Numerical solutions of plasma flow in the presence of absolute vacuum

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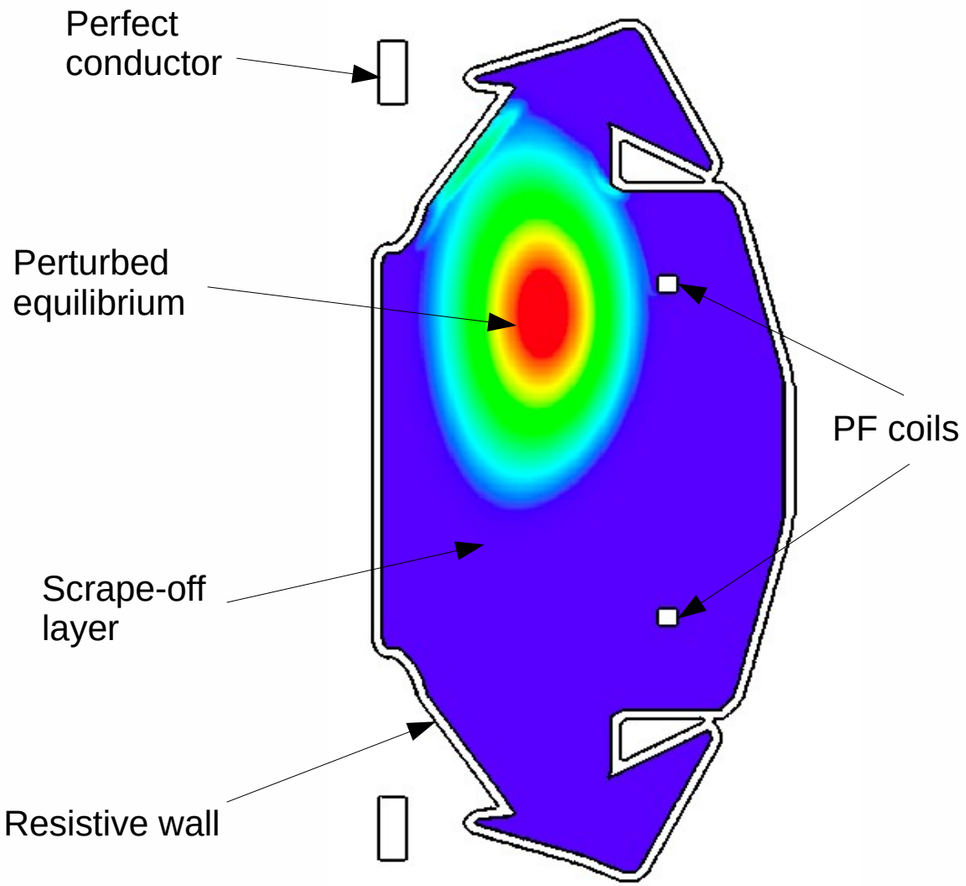
Outlook

- The objective of this work is to account for all regions of the reactor (plasma, scrape-off layer, wall) within the same simulation.
- Material boundary conditions are incorporated using state of the art sharp and/or diffuse interface methods.
- Hierarchical adaptive mesh refinement (AMReX framework) is used to resolve material boundaries and steep gradients.
- The end goal is the investigation of highly unstable events such as VDEs.
- In this presentation we focus on the representation of the scrape-off layer.



Framework

- The ideal MHD equations are solved using a conservative finite volume approach.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \left[\rho \mathbf{v} \otimes \mathbf{v} + \left(p + \frac{1}{2} B^2 \right) \mathbf{I} - \mathbf{B} \otimes \mathbf{B} \right] = 0,$$

$$\frac{\partial U}{\partial t} + \nabla \cdot \left[\left(U + p + \frac{1}{2} B^2 \right) \mathbf{v} - (\mathbf{v} \cdot \mathbf{B}) \mathbf{B} \right] = 0,$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{B} \otimes \mathbf{v} - \mathbf{v} \otimes \mathbf{B}) = 0.$$

- Exact and/or approximate Riemann solvers, such as HLLC, are used to compute fluxes.
- Fluid-vacuum interfaces are fitted using a flux modified approach.

The Riemann problem

- A Riemann problem is defined as an initial value problem made up of a single discontinuity.
- For the MHD equations the solution to a RP consists of seven waves, which can either be discontinuities or rarefaction fans.
- Any hyperbolic system can be solved by computing the solution to a series of local RP at each cell interface.

