  of singular values are retained)
  - ...
- Produce Initial Data via CARIDDI
- Tests with standard and compressed matrices

$r_r$	Size n=0 [GiB]	Size n=9 [GiB]	Size n=19 [GiB]
-	0.0273	0.245	0.518
1.0	0.0275	0.265	0.605
0.75	0.0206	0.199	0.454
0.5	0.0137	0.132	0.302
0.4	0.0110	0.106	0.242
...			





- Performance issues ( $r_r=1.0$  required much more time with respect to standard)
  - ✓ *Factorized matrix distribution across MPI tasks + OpenMP parallelization of loops*
- $r_r = 1.0$  results matching the standard code
  - ✓ *Implementation is validated*
- The  $n=0$  part of the matrix is very sensitive to the compression
  - ✓ *increasing  $n$  slightly improves “compressibility”*

## **TAKEAWAY:**

Implementation **optimized** and **validated**, efficiency is still **improvable**



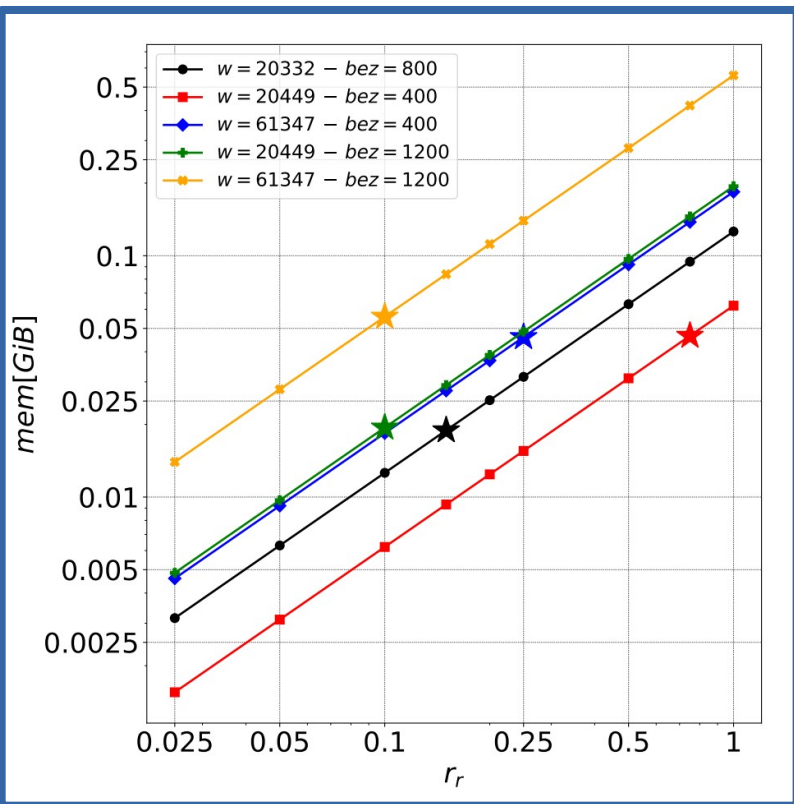


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# Memory compression for TM test cases



$r_r$  = rate of retained singular values  $\rightarrow$  smaller = more compression



Here, the matrices do not contain the n=0 part

**NOTE:** ★ indicates the smallest  $r_r$  providing accurate results, i.e. the more it is towards the left the more the compression is efficient

- First case if the **black** curve (shows good compression)
- Started a systematic study on resolution:
- **Red** curve taken as basis
  - Tripling  $w$  in the **blue** curve
  - Tripling  $bez$  in the **green** curve
  - Tripling both  $w$  and  $bez$  in the **orange** curve

**TAKEAWAY:**

Compression is more efficient on large matrices, in particular for larger resolutions on the plasma side