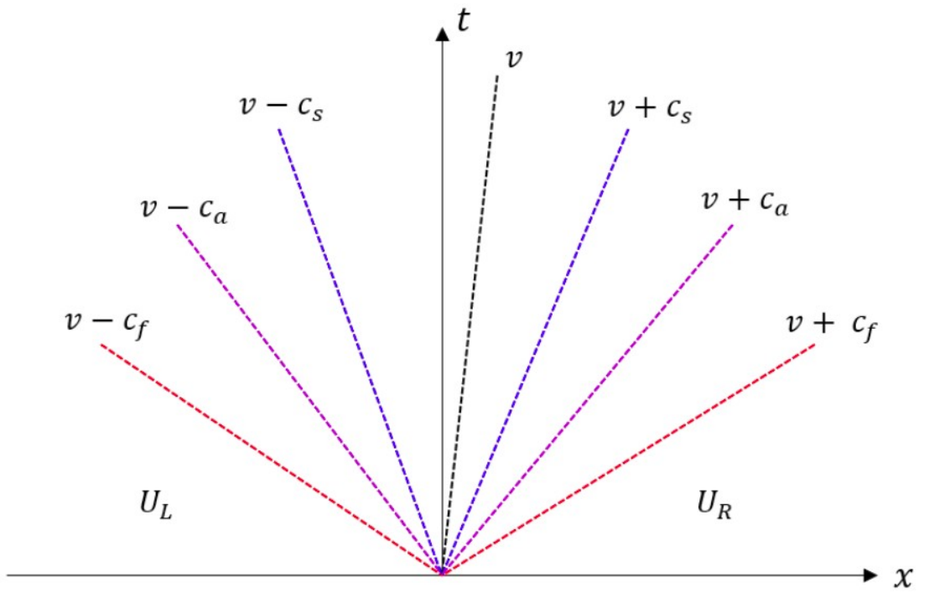


Figure: The characteristic wave pattern for a one-dimensional ideal MHD system, consisting of seven waves.

Exact Riemann solver for MHD

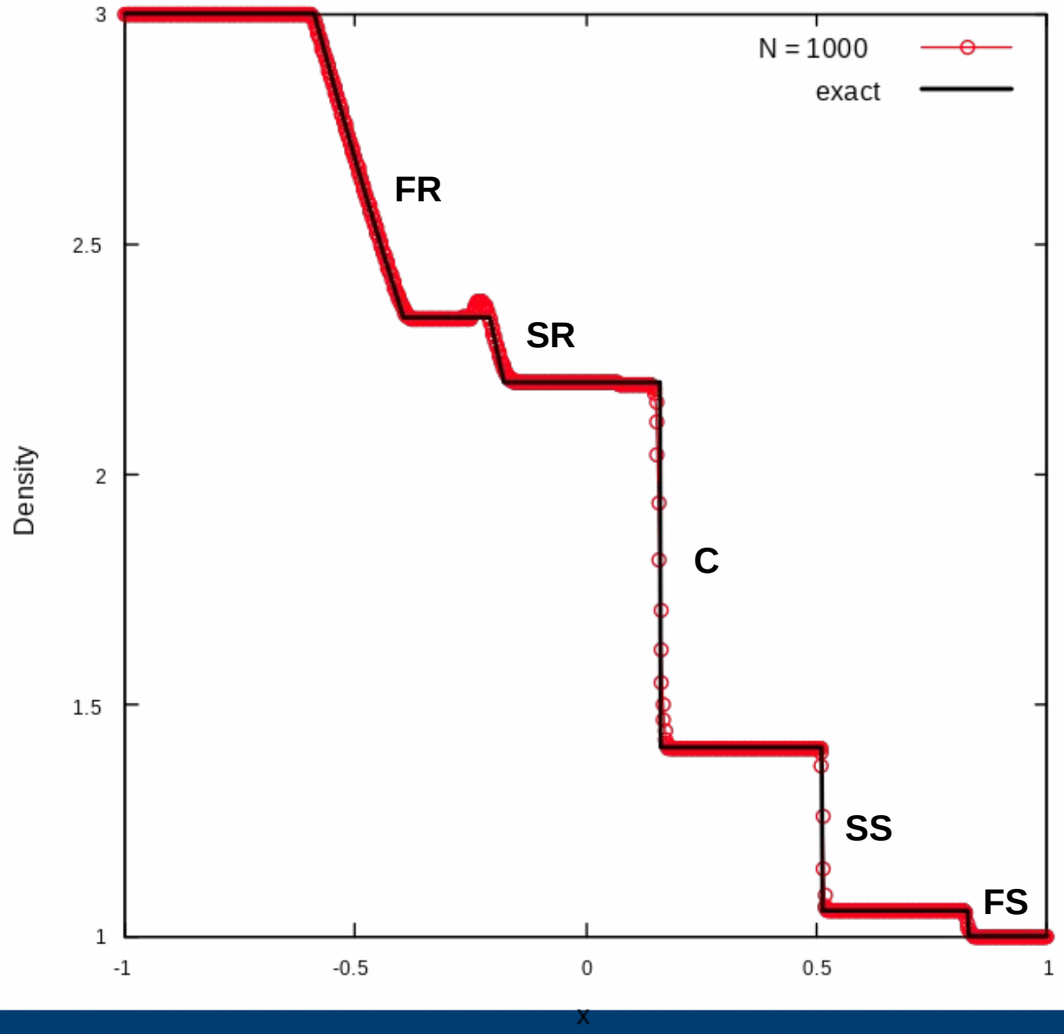
- The solution to a one-dimensional ideal MHD Riemann problem can admit up to seven waves.
- Each wave is mapped onto a path variable, resulting in a nonlinear system of equations.
- The equations can be reduced into a 5×5 system of the form

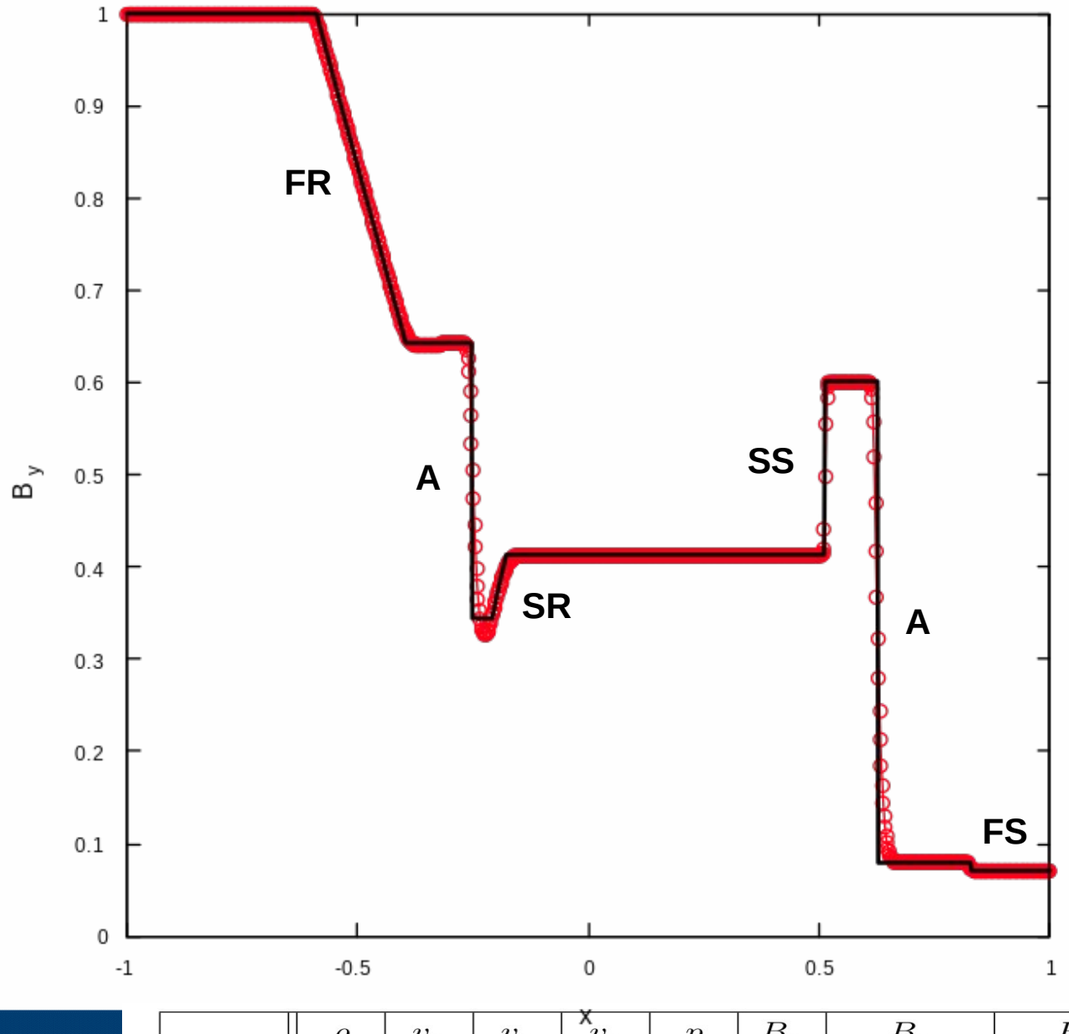
$$\mathrm{RP}_U(\boldsymbol{\Psi}) = \mathrm{RP}^*_{\mathbf{U}_0}(\psi^-_{s,f}, \alpha_{R0}) - \mathrm{RP}^*_{\mathbf{U}_1}(\psi^+_{s,f}, \alpha_T - \alpha_{R0}) = \mathbf{0} \in \Re^5,$$

where RP_U represents the Riemann problem for initial conditions $\boldsymbol{U} = (\boldsymbol{U_0}, \boldsymbol{U_1})$ and $\boldsymbol{\Psi}$ is the vector of path variables $\left(\psi^{\pm}_{s,f}, \alpha_{R0}\right)$.

- This is solved using a multidimensional Newton-Raphson procedure in conjuction with Broyden's method.

Exact and numerical solutions for MHD RP





	ρ	v_x	v_y	v_z	p	B_x	B_y	B_z
Left	3.0	0.0	0.0	0.0	3.0	1.5	1.0	0.0
Right	1.0	0.0	0.0	0.0	1.0	1.5	$\cos(1.5)$	$\sin(1.5)$

Fluid – vacuum Riemann problem

- The fluid-vacuum RP is a special case RP, which poses important numerical challenges, due to the presence of a strong rarefaction and the disparity between the very small densities around the fluid-vacuum interface and the very large velocity of the vacuum front.