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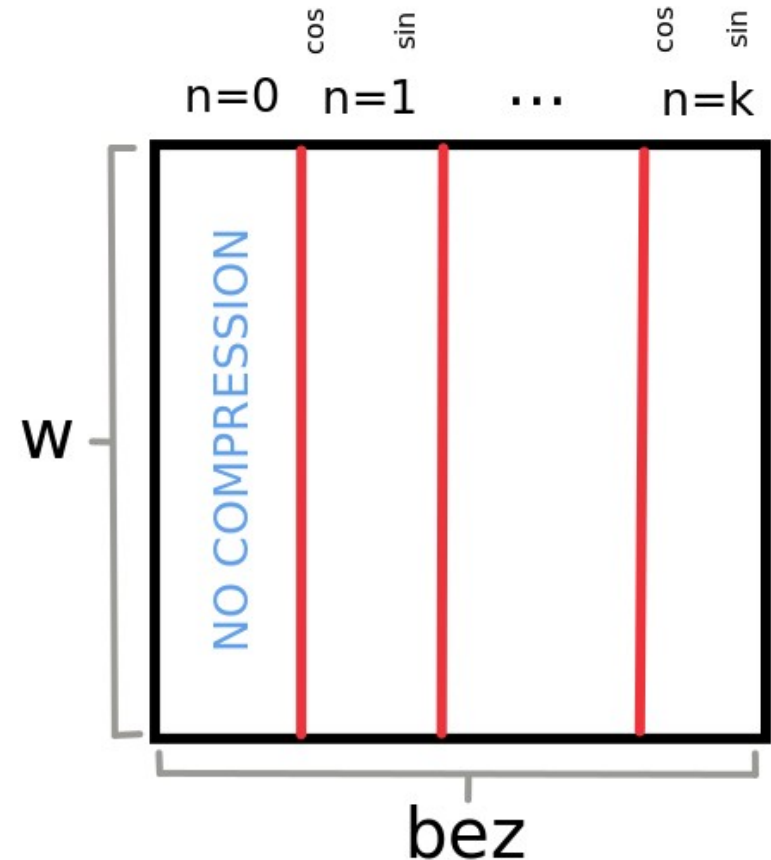
# Toroidal harmonics representation and compression



Issues shown for the previous tests of **VDE** are probably due to the fact that the  $n=0$  part is sensitive to compression

Given a response matrix  $A$ , it is composed of blocks as the sketch on the side, for each node element at the boundary of the plasma

The current development is testing the possibility to leave the  $n=0$  parts of the matrices uncompressed





Limitations of efficiency of compression in the **TM** tests with respect to  $w$  is given by the definition of SVD and the depicted geometry

From the dimensions point of view:  
$$A(m,n) = U(m,k) \times \Sigma(k,k) \times V^T(k,n)$$
With  
$$k \leq \min(m,n)$$

1.  $\min(m,n)$  “rules” the amount of compression
2. In JOREK cases studied  $bez < w$

Need to develop of way of selecting “worthy” regions of the wall to be compressed



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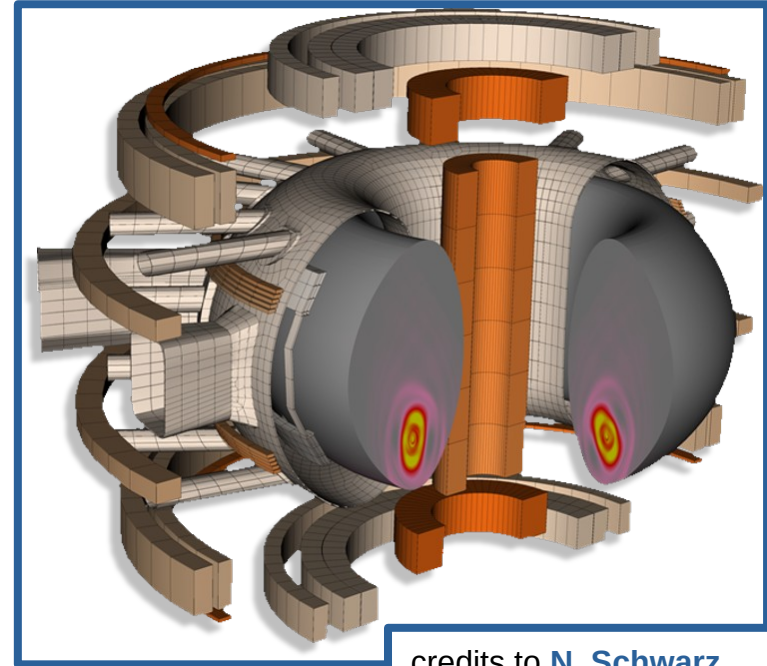


## Summary

- First implementation of a tool for the compression of response matrices have been provided, along with its validation and assessment

## Future

- Treat the  $n=0$  and  $n>0$  parts separately (possibly treat each mode separately)
- Assess effectivity of the new implementation via *VDE tests*
- Improve efficiency in compressing in *w* “direction” (e.g. *AMG*, *H-matrices* methods or similar)
- Implement and test compression for other matrices (including **full MHD** and **halo current** couplings)



credits to **N. Schwarz**