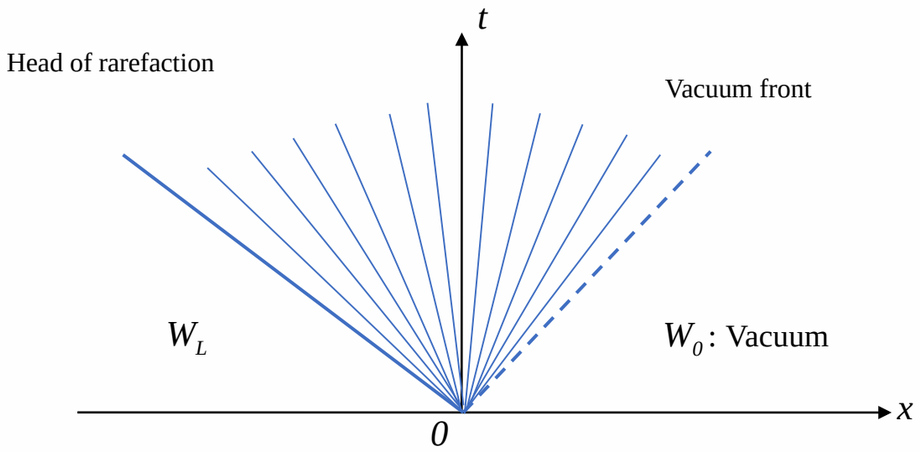


Figure: Riemann problem solution for a right vacuum state.

- Therefore, a major development of this work is the modification of the system of equations to enable realistic and physically consistent simulations of vacuum in the presence of magnetic fields.

- The first step is to consider the exact solution for the vacuum RP in the presence of magnetic fields.

Plasma-vacuum RP for varying magnetic field strengths

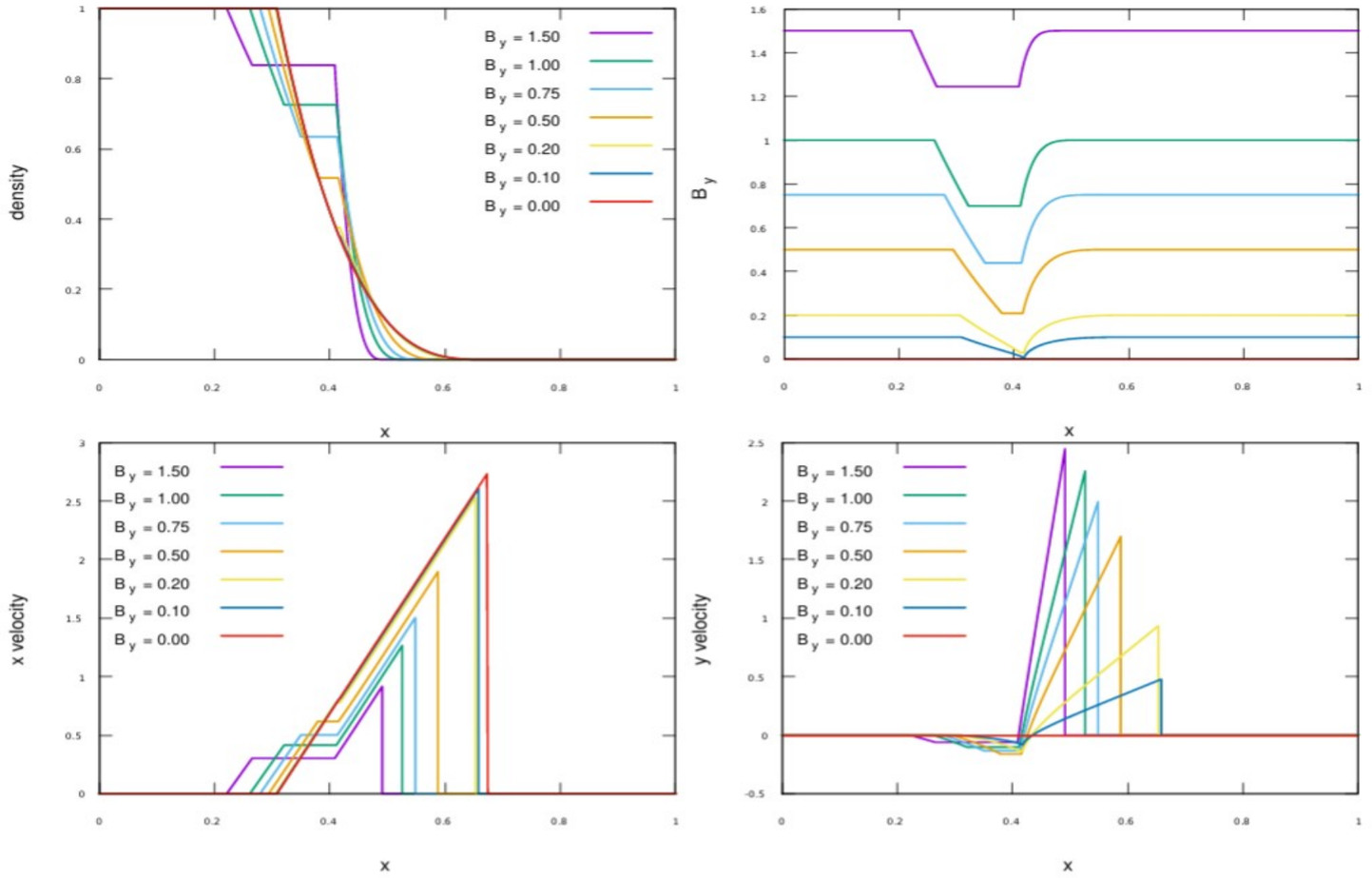


Figure: Exact solutions for plasma-vacuum Riemann problem, consisting of a fast rarefaction, a slow rarefaction and a contact wave at the plasma-vacuum interface.

	ρ	v_x	v_y	v_z	p	B_x	B_y	B_z
Left	1.0	0.0	0.0	0.0	0.5	0.375	$(0.0, 0.1, 0.2, 0.5, 0.75, 1.0, 1.5)$	0.0
Right	0.0	0.0	0.0	0.0	0.0	0.375	$(0.0, 0.1, 0.2, 0.5, 0.75, 1.0, 1.5)$	0.0

Fitting the plasma-vacuum interface

- We implement a novel diffuse interface method by Wallis *et al.* [2021] based on flux modifiers and interface seeding routines.
- This requires the introduction of a new evolution equation for the void volume fraction variable.

$$\frac{\partial \nu}{\partial t} + \nabla \cdot \nu \mathbf{v} = \nu \nabla \cdot \mathbf{v}$$

- The entire system of equations is solved using a conservative monotone finite volume method which can accurately capture discontinuous solutions (HLLC solver with MUSCL reconstruction on primitive variables, RK3 in time.)
- MUSCL-BVD-THINC reconstruction is also implemented for minimising numerical diffusion around the interface.

Numerical method

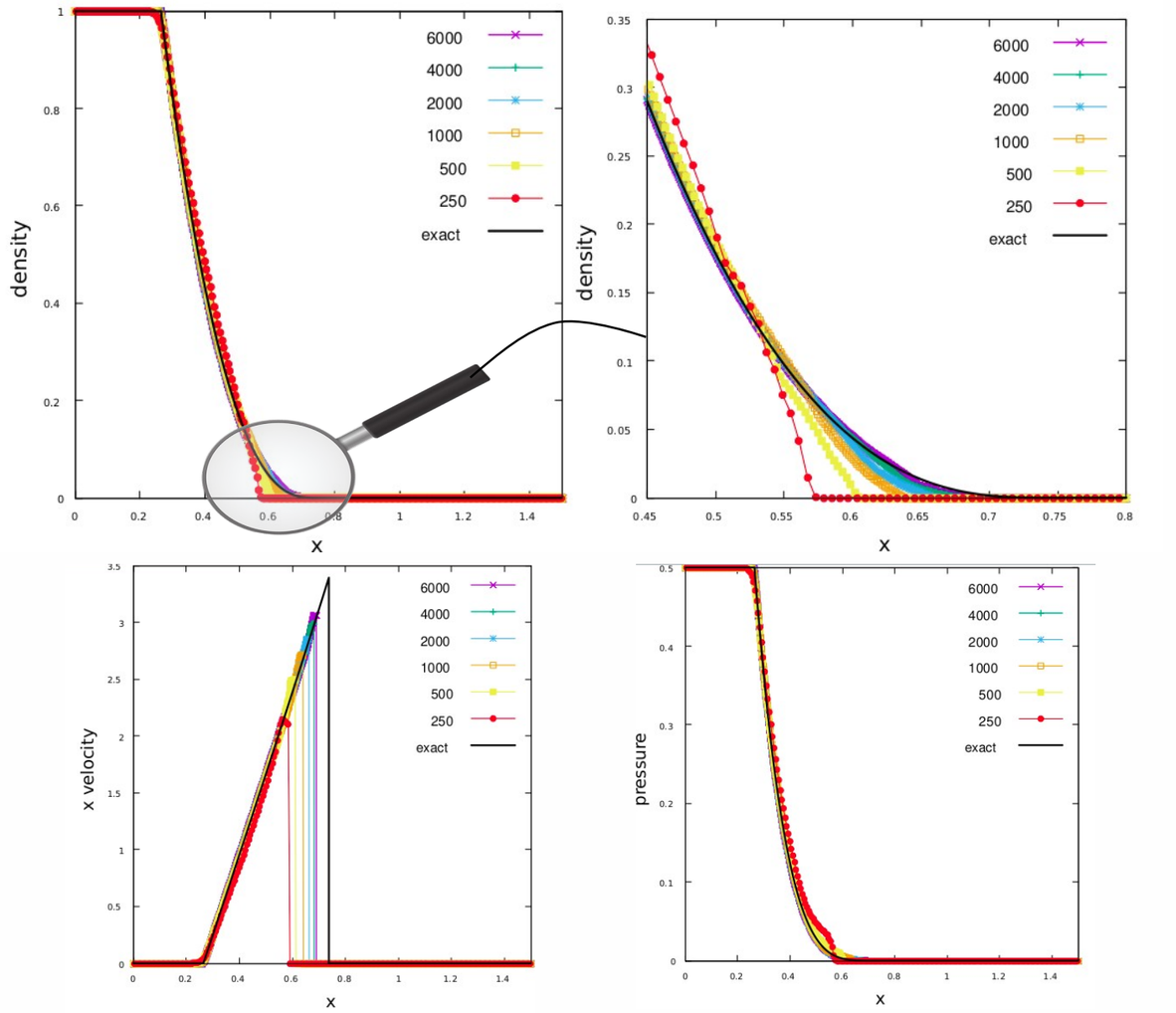
- Void seeding routine:
  1. In cells where $\nu \geq 0.9$, the void normal is calculated using Young's/ELVIRA method.
  2. A probe is sent along the normal and a new state $\boldsymbol{U}_{interp}$ is interpolated.
  3. If $\nu_{interp} < v$, a new state is defined, so that $\boldsymbol{U}_{new} = \boldsymbol{U}_{interp}$.
  4. The new state is linearly combined with the original state, $\boldsymbol{U} \leftarrow (1 - v)\boldsymbol{U} + \nu \boldsymbol{U}_{new}$, but keeping the magnetic field variables and the void volume fraction variable constant.
- Flux modifier (improvement based on Munz's work [1994]):
  1. Cells are separated into three different types, fluid-fluid interface, fluid-void interface and void-void interface cells.
  2. Normal HLLC flux is computed for fluid-fluid interface cells and a modified flux is computed for fluid-void interface cells given by:

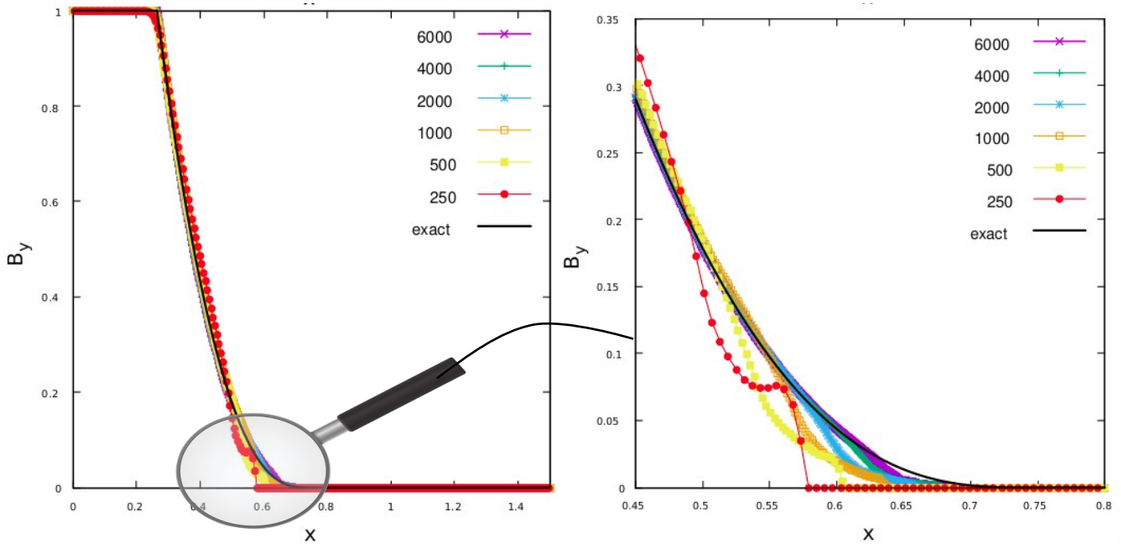
$$\boldsymbol{F}_{fi} = \begin{cases} \boldsymbol{F}_{fluid}^{HLLC} & \text{if } S_l \geq 0, \\ \dfrac{(S_r \boldsymbol{F}_{fluid}^{HLLC} - S_r S_l \boldsymbol{U}_{fluid})}{S_r - S_l} & \text{if } S_l < 0, \end{cases}$$

where $S_l = v_n^{fluid} - c_f^{fluid}$ and $S_r = v_n^{fluid} + \frac{2c_f^{gas}}{\gamma - 1}$.

fi: fluid-vacuum interface

Free expansion test case





	ρ	v_x	v_y	v_z	p	B_x	B_y	B_z
Test 2 Left	1.0	0.0	0.0	0.0	0.5	0.0	1.0	0.0
Test 2 Right	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Free expansion vs low-density approximation

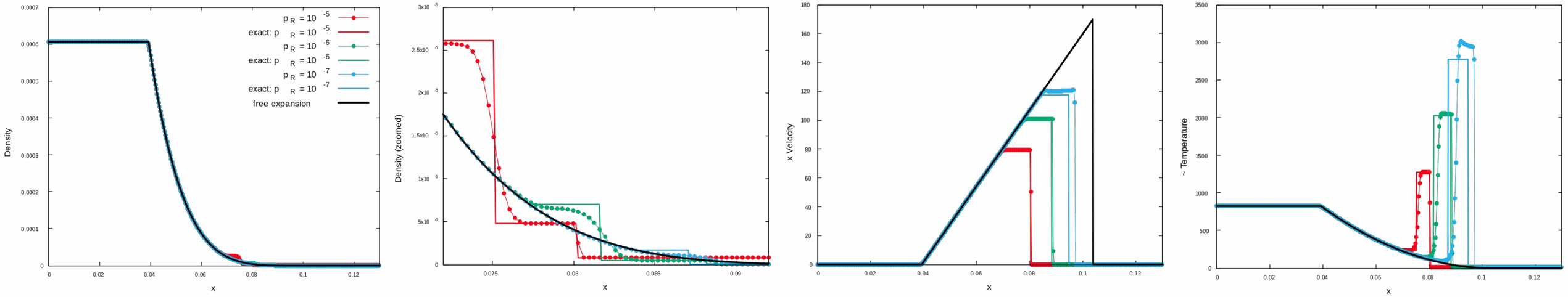
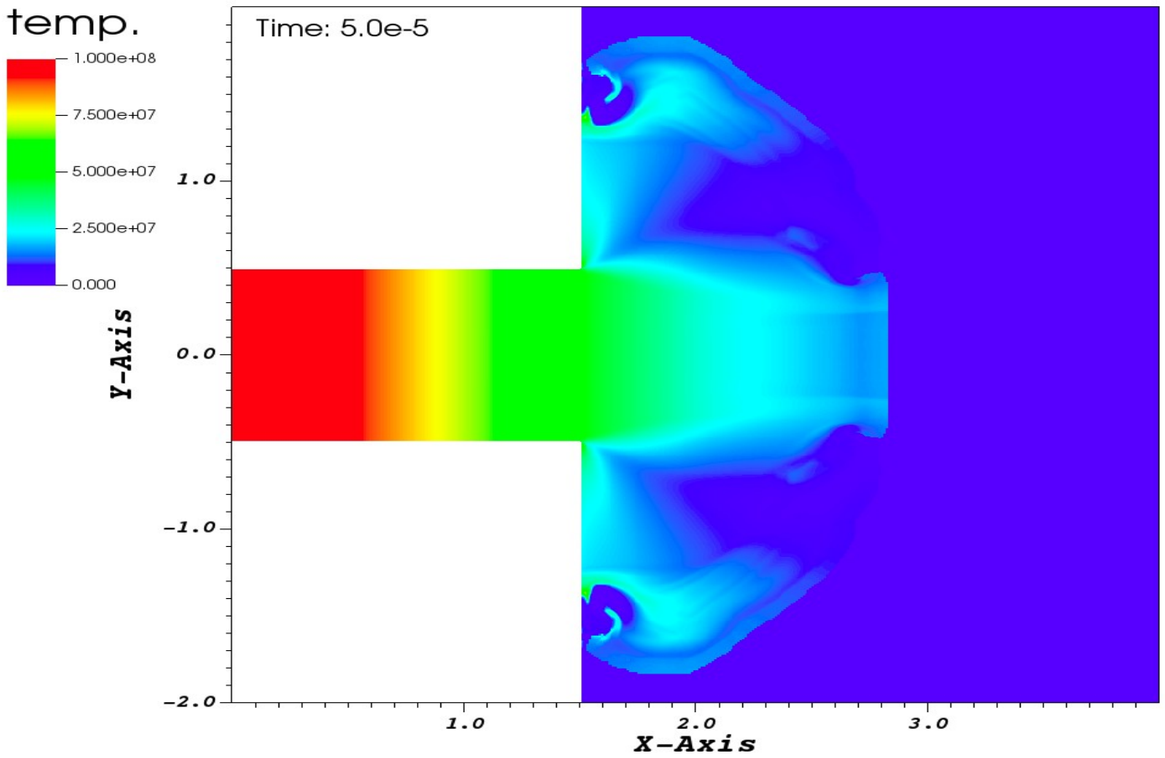


Figure: Exact solution for free expansion plotted against various low-density approximations.
An **additional shock** is observed in the presence of a finite density in the background state.

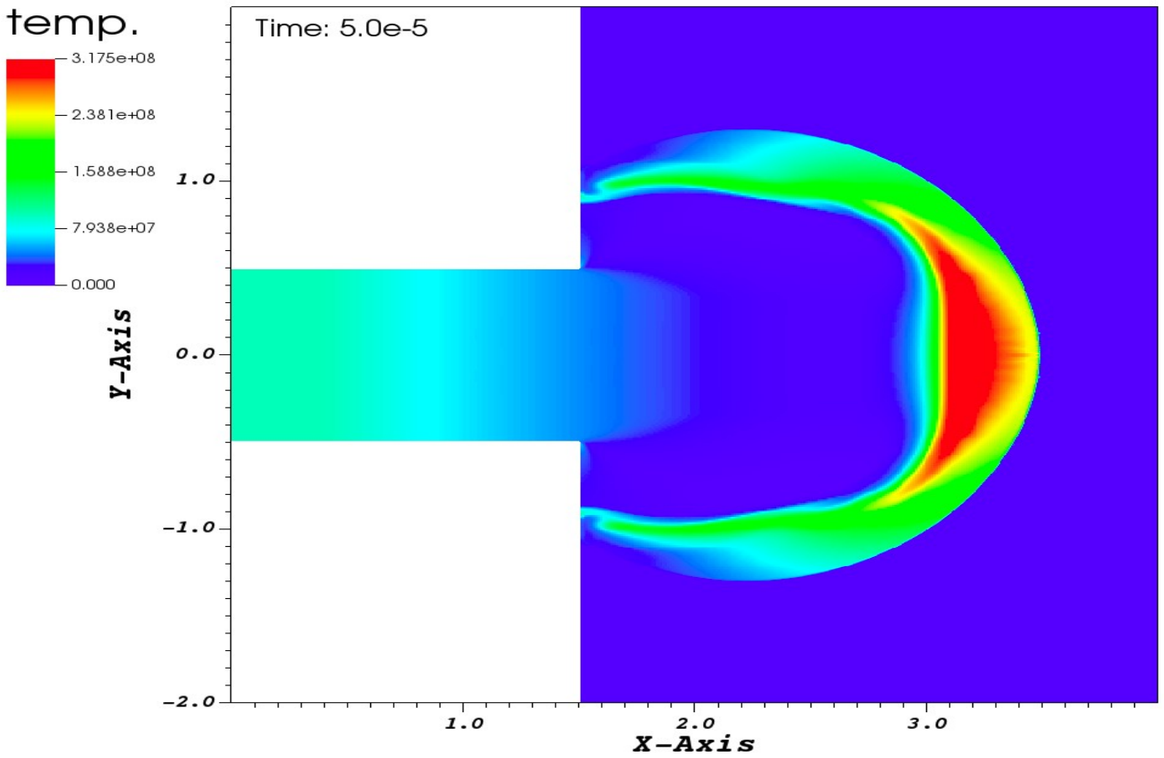
	ρ	p
Driver state	6.06×10^{-4}	0.5
Background state 1	8.08×10^{-7}	1×10^{-5}
Background state 2	8.08×10^{-8}	1×10^{-6}
Background state 3	8.08×10^{-9}	1×10^{-7}

Plasma accelerator test case

TRUE VACUUM SIMULATION

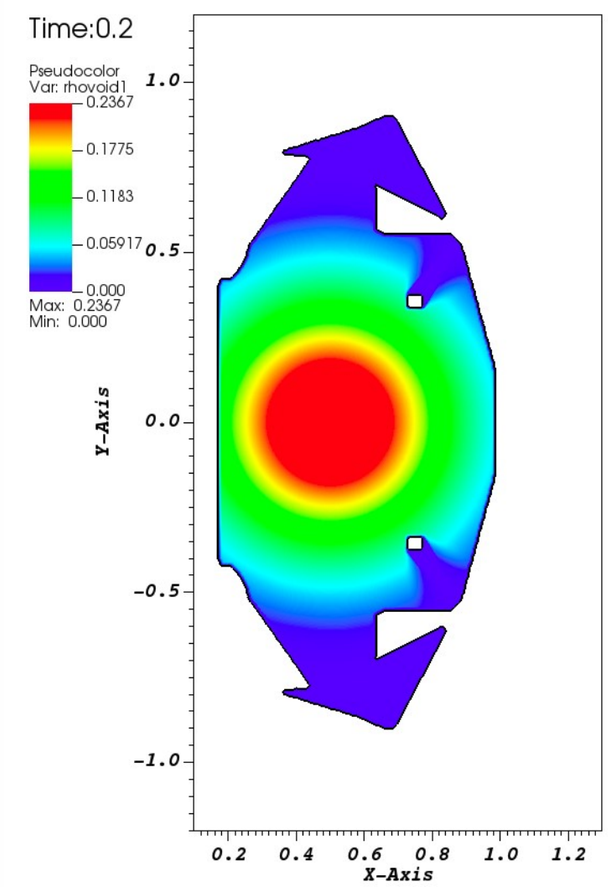


LOW DENSITY SIMULATION

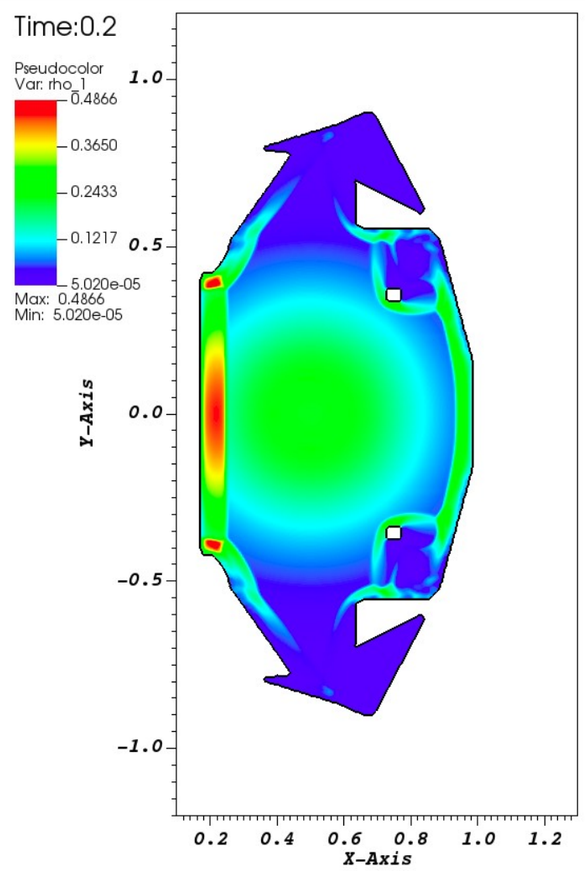


Plasma expansion in low density and vacuum background state within ST40 geometry

TRUE VACUUM SIMULATION



LOW DENSITY SIMULATION



Concluding remarks

- Considering alternative methodologies for the accurate and physical representation of the SOL.
- Demonstrated that low density approximations produce solutions that are physically inconsistent with the theory of free expansion.
- True vacuum solutions and low density approximation solutions result in distinct wave patterns which interact differently in the presence of a true material boundary.

- Addition of more physics (full model) :
  - Multi-species system
  - Kinetic closure relations
  - Elastoplastic reactor wall model (for computing heat loads on divertor)
- Enforcement of current free condition in vacuum region.
- Consideration and validation of unstable events within fusion reactors (e.g. ELMs and VDEs).